

PREDICTION OF THE TURBULENT NEAR WAKE OF AIRFOIL SECTIONS

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التنبؤ بغوامر منطقة اثر السريان القريبة من مقاطع الاسطوانات الانسيابية

الخلاصة:

يقدم هذا البحث طريقة جديدة للتنبؤ بغوامر منطقة اثر السريان القريبة من مقاطع الاسطح الانسيابية الحاملة باستخدام نموذج رياضي جديد للاضطراب "مؤلف زمن الاسترخاء" والذي يعتمد على تغيير معامل اللزوجة من مقطع الى مقطع وفقا لخواص الانسياب. استخدم في هذا البحث مقطعين انسيابين عند سرعات مختلفة وزوايا انحراف مختلفة. تم عمل مقارنة بين النتائج التي تم الحصول عليها بواسطة النموذج الجديد والنموذج الاصلى $k-\epsilon$ والنتائج المعملية. أظهرت النتائج أن النموذج الجديد يتتبع بطريقة صحيحة بمنحنيات السرعة ومعدل الانتشار. كما تتبنا النموذج الجديد بقم اجهاد القص أعلى قليلا من النتائج المعملية. تتبنا النموذج الجديد بصورة صحيحة بطريقة تغيير معدل الانتشار لمنطقة التضاعف مع تغيير زوايا الانحراف وسرعة السريان. كما تم تحديد التقسيمات المختلفة لمنطقة التضاعف وفقا لخواص كل منطقة.

ABSTRACT:

This paper presents a numerical prediction to the wake flow after airfoil sections using a finite difference program. Two turbulence models ($k-\epsilon$ model and Relaxation Time Model) were employed. Two airfoil sections (NACA 0012 and NACA 2415) were tested at different angles of attack and different free stream velocities. Comparison between the results obtained by the two models and the experimental results were made. The predicted longitudinal mean velocity and spreading rate by the modified model agree with experimental results. The predicted turbulence quantities ($\overline{u^2}$, \overline{uv}) by RTM were found to be slightly higher than experimental results in the near wake region. Self-similarity for mean velocity, fluctuating velocity and shear stress were predicted in the far wake region. The effect of increasing the angle of attack and Reynolds number on the wake development by the airfoil sections was predicted correctly by the modified model. The results achieved show that this model is close to the truth for wake flows in both near wake and far wake regions.

1. INTRODUCTION:

The flow in the near wake of a streamlined two-dimensional body provides a simple and yet a critical test of the generality of turbulence models and calculation procedures. Since it is a region of relaxation from wall-dominated

turbulent boundary layers to a single equilibrium shear layer (the asymptotic or far wake) at large distance from the trailing edge. Almost all turbulence models evolved from the extensive data bases in boundary layers and fully free shear flows such as wakes and jets. The near wake, being the region of adjustment between two extreme states, therefore offers an independent test of the generality of turbulence models. The prediction of the near wake flows is of considerable practical interest because the near wake participates in the viscous-inviscid interaction near the trailing edge of the airfoil.

The $k-\epsilon$ model has been shown to simulate well the near field of symmetric wakes behind thin plates [1, 2]. Patel and Scheuerer [2] have applied the $k-\epsilon$ model to the calculation of the asymmetric wakes past the trailing edge of flat plates with one smooth and one rough wall. The calculated velocity and shear stress profiles at various distances from the trailing edge are compared with those studied experimentally by Ramaprian et al [3]. The velocity profiles are predicted well by the model. The shear stress profiles are also in reasonable agreement with the data in the near field, but deteriorates further downstream where the shear stress level is underpredicted and the profiles become symmetrical faster than in the experiment. It is noteworthy that the initial shear stress peak on the smooth-wall side is reproduced well by the model. In the far field, the model predicts low shear stress level which is consistent with the observation that the standard $k-\epsilon$ model underpredicts the asymptotic spreading rate of plane wakes [3, 4].

A viscous-inviscid algorithm is developed by Baker et al. [5] for the prediction of two-dimensional mean and fluctuating velocity distributions in the wake immediately downstream of the trailing edge of airfoil NACA 63-012. They defined a composite pressure field and a Poisson equation is solved for transverse pressure distributions. A parabolized form of the time-averaged steady Navier-Stokes equations is solved in conjunction with a viscous-augmented, two-dimensional potential flow analysis. They applied a tensor constitutive equation to predict Reynolds stress distributions from solutions of turbulent kinetic energy two equation closure model. The algorithm is equally applicable to nonseparated wake flow predictions at nonzero angle of attack. They compared the experimental and the predicted profiles in the wake at zero angle of attack. Their comparison was confined to a very narrow region between 0 and 0.1 chord length behind the airfoil.

Horstman [6] used the low-Reynolds-number version of the $k-\epsilon$ model to predict successfully the asymmetric wake behind the trailing edge of an airfoil, involving a small separation region. The calculation were carried out with an elliptic solution procedure. The resulting profiles of mean velocity, shear stress and turbulent kinetic energy agree fairly well with the measurements.

Launder et al [1] calculated the axisymmetric wake measured by Demetriades [7] using various turbulence models which are: one equation model, mixing length hypothesis, standard $k-\epsilon$ model and modified $k-\epsilon$ model with C_μ function of (P/ϵ) [4]. The resulting development of the center-line velocity were compared with the experimental data [7]. When they used C_μ function, given by Rodi [4], the data were well simulated. It should be mentioned here that the mixing length and one-equation model were seen to fail dramatically in this case.

Voutsinas et al [8] has used an approach which can be termed as multi-parametric wake modeling. They has coupled a linear vortex model with a $k-\epsilon$ viscous model to analyze the characteristics of the near wake region of a rotor. The far field profiles are then obtained through matching across the inlet plane of the far wake region selected at an axial distance of approximately 3-4 rotor diameters. The use of a vortex method for the evaluation of the velocity profiles, just downstream the rotor, gives an account of the effect of the rotor geometry. As a result, the far wake self-similar velocity profiles are no longer depending on only one parameter, i.e. the initial deficit. Instead they depend on the circulation distribution defined by means of the vortex model.

Suryavamshi et al [9] investigated the prediction of the flow field including wakes and mixing in axial flow compressor rotors. The wake behavior in a moderately loaded compressor rotor has been studied numerically using a three-dimensional incompressible Navier-Stokes solver with a high Reynolds number form of the $k-\epsilon$ turbulence model. The equations are solved using a time-dependent implicit technique. The agreement between the measured data and the predictions is good, including the blade boundary layer mean velocity profiles, the near and far wake profiles and its decay characteristics. An analysis of the passage -averaged velocities and the pressure coefficients shows that the mixing in the downstream regions away from the hub and annulus walls is dominated by wake diffusion. In regions away from the walls, the radial mixing is predominantly caused by the transport of mass, momentum and energy by the radial component of velocity in the wake.

