

HORIZONTAL TRANSITIONS IN SUBCRITICAL FLOW.

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الانتقالات الأفقية في السريان تحت الحرج

خلاصة:

تستخدم الانتقالات عندما يتغير حجم أو شكل القطاع المائي وهذه التغيرات غالباً ما تكون في المجاري المائية الطبيعية أو الصناعية عند المنحدرات المائية وذلك لغرض اقتصادي أو لغرض عملي . والانتقالات إما أن تكون رأسية كالانقباض في منحور قطاع المجري المائي فحاشياً أو تدريجياً أو أفقية كحالة اتساع أو تضيق عرض القطاع المائي. وهذا التضيق أو الاتساع إما أن يكون فحاشياً أو تدريجياً . لذلك فإن هذه الانتقالات تعتمد أساساً على الخصائص الهندسية للقطاع المائي ، التصرف وكذلك حالة السريان في المجري المائي سواء كان سريان تحت حرج أو فوق حرج ونسبة لمعومة هذه المشاكل الناجمة من هذه التغيرات فقد أجريت دراسة نظرية لاستنتاج أهم المعادلات الرياضية التي تحكم خصائص السريان خلال هذه الانتقالات الأفقية واتحاد حل نظري لجميع الحالات التي يمكن حدوثها كما شمل هذا البحث دراسة معمّلة لتحقيق تلك النتائج النظرية .

ABSTRACT

The present research deals with open channel transitions (sudden contraction and limited constriction) in subcritical flow. The flow pattern is usually complicated to be analytically solved, so practical solution is possible through systematic experimental investigations.

Interrelationships between parameters in dimensionless forms, which govern the flow characteristics are presented. Effects of contraction ratio, flared entrance and friction on the flow pattern, especially the oblique waves and choking phenomenon are given.

INTRODUCTION

Transitions are provided whenever the size or shape of cross section of an open channel changes. Such changes are often required in natural or artificial channels at irrigation structures for economic as well as practical reasons.

The problem of horizontal transition may be solved by writing the specific energy equation before and after the transition sections.

The existing method for solving the problem of open channel transitions involves a trial and error solution of higher degree equation. There are two possible answers for the depth in transition. The correct answer can be determined only by knowing in advance the state of flow which will be either subcritical or supercritical condition.

The phenomenon is usually complicated that the resulting flow pattern is not readily subject to any analytical solution, so a practical solution is possible through systematic experimental investigation. The present research deals with horizontal transitions in the case of subcritical flow. Such transitions can be solved more confidently with the use of the specific energy equation than the momentum concept.(7).

The main objective of this research is to develop a graphical solution for horizontal contraction and limited constriction for practical purposes i.e. to establish interrelationships between hydraulic parameters which affect the phenomenon in the form of dimensionless terms.

THEORETICAL ANALYSIS

From the concept of specific energy equation:

$$E = Y + \alpha Q^2/2y A^2 \dots\dots(1)$$

in which;

- E = specific energy;
- Y = depth of flow;
- α = energy coefficient;
- Q = Water discharge;
- A = cross sectional area of flow; and
- g = acceleration due to gravity.

For rectangular section $A = b.Y$

Where: b = breadth of section.

Considering $\alpha = 1.0$ equation (1) becomes

$$Q^2/2g b^2 E^3 = (Y/E)^2 (1 - Y/E)$$

or $Q_* = Y_*^2 (1 - Y_*) \dots\dots(2)$

in which;

- $Q_* = \frac{\text{dimensionless discharge}}{\sqrt{Q^2/2g b^2 E^3}}$ and
- $Y_* = \text{dimensionless depth} = Y/E$

Equation (1) can be written in the following form:

$$E = Y(1 + F^2/2)$$

or $Y_* = 1/(1 + F^2/2) \dots\dots(3)$

From equations (2) and (3)

$$Q_* = (f/\sqrt{2})(1 + f^2/2)^{3/2} \dots\dots(4)$$

The specific energy equation for section upstream and downstream the transition can be written as:

$$E_1 = E_2 + \Delta E$$

$$\text{i.e. } Y_1 + \frac{Q^2}{2g b_1^2 Y_1^2} = Y_2 + \frac{Q^2}{2g b_2^2 Y_2^2} + \Delta E \quad \dots\dots\dots(5)$$

Equation (3) can be arranged to give

$$F_1^2 = \frac{2((Y_2/Y_1 - 1) + \Delta E/Y_1)}{(1 - 1/(\frac{b_2 Y_2}{b_1 Y_1})^2)} \quad \dots\dots\dots(6)$$

$$\text{or } F_1^2 = \frac{2((Y_g - 1) + h_2/Y_1)}{(1 - \frac{1}{Y_g^2} (1 - \frac{\Delta b}{b_1})^2)}$$

Where: $Y_g = Y_2/Y_1$

Again equation (5) can be written as :

$$E = Y_1(1 + F_1^2/2) - Y_2(1 + F_2^2/2)$$

or

$$\frac{1 + F_2^2}{1 + F_1^2/2} = \frac{1}{Y_2/Y_1} (1 - \frac{\Delta E/Y_1}{1 + F_1^2/2}) \quad \dots\dots\dots(7)$$

From the continuity equation:

$$Q = \text{constant} = A_1 V_1 = A_2 V_2 = b_1 Y_1 V_1 = b_2 Y_2 V_2$$

$$\text{or: } \frac{b_1^2 Y_1^3 V_1^2}{g Y_1} = \frac{b_2^2 Y_2^3 V_2^2}{g Y_2} \quad \dots\dots\dots(8)$$

$$F_1^2 = (\frac{b_2}{b_1})^2 (\frac{Y_2}{Y_1})^3 F_2^2 \quad \dots\dots\dots(9)$$

From equation (4)

$$(Q_{*2}/Q_{*1}) = (F_2/F_1)^2 (\frac{1 + F_1^2/2}{1 + F_2^2/2})^3 \quad \dots\dots\dots(10)$$

The relationships between Q_{*2} and F_1 with the parameter b_2/b_1 are derived from equations (9) and (10) the following equation:

$$Q_{*2} = F_1^2/2 (b_2/b_1)(1 + F_1^2/2 - \Delta E/Y_1)^3 \dots\dots\dots(11)$$

Again equation (5) can be written as follows:

$$\Delta E/Y_1 = (1 + F_1^2/2) - (Y_2/Y_1 + F_1^2/2 (b_2/b_1)^2 (Y_2/Y_1)^2)$$

or $\Delta E/E_1 = Y_1/E_1(1 + F_1^2/2) - (Y_2/Y_1 + F_1^2/2 (b_2/b_1)^2 (Y_2/Y_1)^2)$

$$= \frac{1}{(1 + F_1^2/2)} (1 + F_1^2/2) - (Y_2/Y_1 + F_1^2/2 (b_2/b_1)^2 (Y_2/Y_1)^2) \dots\dots\dots(12)$$

The above equation represents the relationship between the efficiency ($E_2/E_1 = 1 - \Delta E/E_1$) and F_1 with (b_2/b_1) and (Y_2/Y_1) as parameters.

EXPERIMENTAL WORK

Experiments were conducted in flow visualization tank. The general arrangement of this equipment is shown in Fig.(1). The channel has two steel parallel walls and cross section of 61 cm breadth and 20 cm depth, along its two metres length. For the purpose of the present study, in order to have more accurate results, an extended part of other two metres having the same cross section was fabricated.

The channel is firmly supported on two steel tanks inlet tank (7) and outlet tank (1) the two steel water tanks are predisposed for closed circuit. The inlet tank was provided with grading gravel rest on horizontal screen to minimise the energy of the coming flow. A suitable pump (4) was provided to the apparatus to supply water in a closed circuit, suction pipe (3) and delivery pipe (5) from the outlet tank to the inlet tank. An inferential mechanical meter (2) which measure the flow rate into the channel is placed on the suction pipe. Because of high head loss this type of meter is not often used for measuring flow rate above 0.3 m³/sec. Valve (6), control the flowrate, is placed on the delivery pipe. The outlet water from the channel falls freely over steel spillway (11) back to the outlet tank for recirculation. The water depth is controlled by screw wheel (10) which is used to adjust the spillway inclination. The water depth was measured by a hook gauge mounted on carriage (9) which could slide across the breadth of the channel. Also it could slide along two rails above the horizontal surface of the channel.

The velocity through the channel was measured by using the current meter. Some readings of the current meter were checked by using the pitot tube.

Apair of steel side wells were erected in both sides of the channel providing an equal distance to the centre line giving a transition part for the purpose of study. They were 2.0 m in

length and 0.2 m height (sudden contraction), plate (1). Another pair of steel walls were used with 0.5 m long and 0.2 m height (limited constriction), plate (2), four shapes of transitions are shown in Fig. (2).

The widths used for transition section having contraction ratio $\Delta b/b = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6 .

The transition section, under test was fixed at the channel by using water tight material. The water depth at the upstream of transition was fixed to the required depth at a steady flow condition.

RESULTS AND ANALYSES

The analysis of transition is based on the assumptions, the flow is considered one dimensional flow, both the energy coefficient and momentum coefficient are unity, the pressure is hydrostatically distributed and the channel is considered in horizontal position.

The following figures, from Fig.(1) to Fig.(28) show the relationships between these parameters:

$$F_1 = V_1 / \sqrt{gY_1}, \quad F_2 = V_2 / \sqrt{gY_2}, \quad Y_{*2} = Y_2/E_1,$$

$$Q_{*2} = \sqrt{Q^2/2g b_2^2 E_2^3}, \quad Y_s = Y_2/Y_1, \quad B_{*2} = b_2/E_2$$

$Y_{*1} = Y_1/E_1, \quad Q_{*1} = \sqrt{Q^2/2g b_1^2 E_1^3}, \quad B_{*1} = b_1/E_1$ and E_2/E_1 for contraction ratio $\Delta b/b = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6 .

The relationships between the following parameters are the same for both the sudden contraction and limited constriction cases:

$$F_1 \rightarrow F_2; \quad Y_{*1} \rightarrow F_2; \quad Y_{*1} \rightarrow B_{*2}; \quad Q_{*1} \rightarrow F_2; \quad Q_{*1} \rightarrow B_{*2}$$

and $B_{*1} \rightarrow B_{*2}$.

The relation between upstream Froude number F_1 and the submergence Y_s , dimensionless upstream depth Y_{*1} and Y_s , and the upstream dimensionless breadth B_{*1} and Y_1 have shown bigger values in the case of limited constriction. A limited constriction exhibited bigger value of the downstream dimensionless discharge Q_{*2} than the corresponding one in a sudden contraction case for the same contraction ratio and F_1 , the limited constriction has bigger downstream Froude number F_2 , Q_{*2} , Y_{*2} and Y_s than the corresponding sudden contraction for the same value of B_{*1} and $\Delta b/b$. Slightly bigger values of downstream dimensionless breadth B_{*2} are given by limited constriction than sudden contraction for Froude number less than 0.25 and for contraction ratio $\Delta b/b = 0.4, 0.5$ and 0.6 .

Values of Q_{*2} are bigger in the case of limited constriction than the case of sudden contraction for the same value of Y_{*1} and

contraction ratio. On the contrary the value of Y_{*2} is bigger in the sudden contraction for $\Delta b/b = 0.4, 0.5$ and 0.6 .

A limited constriction has bigger value of Q_{*2} and smaller value of Y_{*2} for the same value of Q_{*1} under the same contraction ratio $\Delta b/b = 0.4, 0.5$ or 0.6 .

The value of efficiency E_2/E_1 increases with the decreasing value of contraction ratio under the same value of both the upstream Froude number F_1 , depth Y_{*1} and discharge Q_{*1} . Also with the same value of contraction ratio, the value of efficiency E_2/E_1 increases with the decreasing value of both the upstream Froude number and Q_{*1} and with the increasing value of Y_{*1} .

The sudden contraction case exhibited higher values of efficiency E_2/E_1 than the limited constriction mainly due to less energy losses.

Velocity Distribution

Change in shape and values of velocity contours (isovels) was observed by changing the contraction ratio Figs. (29 a and 29 b) or the channel bed from plain to gravel bed Figs. (30 a and 30 b).

Calculation of energy coefficient α varies with the contraction ratio, bed roughness and position of section.

The momentum coefficient β is slightly affected by both contraction ratio and bed roughness.

Effect of Flared Entrance

An improvement of flow through constriction has emerged due to increasing the degree of flaring at entrance from 1:1 sudden, 1:2, 1:3 and 1:4. It is noticed from Fig. (31 a) that increasing the degree of flaring decreases the value of $\Delta E_1/E_1$ as the flow in the flared entrance will deflect gradually to pass downstream than the case of sudden entrance plate (3). Fig. (31 b) gives the relationship between degree of flaring and the downstream Froude number for different values of water depths.

Choking Phenomenon

Various profiles of water surface occur in horizontal transition problems. These different profiles are dependent on the state of approach of flow and horizontal transition conditions.

If the contraction becomes critical at the downstream section, the flow will be in a critical condition and the characteristics of the upstream flow, depth and velocity will not be affected at this stage. When the contracted width is less than the critical breadth, choking phenomenon will occur i.e. the breadth of the channel will not be able to pass the energy per unit width of the channel and the upstream flow condition will be influenced.

The contraction ratios $\Delta b/b = 0.2, 0.3, 0.4, 0.5$ and 0.6 were used to cause backwater the transition section. Oblique standing waves (shock waves) within the transition were observed. The relationship between upstream Froude number F_1 and contraction ratio b/b are given in Fig. (32). It is observed from the figure that the theoretical values of contraction ratio are bigger than the corresponding practical values which causes the choking phenomenon mainly due to energy losses covering the transition in practical cases.

The pattern of standing waves was found approximately at centre line of the channel for contraction ratio more than the critical contraction ratio which causes the choking. It moves to the upstream position with an increasing value of contraction ratio b/b , Fig. (33). Also the deflection angles of oblique standing waves are shown between 30 and 35° , Fig. (33).

Bed roughness (gravel bed), plate (4) causes the junction to move in the upstream position, Fig. (34) and decreases the negative disturbances at the downstream in comparison with the same contraction ratio for plain bed and it was observed that the deflection angle increases with $\Delta b/b = 0.6$.

Figure (35) shows the flared entrance decreases the water depths upstream section (2), Fig. (2). Water depths increases with flared entrance from 12.5 cm downstream section (2) to about 37.5 cm at the constriction and downstream the transition, as the energy per unit width in case the flared entrance will have more chance to pass the downstream. The same behaviour, as before, can be obtained for flared entrance of gravel bed Fig. (36). This could mean that the flared entrance is more effective in the choking than the bed roughness.

CONCLUSIONS

A set of graphical relationships has been deduced which could be used for design purposes to provide an accurate solution to the theoretical one.

Experimental value for the critical width of a channel is bigger than the corresponding theoretical value given by the equation:

$$(b_c/b)^2 = F_1^2 (1.5/(1 + F_1^2/2))^3$$

and the difference between the two values increases with the increasing value of the upstream Froude number.

It is hoped that this paper will be of value in the solution of problems involving horizontal transitions especially choking phenomenon and cross waves (oblique standing waves). However the successful design of transition depend on the designer ability to predict the deflection angle with reasonable degree of accuracy.

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NOTATION

The following symbols are used in this paper:

- A = cross sectional area;
- b = breadth of section;
- b^c = critical contraction breadth;
- b* = dimensionless breadth;
- E* = specific energy;
- F = Froude's number;
- g = acceleration due to gravity;
- Q = water discharge;
- Q* = dimensionless discharge;
- V = water mean velocity;
- Y = water depth;
- Y* = dimensionless depth;
- Y_a = Y₂/Y₁.
- α = energy coefficient;
- β = momentum coefficient;
- Δ b = contraction in breadth;
- Δ E = energy loss; and
- θ = angle of flared entrance.

Subscripts

- 1 = upstream; end 2 = downstream.