Agoropoulos and Squire [10] investigated the interaction between turbulent wakes and boundary layers by two models. They applied the Algebraic Stress Model (*ASM*) and an improved version of the $k-\epsilon$ model to predict five cases of two-dimensional turbulent wakes mixing with boundary layers. They compared the predicted profiles with their measurements. They found that the *ASM* predicts the mean velocity U more accurately than the $k-\epsilon$ model.

The results obtained by previous investigators suggest that the major characteristics of the symmetric as well as asymmetric near wake can be predicted with satisfactory accuracy. The agreement between prediction and experiment deteriorates with distance from the trailing edge, and the asymptotic growth rates are underestimated. The purpose of this paper is to predict the turbulent near wake characteristics of airfoil sections by using the Relaxation Time Model (RTM) introduced by Mohamed [11]. The RTM represents a new modification to the standard $k-\epsilon$ that simulates the jet flow in stagnant surrounding. The Relaxation Time Model recommends the use of the "Eddy Relaxation" effect which has the same "Eddy Viscosity" assumption with C_ν constant in the transverse direction, but C_μ adjusted at each downstream station by the local flow parameters.

2. GOVERNING EQUATIONS:

The mean flow field is governed by the continuity and stream wise momentum equations. Assuming the mean flow to be steady, plane, two dimensional and incompressible flow; the thin shear-flow equations can describe this flow. These equations apply:

Continuity equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad \dots\dots\dots (1)$$

U-momentum:

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = - \frac{\partial P}{\partial x} - \rho \frac{\partial \overline{u^2}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \quad \dots\dots\dots (2)$$

The term $\frac{\partial P}{\partial x}$ is equal to zero since there is no pressure gradient, while the term $\rho \frac{\partial \overline{u^2}}{\partial x}$ is usually neglected in boundary layer calculations since it is small compared to the other terms in equation (2). Equation (2) in this case takes the form:

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = \frac{\partial \tau_{xy}}{\partial y} \quad \dots\dots\dots (3)$$

where the shear stress $\tau_{xy} = \mu_t \frac{\partial U}{\partial x} + (-\rho \overline{uv})$ is the sum of two terms, the viscous and turbulent stresses. The latter \overline{uv} normally called the Reynolds

shear stress, represents the rate of transport of x-momentum in the y-direction produced in the flow as a result of turbulent eddy motion. The purpose of a "turbulent model" is to determine the level of this important correlation.

The standard $k-\epsilon$ model [12] calculates the shear stress with the eddy viscosity hypothesis in the following way:

$$\overline{uv} = \nu_t \frac{\partial U}{\partial y} \quad \text{where} \quad \nu_t = C_\mu \frac{k^2}{\epsilon} \quad \dots\dots\dots (4)$$

where k is the turbulent kinetic energy, ϵ is its dissipation rate and C_μ is an empirical constant. The local values of k and ϵ are determined from transport equations, which have the form:

$$U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial y} \right) + P_k - \epsilon \quad \dots\dots\dots (5)$$

$$U \frac{\partial \epsilon}{\partial x} + V \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial y} \left(\frac{\nu_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial y} \right) + \frac{\epsilon}{k} (C_{\epsilon 1} P_k - C_{\epsilon 2} \epsilon) \quad \dots\dots\dots (6)$$

the term P_k is the production term of k and is given by

$$P_k = -\overline{uv} \frac{\partial U}{\partial y} - \underbrace{(\overline{u^2} - \overline{v^2}) \frac{\partial U}{\partial x}} \quad \dots\dots\dots (7)$$

The second (underlined) term in equation (7), which involves the irrotational strain rate $\partial U/\partial x$, is neglected. For the empirical constants the values $C_\mu = 0.09$, $C_{\epsilon 1} = 1.44$, $C_{\epsilon 2} = 1.92$, $\sigma_k = 1.0$ and $\sigma_\epsilon = 1.3$ given for instance by Rodi [4] have been adopted.

The standard $k-\epsilon$ model employs the assumption that C_μ is constant that produces fairly good results for relatively simple flows in which only one Reynolds stress component is of importance in the momentum equation. However, Rodi [4] showed that C_μ cannot ever be considered constant in all simple shear flows. The standard $k-\epsilon$ (employing the eddy viscosity concept with a constant C_μ) lacks the universality required for shear flows because of the inadequate representation in this model of the convective and diffusive transport of \overline{uv} .

The Relaxation Time Model uses the Prandtl-Kolmogorov relation between shear and strain:

$$\overline{uv} = C_{\mu} \frac{k^2}{\epsilon} \frac{\partial U}{\partial y} \quad \text{where } \nu_t = C_{\mu} \frac{k^2}{\epsilon}$$

but with changing C_{μ} with the local flow parameters. This produces a variation of C_{μ} in the axial direction but not in the radial direction. This is desirable because C_{μ} is reasonably constant across free shear layers. In *RTM*, C_{μ} takes the form:

$$C_{\mu} = C_{\mu} [1 + \beta (\text{constant})] \quad \dots(8)$$

where β is the relaxation time coefficient which was found to be 1.2 for jet flows, and

$$\text{constant} = \left[\frac{\bar{L}}{k_{max}^{1/2}} \frac{U_{conv}}{uv_{max}} \frac{\partial \overline{uv_{max}}}{\partial x} \right]$$

\bar{L} is the average length, $\overline{uv_{max}}$ and k_{max} are the maximum shear stress and turbulent kinetic energy respectively. U_{conv} is the convection velocity for the eddies:

$$U_{conv} = 0.5 [U_{max} + U_{min}]$$

The scope of the paper is to check the validity of the Relaxation Time Model for wake flows and how it predicts the flow parameters.

3. SOLUTION PROCEDURE:

Equations (1), (3), (5) and (6) represent the governing equation to this flow. The above set of equations is parabolic. A numerical solution to this set is obtained using a computer program developed to calculate the development of the wake in external flows with and without pressure gradient. The partial differential equations were transformed into a system of finite-difference equations. The finite-difference equations governing the consider flow are solved by the computer program in an implicit numerical scheme using an efficient Tri-Diagonal-Matrix Algorithm (TDMA) [13] (or Thomas Algorithm), which involves a once-and-for-all application of a Gaussian Elimination process.

The solution steps start by making a forward step. Upstream information is used to adjust the expansion of the domain over the next forward step based on an approximate balance of momentum over the cell adjacent to the free

boundary. The V-velocity is obtained by solving the continuity equation. The coefficients and sources of the U-momentum equation, the turbulent kinetic energy k and its dissipation ϵ equation are calculated using upstream conditions. The boundary conditions are incorporated into the coefficients and finally the equations are solved. At this stage, the available upstream conditions are already used. Then the Reynolds stress can be obtained using the Boussinesq viscosity concept. The calculated downstream values for U, V, k and ϵ can be stored as upstream information for the next step. If in-step iterations are required, the procedure would be repeated from the calculation of the coefficients and sources of the governing equations, until a converged state has been obtained. For the next forward step, the solution must be started from the beginning.