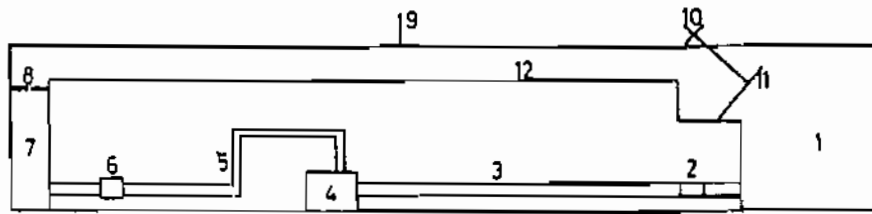
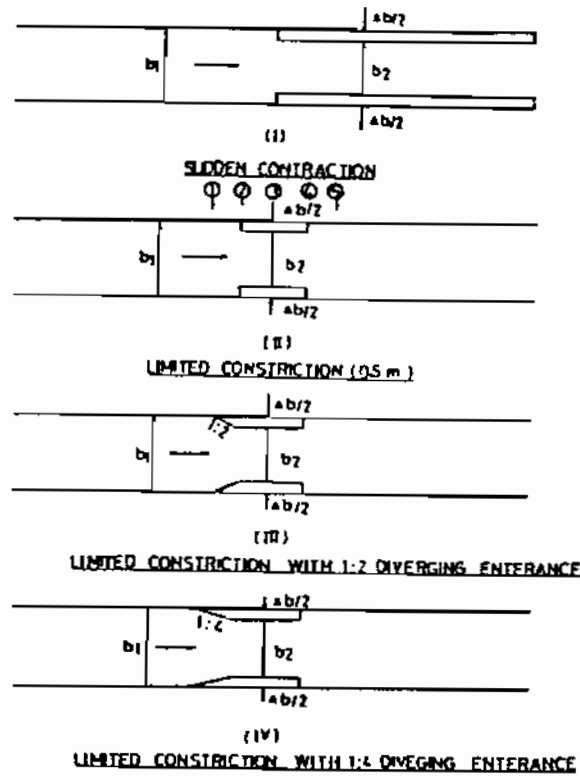


FIG.(1) EXPERIMENTAL APPARATUS.

- | | |
|-----------------|------------------|
| 1)OUTLET TANK | 7)INLET TANK |
| 2)SPARLING | 8)SCREEN |
| 3)SUCTION PIPE | 9)HOOK GAUGE |
| 4)PUMP | 10)SCREW WHEEL |
| 5)DELIVERY PIPE | 11)STEEL SPILWAY |
| 6)VALVE | 12)STEEL CHANNEL |



FIG(2) DIFFERENT TRANSITIONAL SHAPES

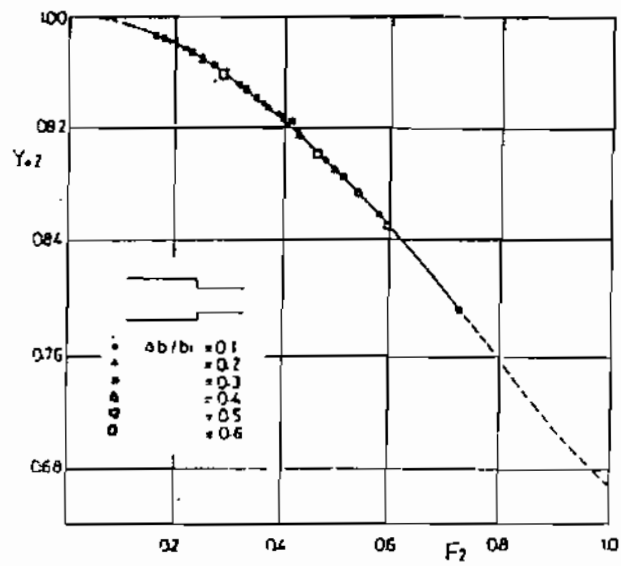


FIG (3) VARIATION OF (Y_2) WITH (F_2) FOR ab/b_1 .

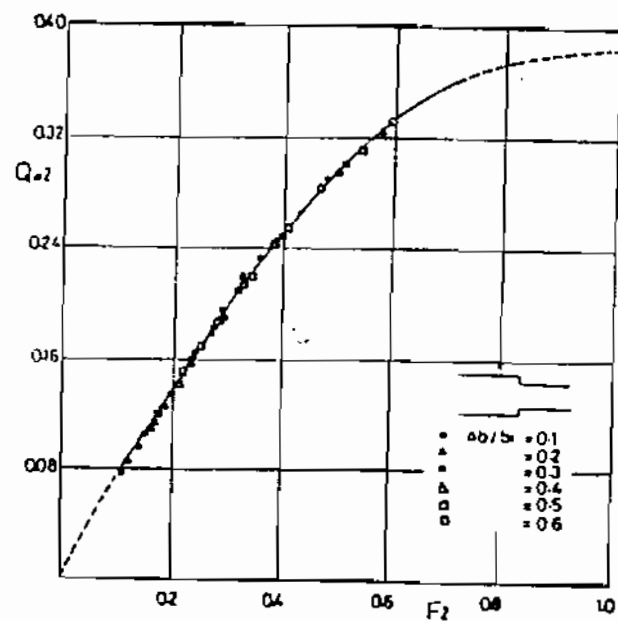
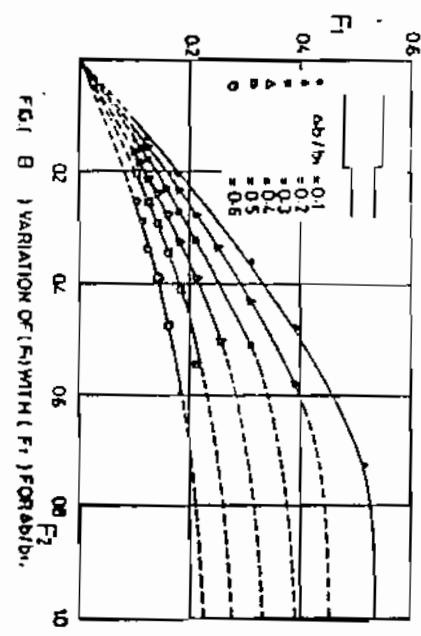
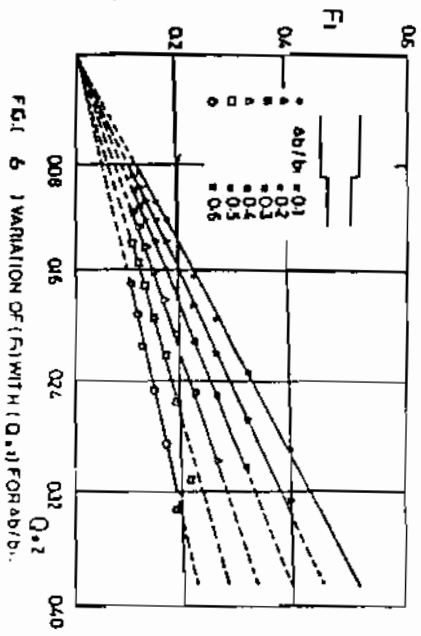
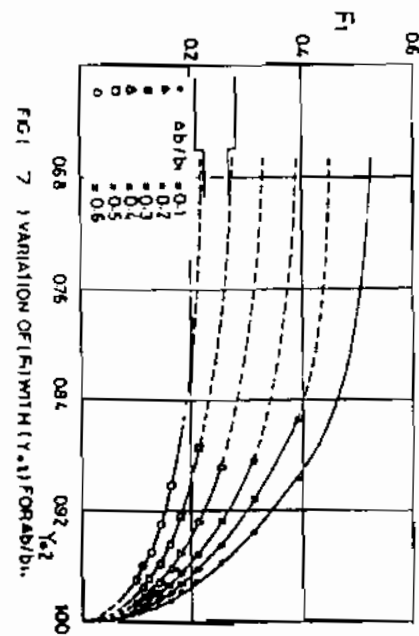
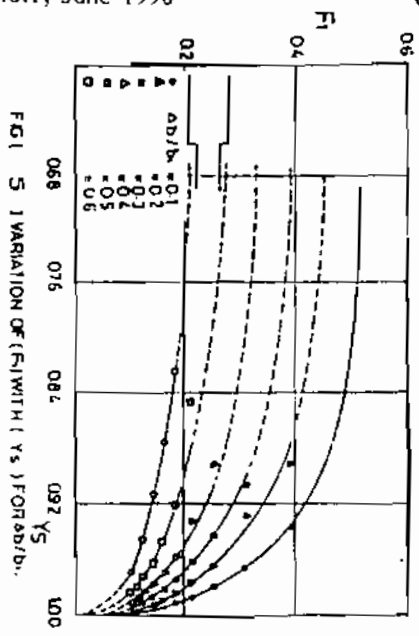
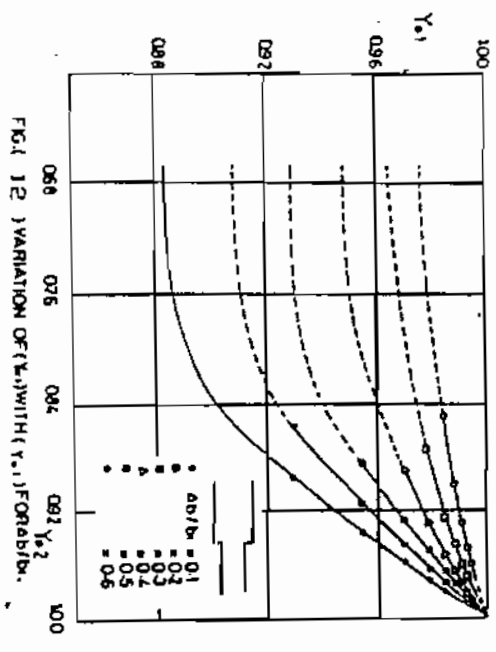
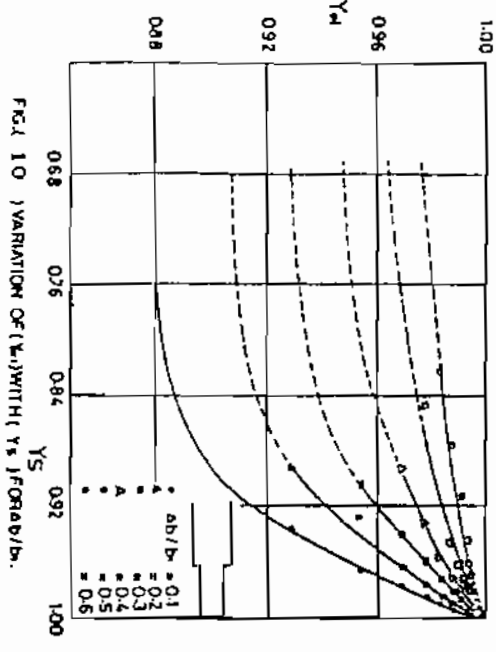
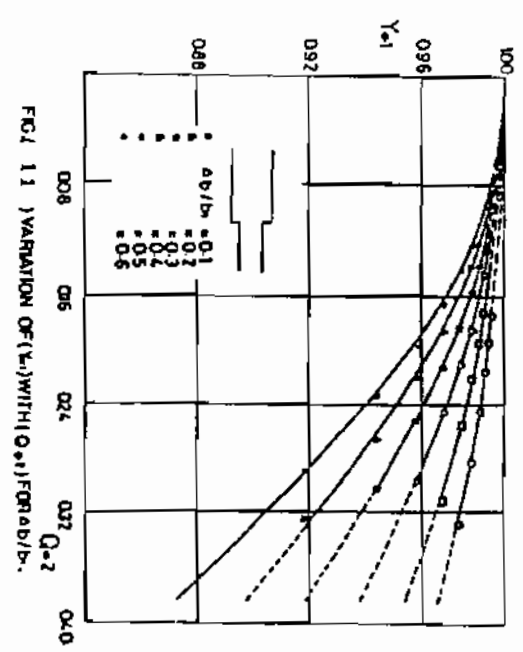
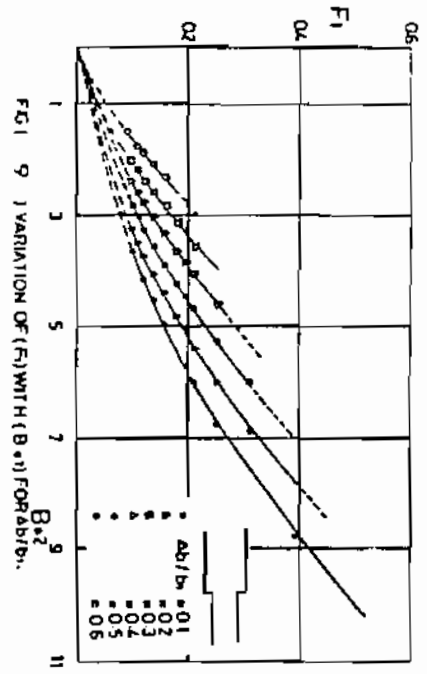
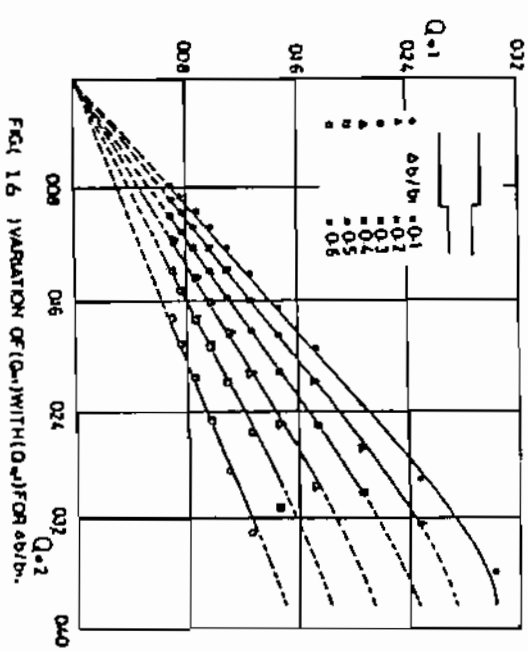
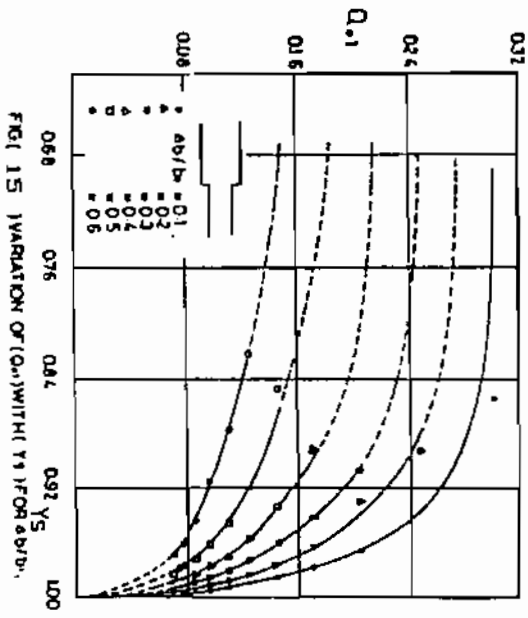
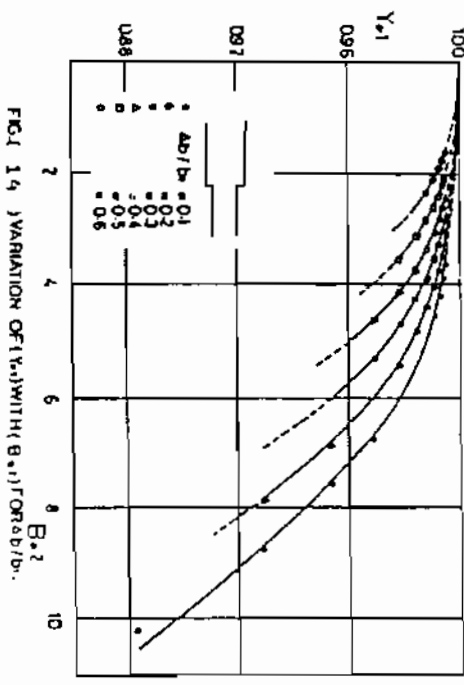
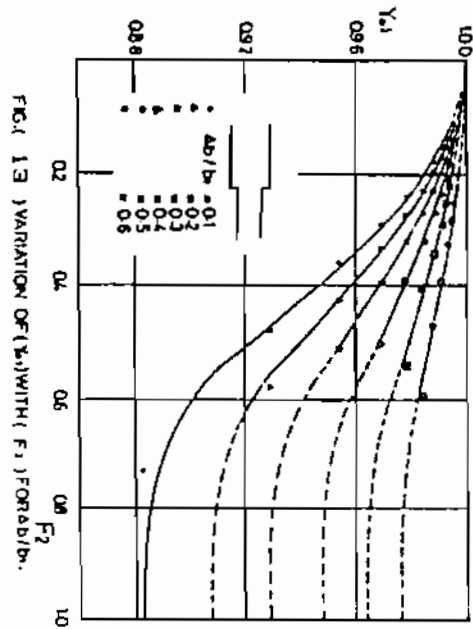
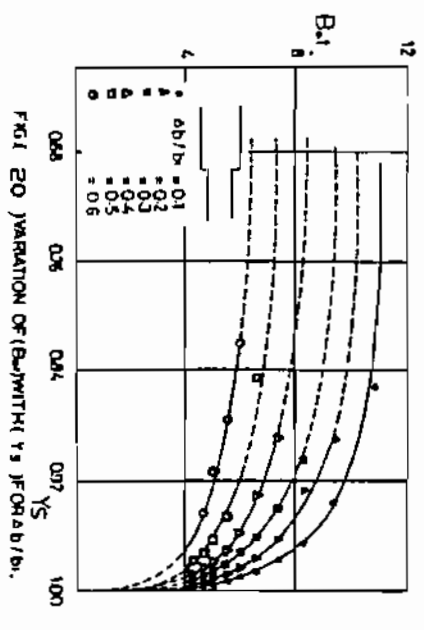
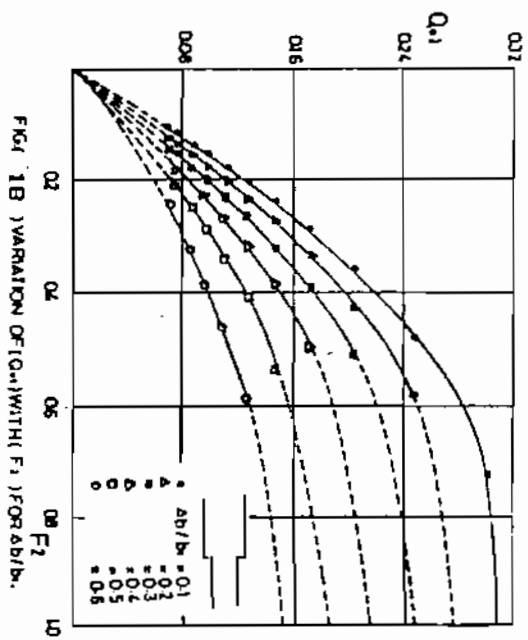
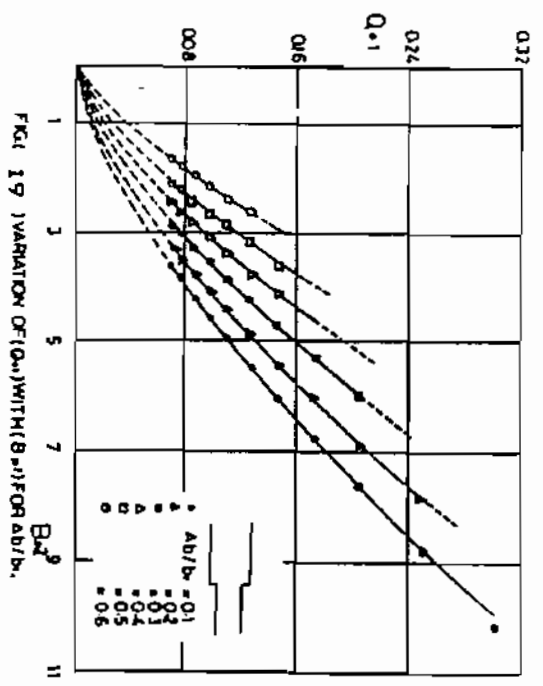
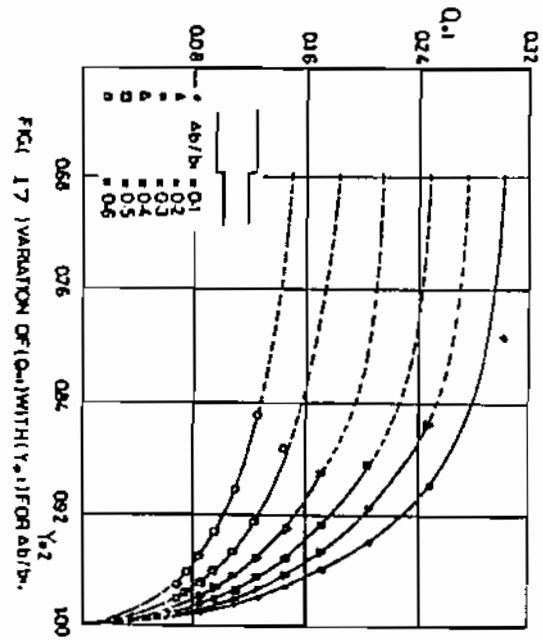