The initial conditions needed in the prediction procedure were prescribed from the experimental measurements carried out by Mohamed et al [14] at $x/c=0.02$ for both airfoil sections NACA 0012 and NACA 2415. The mean and fluctuating longitudinal velocity components (U and $\overline{u^2}$) were available in the measurements. The turbulent kinetic energy k was considered equal to $[k = 3/2 (\overline{u^2})]$ since $\overline{u^2} = \overline{v^2} = \overline{w^2}$. The dissipation rate ϵ was obtained from the approximate expression relating the dissipation rate to the turbulent kinetic energy and the mixing length l given in the $k-\epsilon$ [12] as $\epsilon = k^{3/2} / C_\mu^{3/4} l$. The mixing length l for wake flow is equal to (0.5δ) where δ is the upper or lower boundary layer thickness of the wake.

For the boundary conditions, we need to prescribe the values of U, k and ϵ along the upper and lower edges of the wake. In the present investigation, the external velocity U at the upper and lower boundary is kept constant with x and equal to the free stream velocity. The turbulent kinetic energy k and its dissipation rate ϵ in the external flow are remained as prescribed initially. The initial distribution of the transverse velocity V is zero at all the nodes and the V-field can be obtained from the U-field and the continuity equation.

The grid for the finite difference scheme has got N number of nodes that divides the flow width into (N-2) control volumes. The node positions are at fixed value of (y/δ) where δ is the wake width. The space between the nodes increases with the increase of the shear layer thickness as the flow advances downstream. At all the stations, the first and last nodes lie on the lower and upper boundaries respectively. The grid is not uniform but becomes more dense in the regions where the flow variables change rapidly in the y-direction. Thirty nodes are needed in each test run, fifteen nodes in each boundary layer of the wake. The number of iterations required in each step is specified by

preliminary test. The tests show that ten iterations are large enough to make a converging solution.

4. RESULTS:

Comparisons between the measured and predicted wake profiles in the near wake region of airfoil sections (NACA 0012 and NACA 2415) are made to check the validity of the *RTM* for this flow. The same constants used in jet flow [11] were employed in the present work. Figure (1) represents the comparison between the measured [14] and predicted longitudinal mean velocity U profiles that were predicted by both original $k-\epsilon$ model and *RTM* model for NACA 0012 at free stream velocity $U = 10, 30$ and 50 (m/s) at an angle of attack $= 0^\circ$. Figure (2) represents the comparison between the measured and predicted longitudinal fluctuating velocity $\overline{u^2}$ that was predicted by the two models for NACA 0012 at the same free stream velocities. Figure (3), also, represents a comparison between the measured and predicted shear stress \overline{uv} profiles at the same conditions. Both models predicted correctly the mean velocity profiles, but *RTM* predicted slightly lower center-line velocities specially at the two downstream distances ($x/c = 0.2$ and 1.8). While it predicts higher center-line velocities after $x/c = 2.0$, as shown in figure(4-b). Both models predicted lower center-line velocity than the experimental results. The Relaxation Time Model also predicted higher values of longitudinal fluctuating velocities than standard $k-\epsilon$ model. The values predicted by the *RTM* agrees much better with the experimental results than standard $k-\epsilon$ model specially after $x/c = 2.0$, as shown in figure (4-c). The predicted shear stress values by *RTM* at all stations are over estimated. Original $k-\epsilon$ model predicted reasonable values of shear stress (figure 4-d).

The reason of the above behaviour of the Relaxation Time Model may be explained by examining the eddy viscosity coefficient C_μ employed in this model. Figure (5) shows the variation of C_μ with the downstream distance. The model uses different values of C_μ according to local flow parameters in the downstream direction while being kept constant in the transverse one. At the beginning higher values of C_μ were predicted because of the higher values of the shear stress, then it gradually decreases until it reaches $C_\mu = 0.09$ at $x/c = 5.0$ and stays constant along the downstream distance. Increasing C_μ causes the increase of turbulent kinetic energy and consequently increasing the shear stress at these stations.

In order to indicate the effect of increasing Reynolds Number R_e on the flow development, three velocities were employed $U = 10, 30$, and 50 (m/s) that corresponds to $R_e = 6.66 \times 10^4$, 2.0×10^5 and 3.33×10^5 respectively. Figure

(6) shows the iso-contours for the velocity distributions that shows the development of the wake half-width with the downstream distance as well. The figure indicates that increasing the R_e reduces the wake half-width and that the wake elongates as R_e increases, which agrees with experimental results.

Due to the unusual behaviour of the airfoil, particularly in the low angle of attack, four angle of attacks ($\alpha = 0^\circ, 2^\circ, 4^\circ$ and -2°) were investigated in an attempt to test the wake. Figure (7) shows the iso-contours for the velocity distribution in the wake for airfoil NACA 0012 at the above angles of attack and at free stream velocity $U = 30$ (m/s). It represents the effect of increasing the angle of attack on the development of wake half-width $Y_{1/2}$ with the downstream distance x . The figure indicates that the wake half-width increases with the increase of angle of attack. The faster separation that occurs as the angle of attack increases, causes wider wake-half width $Y_{1/2}$. It also indicates that the wake disappears much faster as the angle of attack α increases.

One of the interesting and important issues that arise in the discussion of wake development is concerned with the downstream distance from the trailing edge beyond which the wake can be regarded as having reached an asymptotic state. Associated with this issue the labeling of the wake as "near wake" and "far wake". Intuitively, the wake can be said to have reached an asymptotic far-wake state if all flow properties, both mean and turbulent, attain universal distributions independent of trailing edge conditions. Figure (8) shows that longitudinal mean velocity, longitudinal fluctuating velocity and shear stress profiles become self-similar after $x/c = 7.0$. It was therefore concluded that the region $x/c < 7.0$ could be regarded as "near wake" and the rest as "far wake".

Figure (9) indicates the response surface for the predicted longitudinal mean velocity U , longitudinal fluctuating velocity $\overline{u^2}$ and shear stress \overline{uv} by *RTM* for airfoil section NACA 0012 at free stream velocity $U = 50$ (m/s) and angle of attack 2° . The figure shows the development of the above values with the downstream distance.

similar results were obtained for the airfoil section NACA 2415, but they are not shown here because of the limited space.

5. DISCUSSION:

The models that use the eddy viscosity concept with constant eddy viscosity coefficient C_μ can predict consistent results for mixing layers and for plane jet as well as wall boundary layers, but the round jet and particularly the near wake requires different value. The new model (*RTM*) suggested the

employment of a modified eddy viscosity coefficient C_{μ}' that varies with the local flow parameters in the downstream direction while being kept constant in the transverse one.

In the asymptotic wake, the ratio of the production to dissipation of kinetic energy drops to considerably below unity if C_{μ} is taken constant and equal to 0.09. In order to improve the predictions, Rodi [4] introduced an empirical correction that makes the coefficients C_{μ} and $C_{\varepsilon 2}$ functions in the centerline velocity. According to these functions, the overall production of k differs significantly from the dissipation and the coefficient C_{μ} was found to differ from 0.09. Therefore, Rodi [4] proposed to make C_{μ} as a function of the average value of (P/ε) across the free shear layer, that is of practical importance in the improvement of the simulation of axisymmetric wakes.

Launder et al [1] found that, the average ratio of production to dissipation of the kinetic energy, P/ε is only about 0.15, so that the value $C_{\mu} = 0.09$ determined from the local equilibrium shear layers (where $P/\varepsilon = 1$), gives rather poor results. When the C_{μ} function, given by Rodi [4], is used instead, the wake flows were well simulated.

Agoropoulos and Squire [10] found that along the centerline of the wake, where the difference between production and dissipation of k is large and increases under flow deceleration, the *ASM* predicts the mean velocity U more accurately than the $k-\varepsilon$ model. They attributed this to C_{μ} because whereas C_{μ} is constant for $k-\varepsilon$, the same parameter effectively becomes a function of P_k and ε under the *ASM*.

From the above results, it is possible to identify the various stages in development of the wake of airfoil sections as follows:

The asymptotic state (far wake) is reached at quit large distances ($x/c > 7.0$) from the trailing edge. Beyond this distance, the turbulent structure becomes independent of the initial conditions and the growth rates are well predicted by classical far-wake analysis and simple turbulent models as $k-\varepsilon$ and Relaxation Time Model (*RTM*). In that region, the eddy viscosity coefficient $C_{\mu}' = C_{\mu} = 0.09$ that means no change in the predicted mean velocity profiles and spreading rate because the original $k-\varepsilon$ responds correctly in this region.

The region between the trailing edge and the beginning of the asymptotic state may be called the "near wake." This region can be further divided into two regions.

The first region, called the "developing wake", is characterized by development of an inner wake and is the region influenced by the wall layer of the initial boundary layers. The rates of growth of $(Y_{1/2}/c)$ and (U_{Min}/U) in this region are large compared with the asymptotic growth rates. The developing wake extends from the trailing edge to $(x/c \approx 2.0)$. The flow in the immediate neighborhood of the trailing edge is also influenced by the geometry and the thickness of the trailing edge of the airfoil section. In this region, the *RTM* produces a desired increase in C_{μ} and thus an increase of the spreading rate of the wake, a region in which the original $k-\varepsilon$ is known to perform badly and predicts slower spreading rates. The increase in C_{μ} increases the shear stress and the turbulent kinetic energy.

The next region ($2.0 < x/c < 7.0$) of the near wake may be called the "intermediate wake." In this region, influence of the upstream wall layers becomes insignificant and the wake evolves as a free turbulent flow. Local "similarity" of the mean-velocity profiles is observed in this region. The intermediate wake is also characterized by lower increase in C_{μ} , therefore the *RTM* predicted slower rates of growth of the wake width and slower decay of the centerline defected velocity than occur either in the developing wake region ($x/c < 2.0$) or in the far wake region ($x/c > 7.0$).

In physical terms, the three zones of wake development may be described by the level of mixing between the two individual boundary layers at the trailing edge. The inner layer of the near wake is the region of small-scale mixing between the upstream wall layers. In this region, mixing is confined to the inner wake, the outer velocity defect layers remaining practically unchanged. The intermediate wake is the region over which mixing occurs between the outer layers. The asymptotic state is reached when this mixing is completed, where all memory of the boundary layers on the body is then destroyed (forgotten). The dynamics of the large-scale mixing that determines evolution of the near wake into the asymptotic far wake have to be well understood in order to predict the flow in this region with reasonable accuracy. The large-scale mixing in the intermediate zone of the wake has not been studied and would be worthy of future investigations.

6. CONCLUSIONS:

The (*RTM*) model predicted correctly the wake flow after the airfoil sections with a realistic behavior even in such details as mean velocity profiles, turbulent kinetic energy profiles and shear stress profiles in both near wake region and the self-preserving region. It can be seen that the predicted minimum velocities are slightly lower and the width of the wake is slightly

higher than the experimental values. Also, it may be seen that the fluctuating velocity and the shear stress profiles are slightly higher than the experimental values. However, considering the uncertainties involved in the measurement of mean velocities, fluctuating velocity and shear stress in a highly turbulent flow with a hot-wire, the agreement between the two results can be considered as very satisfactory overall. The results suggest that the "near wake" region of airfoil sections ends at $x/c = 7.0$ and consists of two internal regions. These regions are the "developing wake" ($x/c \cong 2.0$) and the "intermediate wake" ($2.0 < x/c < 7.0$). The rest of the wake is the "far wake" region where self-similarity for mean velocity and turbulence quantities was achieved.

7. NOMENCLATURE:

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| c | Cord length of airfoil. |
| C_1, C_2 | Hot wire linearizer constants. |
| $C_{\epsilon 1}, C_{\epsilon 2}$ | Constants in the $k-\epsilon$ model. |
| C_{μ} | Eddy viscosity coefficient. |
| C_m | Modified eddy viscosity coefficient. |
| k | Turbulent kinetic energy. |
| l | Mixing length. |
| P_k | Production term in the k transport equation. |
| R_e | Reynolds number ($R_e = cU/\nu$). |
| T_k, T_{ϵ} | Diffusion term in k transport equation and ϵ transport equation respectively. |
| u | Longitudinal turbulence fluctuation component. |
| U_{Max} | External mean velocity. |
| U_z | Longitudinal mean velocity. |
| U_{min} | Minimum longitudinal mean velocity. |
| U_e | Excess velocity ($U_e = U_z - U_{min}$). |
| \overline{uv} | Reynolds shear stress. |
| \overline{uv}_{max} | Maximum Reynolds shear stress. |
| x, y | Longitudinal and transverse dimensional co-ordinates. |
| x/c | Nondimensional distance along chord. |
| $Y_{1/2}$ | Wake half-width. |
| μ | Fluid dynamic viscosity. |
| ν | Fluid kinematic viscosity. |
| ν_t | Turbulent eddy viscosity. |
| τ | Shear stress. |
| ϵ | Dissipation rate of the turbulent kinetic energy k . |
| $\sigma_{k,t}, \sigma_{\epsilon,t}$ | Turbulent Prandtl number in k and ϵ transport equations. |
| α | Angle of attack. |
| β | Relaxation time coefficient. |