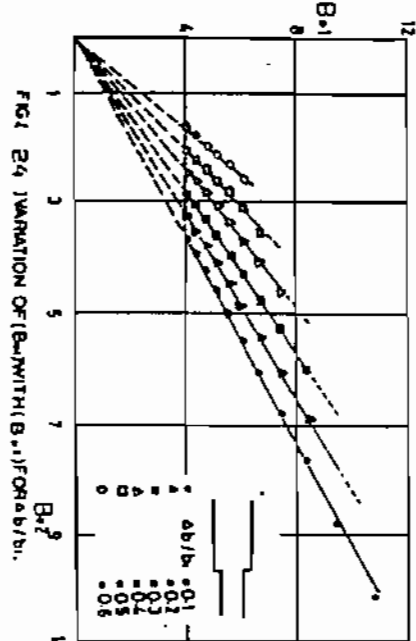
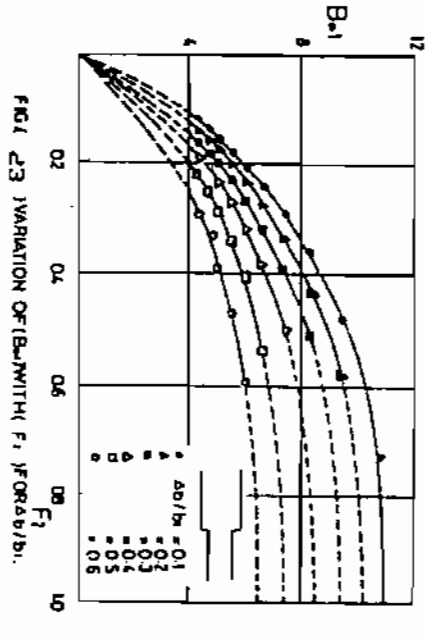
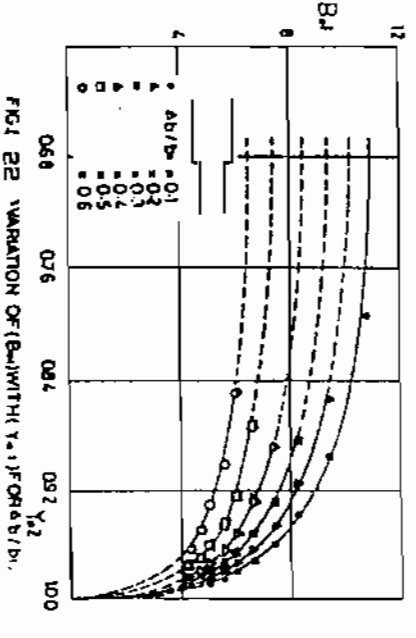
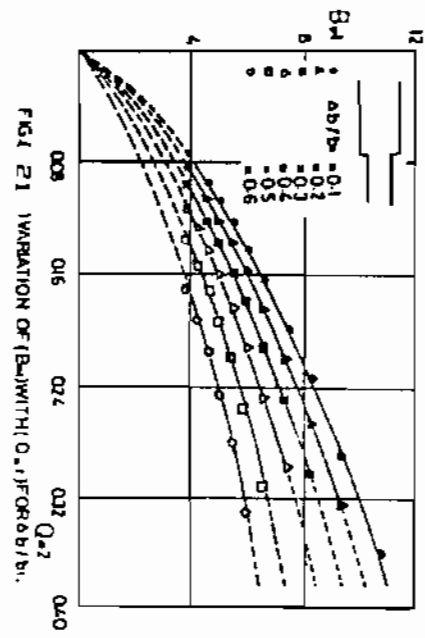
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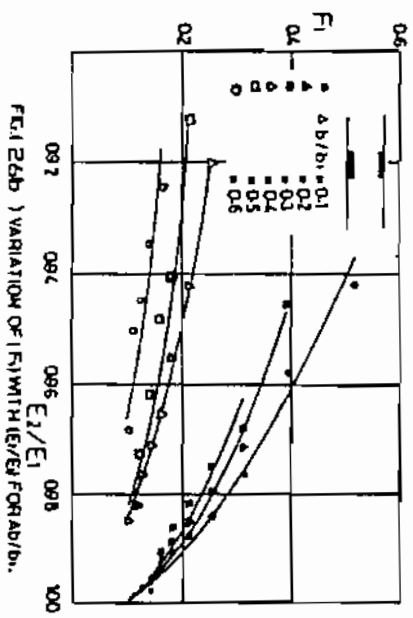
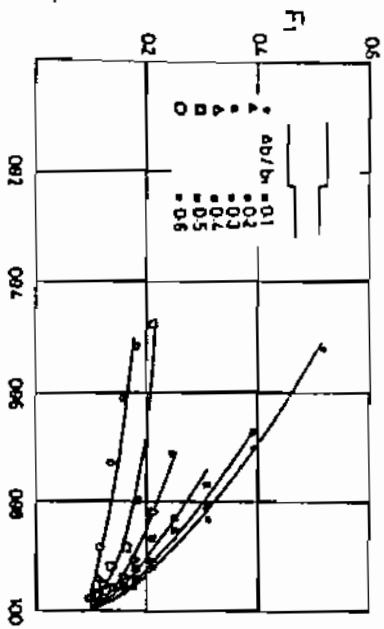
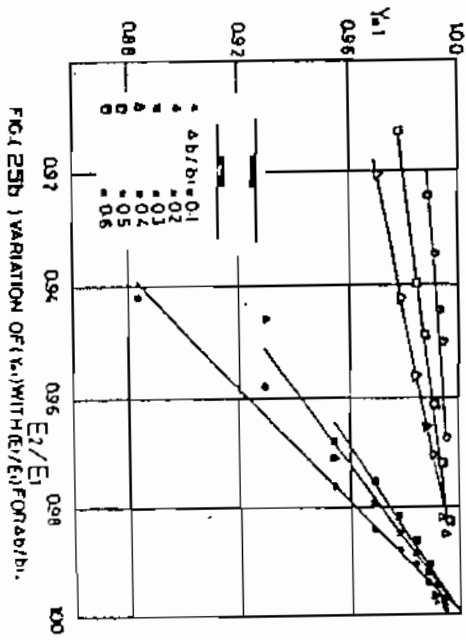
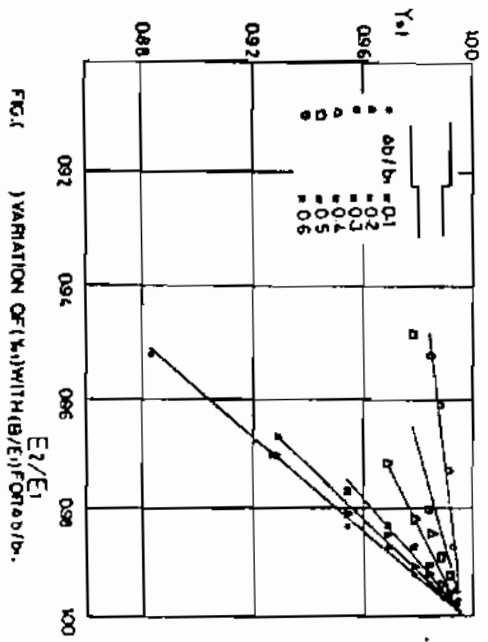