8 Boundary layer thickness.

8. REFERENCES:

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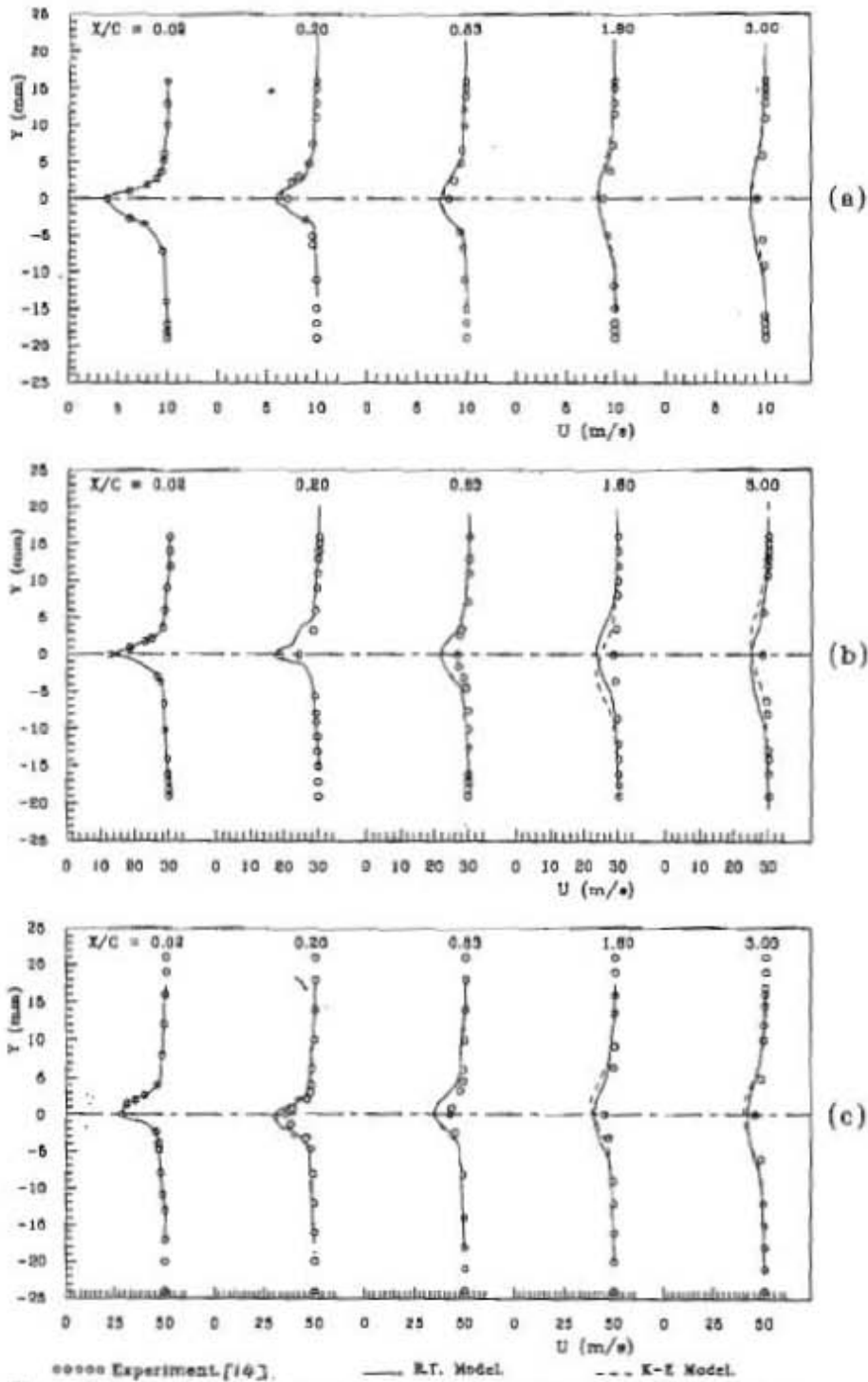


Figure (1) Comparison between the measured and predicted longitudinal mean velocity profiles for NACA 0012 at $\alpha = 0^\circ$.

a) $U = 10$ (m/s)

b) $U = 30$ (m/s)

c) $U = 50$ (m/s).

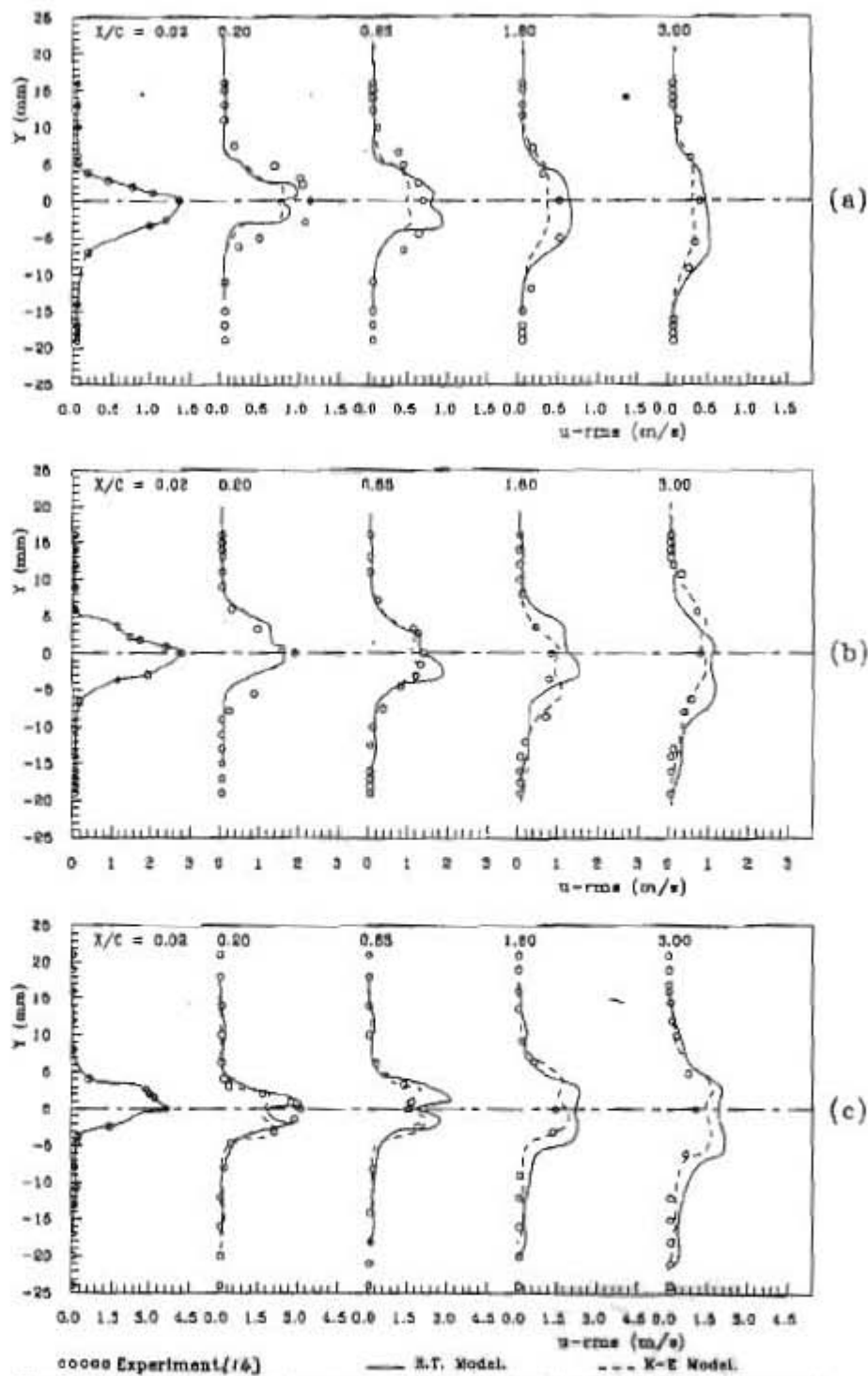
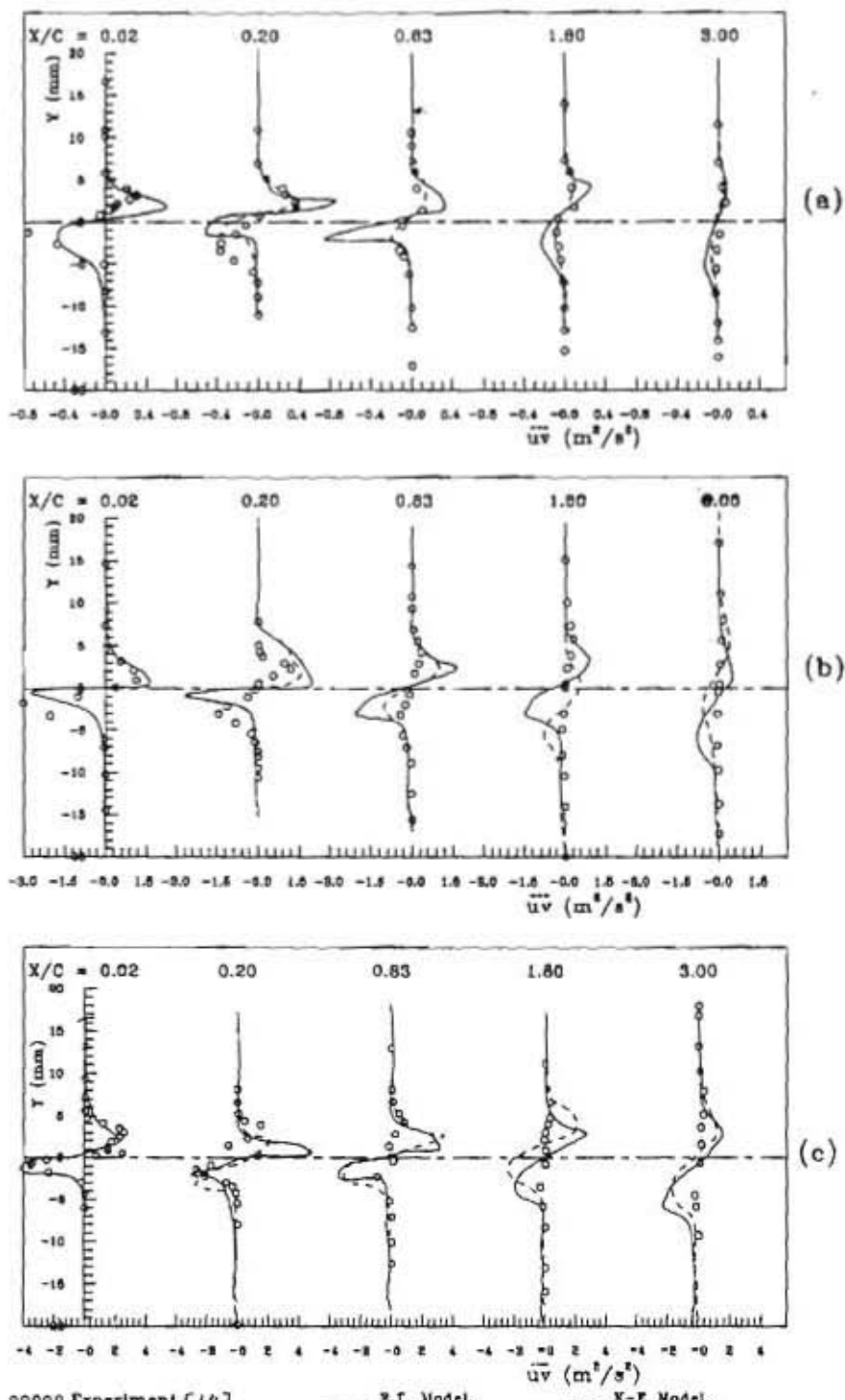


Figure (2) Comparison between the measured and predicted longitudinal fluctuating velocity profiles for NACA 0012 at $\alpha = 0^\circ$.
 a) $U = 10$ (m/s) b) $U = 30$ (m/s) c) $U = 50$ (m/s).



oooo Experiment [14] — Z.T. Model. - - - K-E Model.
 Figure 9) Comparison between the measured and predicted Reynolds shear stress profiles for NACA 0012 at $\alpha = 0^\circ$.
 a) $U = 10 \text{ (m/s)}$ b) $U = 30 \text{ (m/s)}$ c) $U = 50 \text{ (m/s)}$.