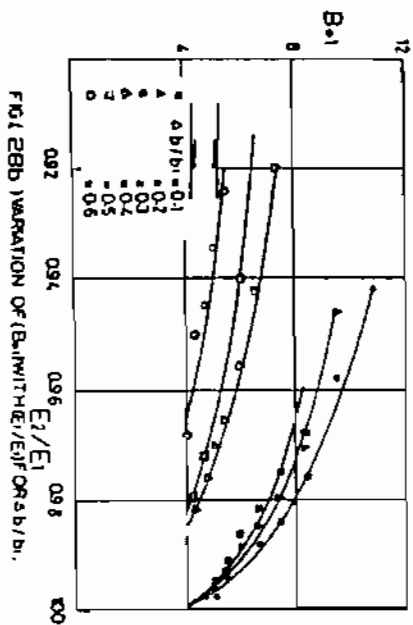
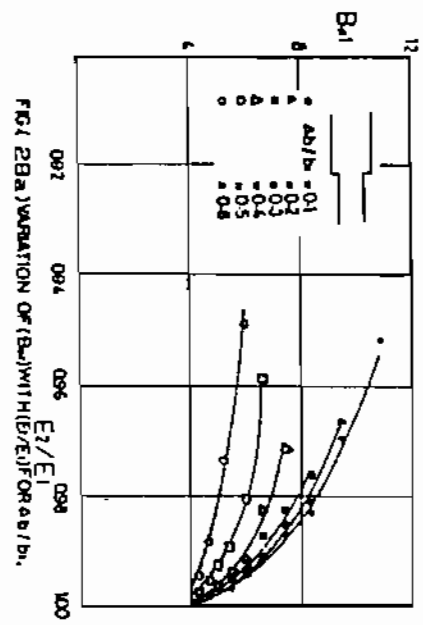
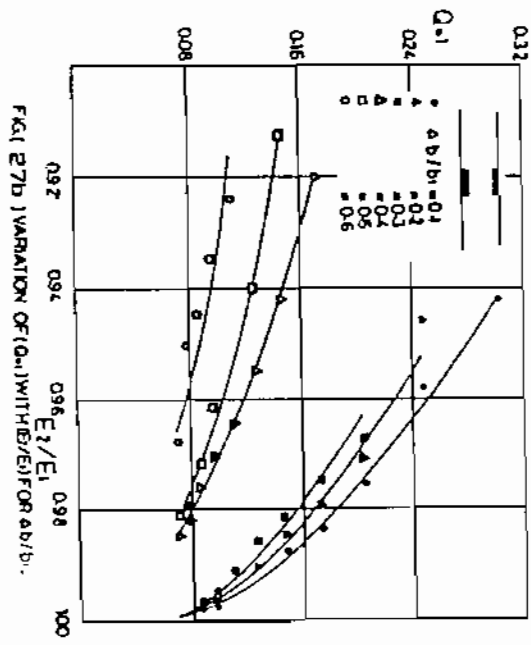
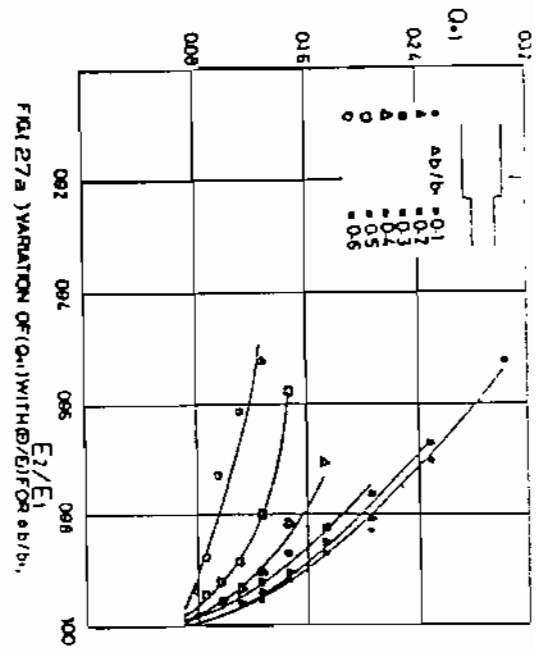












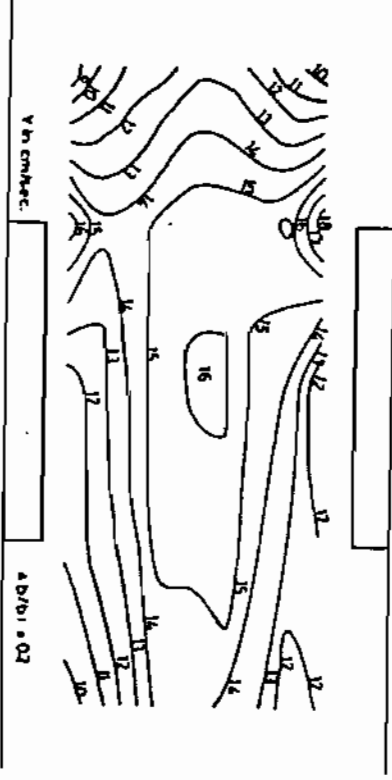


FIG.(29a) VELOCITY CONTOURS AT $y=6$ cm (PLAIN BED)

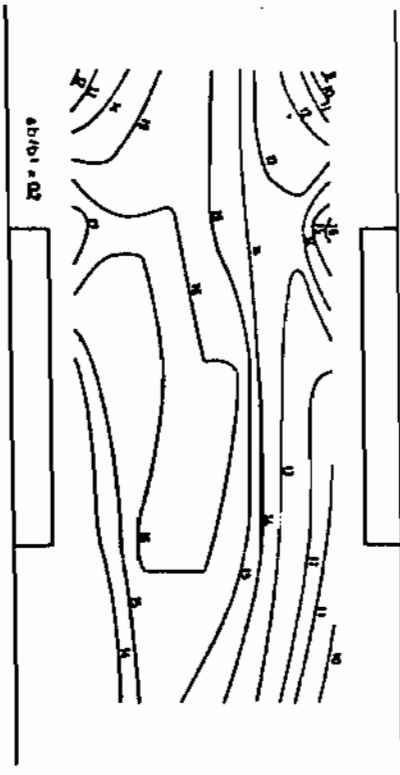


FIG.(30a) VELOCITY CONTOURS AT $y=6$ cm (BED WITH GRAVEL)

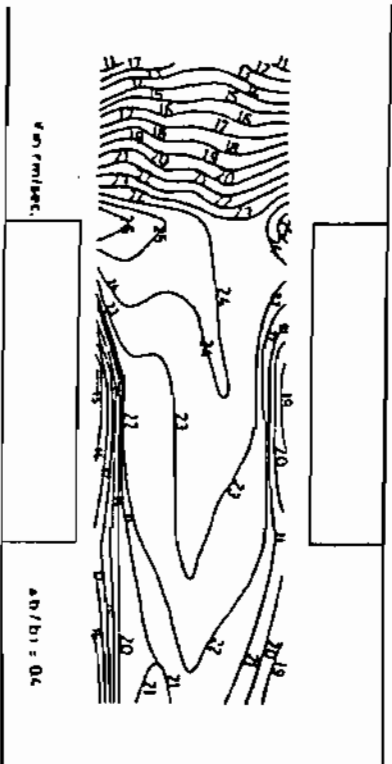


FIG.(29b) VELOCITY CONTOURS AT $y=6$ cm (PLAIN BED)

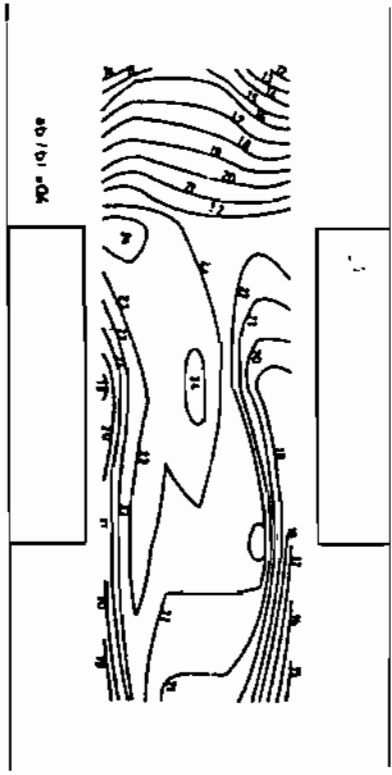


FIG.(30b) VELOCITY CONTOURS AT $y=6$ cm (BED WITH GRAVEL)

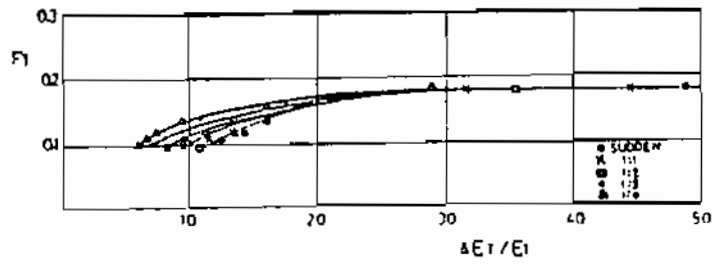
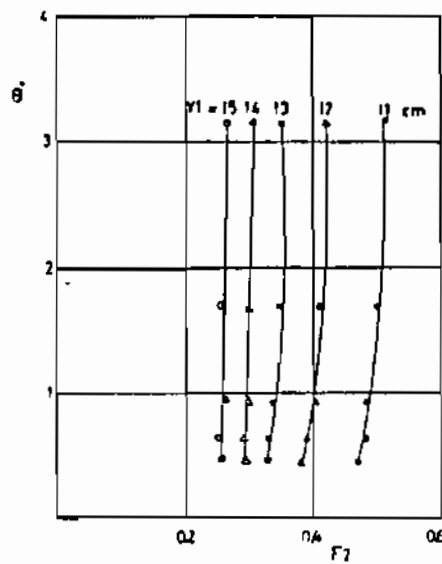


FIG.(31a) VARIATION OF (F_1) WITH ($\Delta E_t/E_t$) FOR (θ)



FIG(31b) VARIATION OF (θ) WITH (F_2) FOR (γ_1)

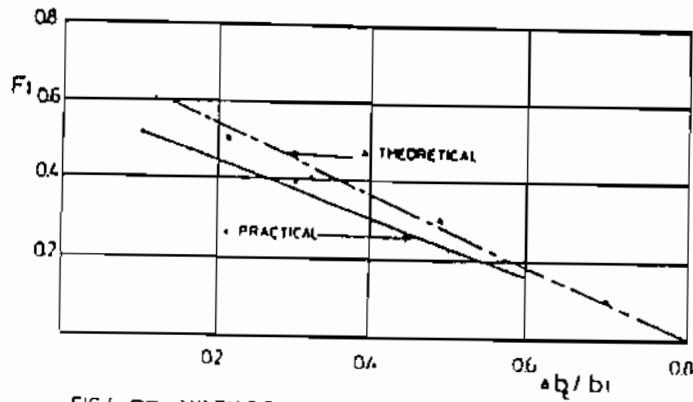
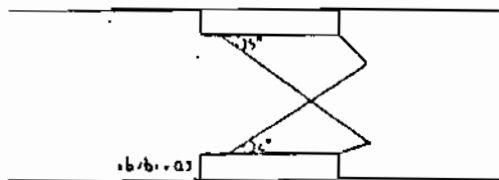
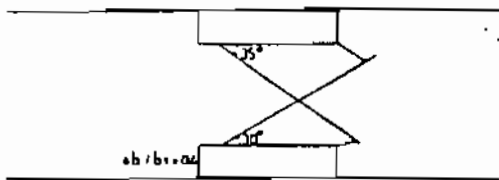


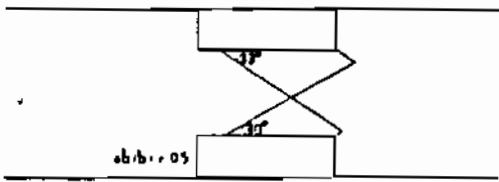
FIG (32) VARIATION OF (F1) WITH ($\Delta q/b_1$)



(a)

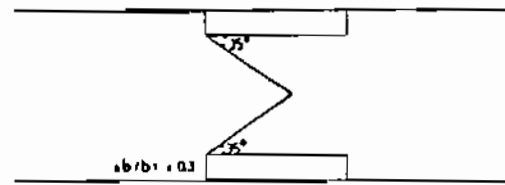


(b)