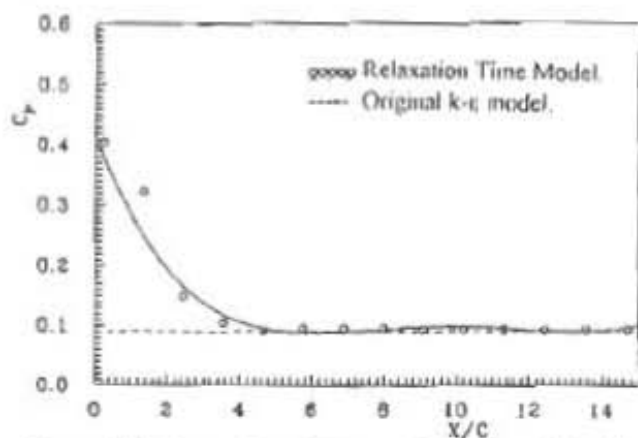
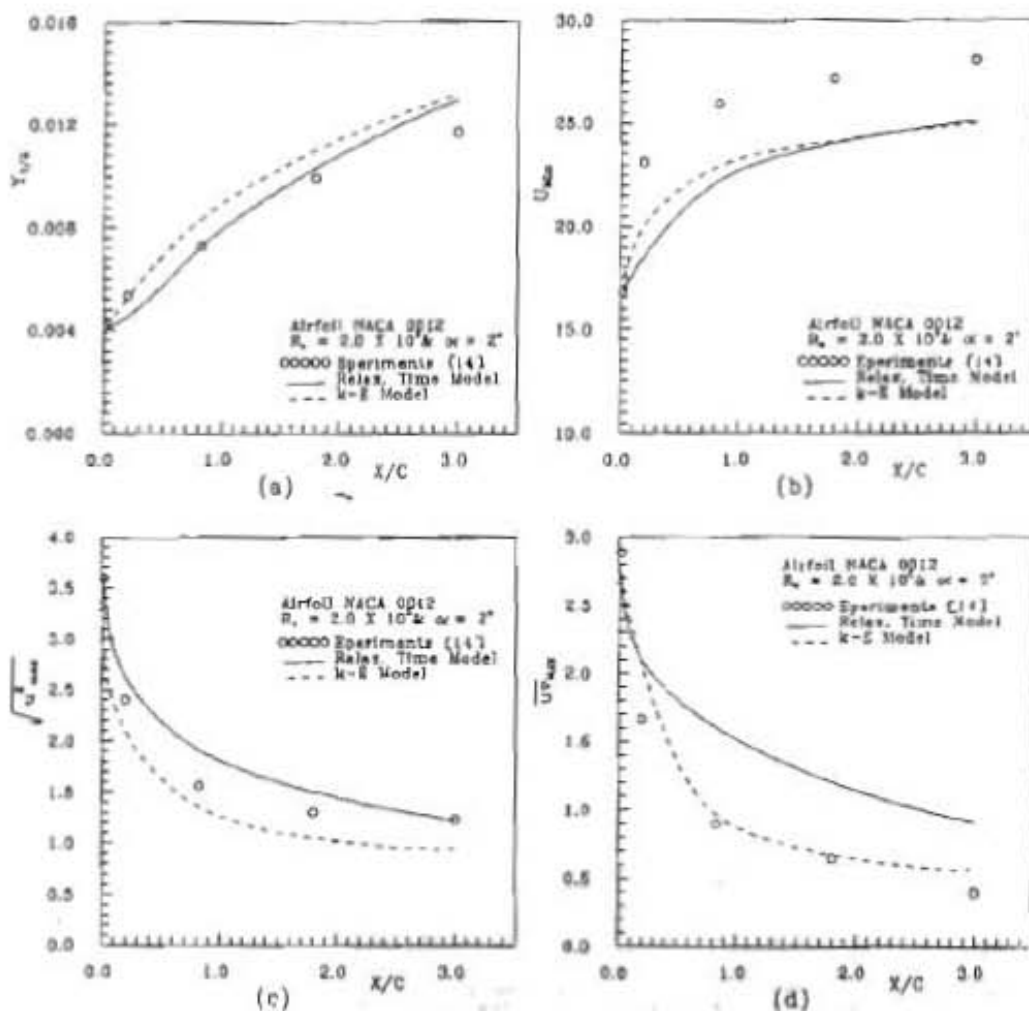


Figure (4) Comparison between C_p employed in the original and modified model.



Figure(5) Comparison between the measured and predicted wake characteristics:
 a) Wake half-width b) Minimum longitudinal mean velocity.
 c) Longitudinal fluctuating velocity. c) Reynolds shear stress.

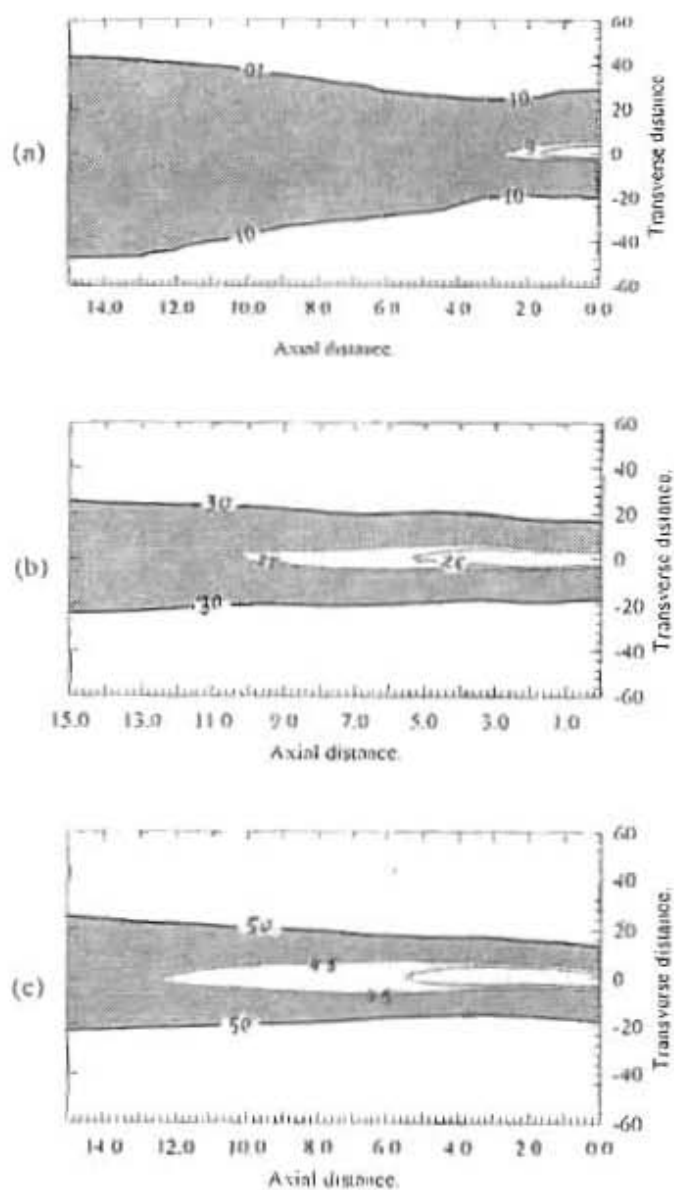


Figure (6) Predicted wake development with the downstream distance for airfoil section NACA 0012 at angle of attack $\alpha = 0$.
 a) $U = 10$ (m/s) b) $U = 30$ (m/s) c) $U = 50$ (m/s)