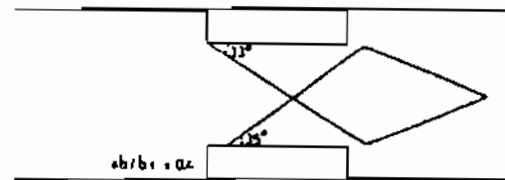


(c)

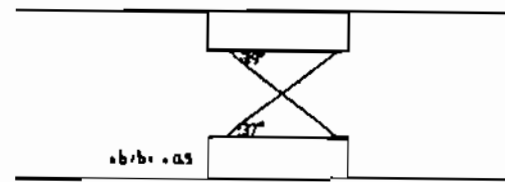
FIG (33) OBLIQUE STANDING WAVES UNDER CHOKING PHENOMENON FOR PLAIN BED



(a)



(b)



(c)

FIG (34) OBLIQUE STANDING WAVES UNDER CHOKING PHENOMENON FOR BED WITH GRAVEL

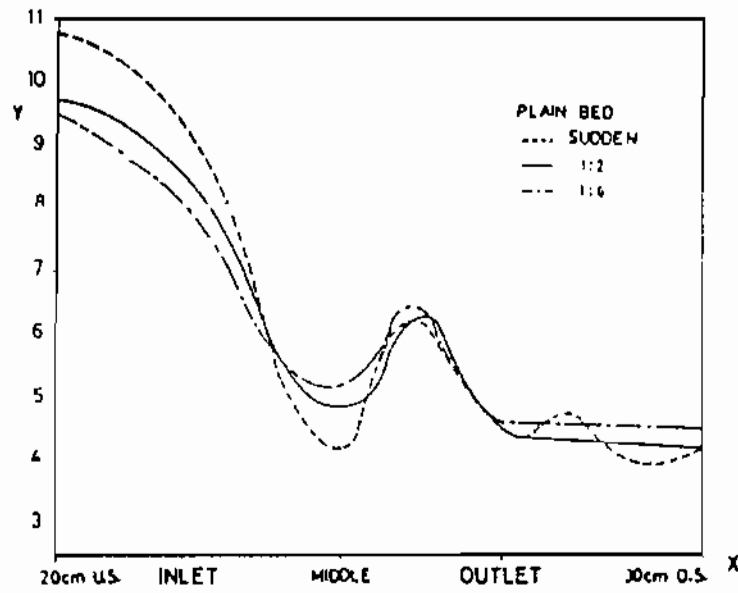


FIG. 35) WATER SURFACE PROFILE UNDER CHOKING STATE

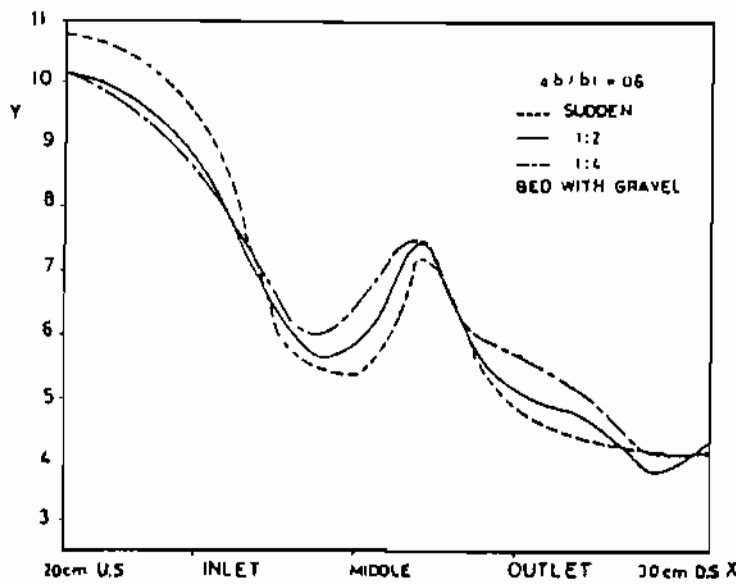


FIG. 36) WATER SURFACE PROFILE UNDER CHOKING STATE

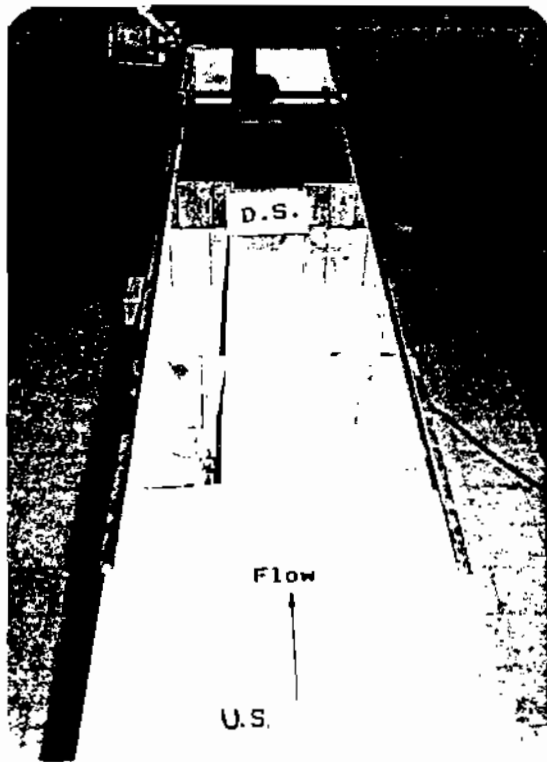


Plate (1) Sudden contraction

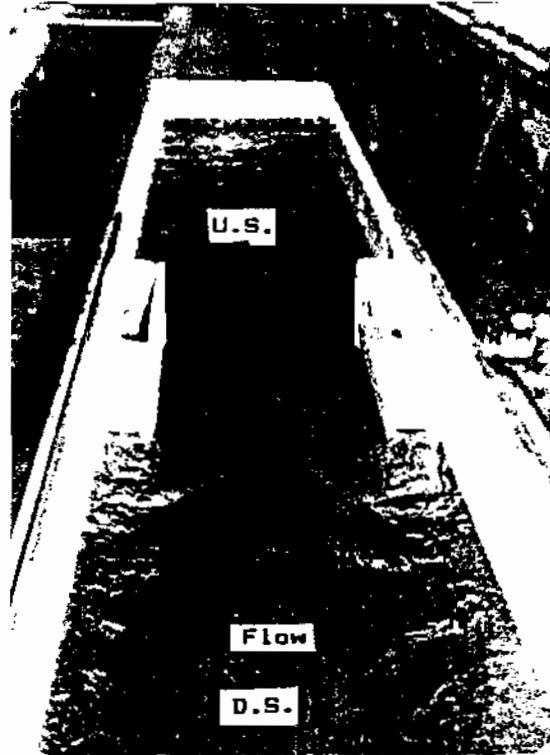


Plate (2) Limited constriction

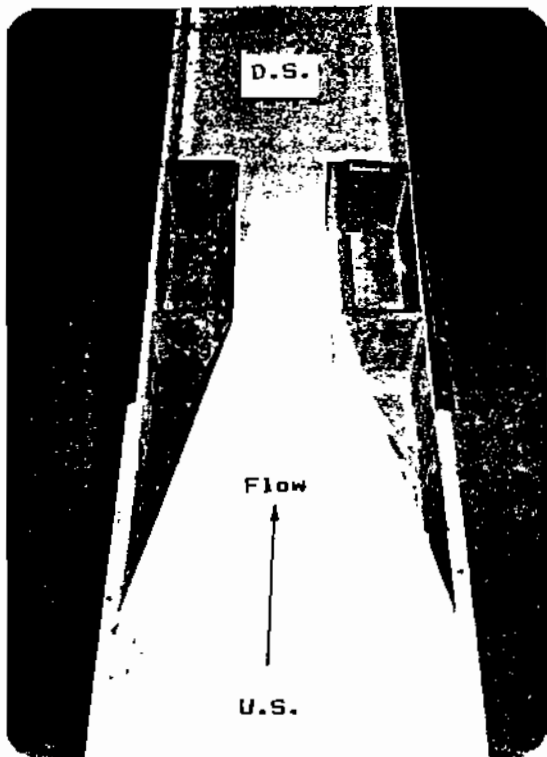


Plate (3) Flared entrance



Plate (4) Gravel bed