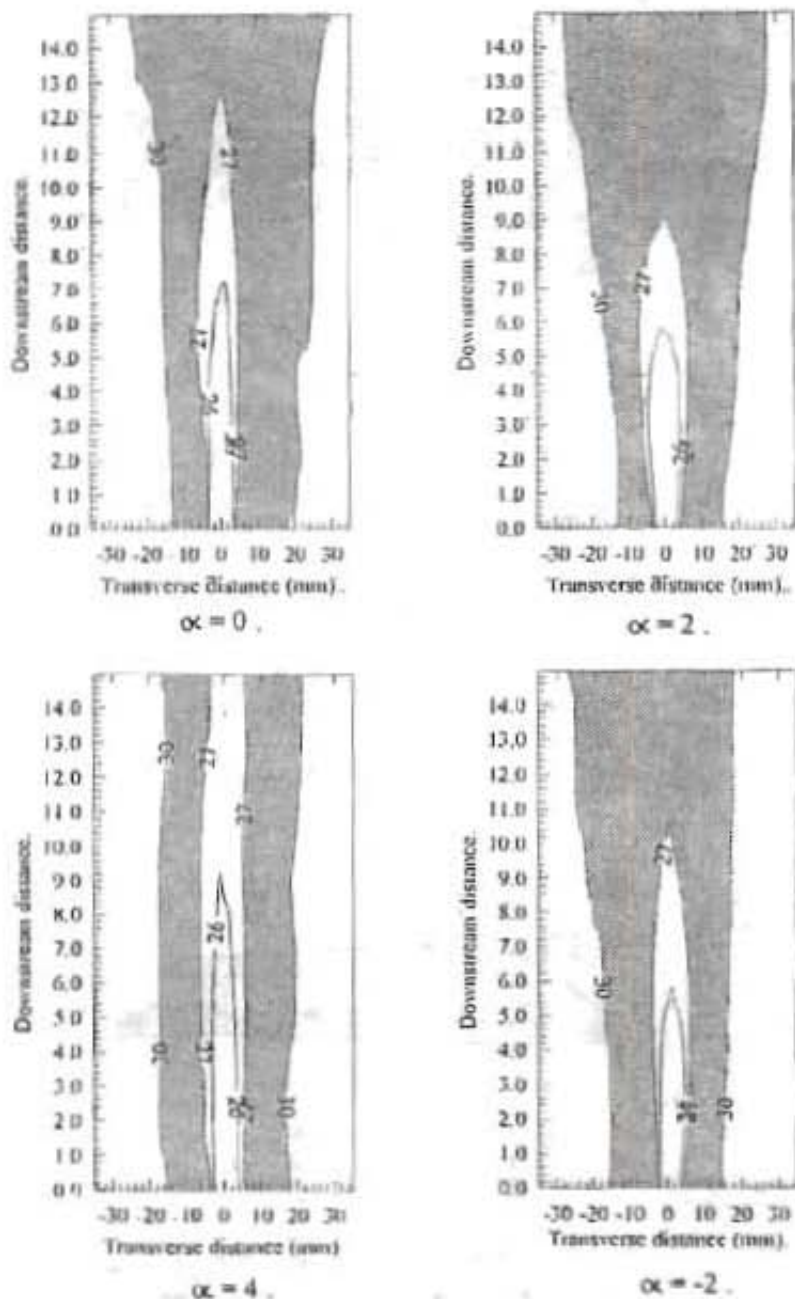
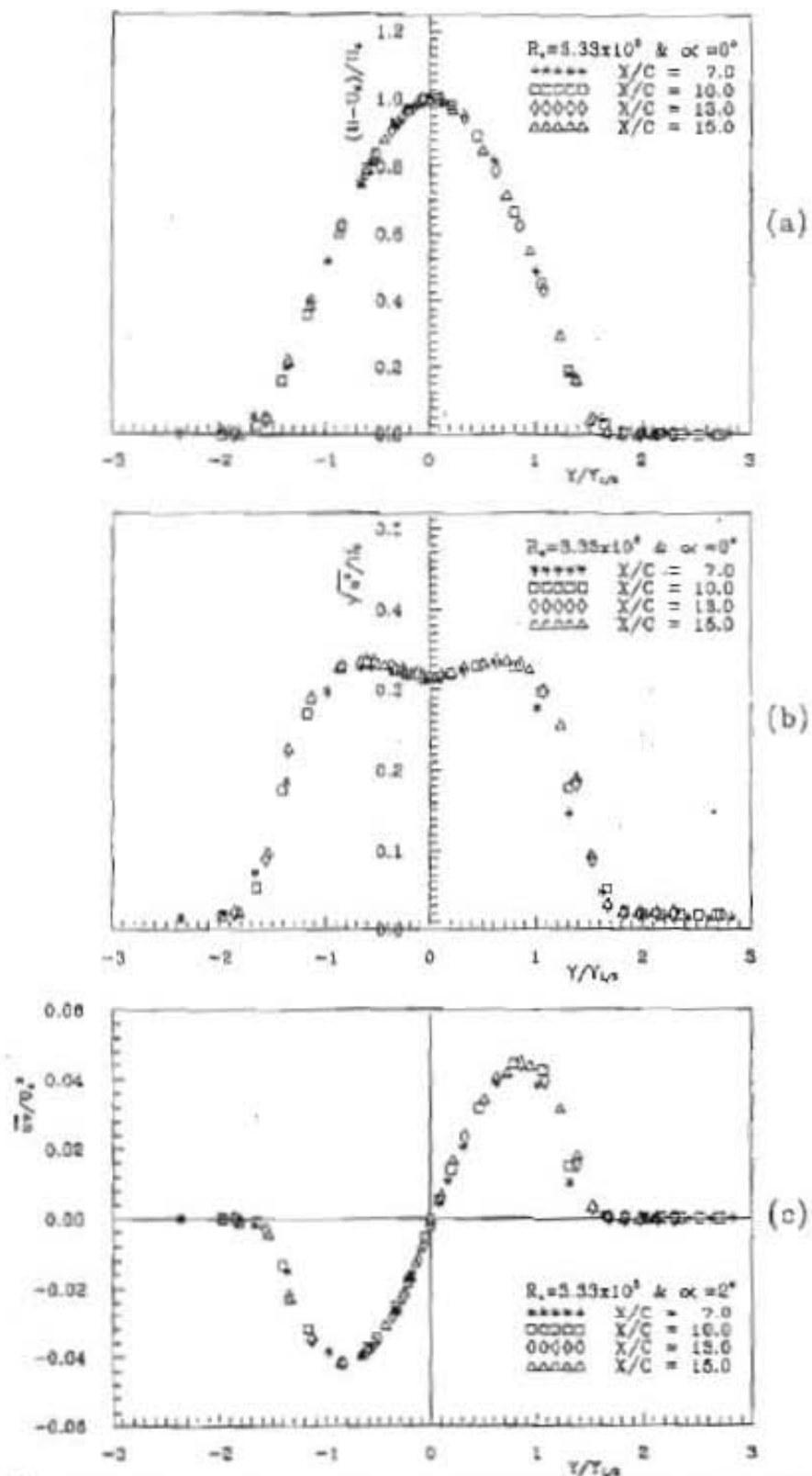


Figure (7) Predicted wake development with the downstream distance for NACA 0012 at $U = 30$ (m/s).



Figure(8) Self-preserving conditions for NACA 0012 at $U = 50$ (m/s):
 a) Longitudinal mean velocity. b) Longitudinal fluctuating velocity.
 c) Reynolds shear stress.

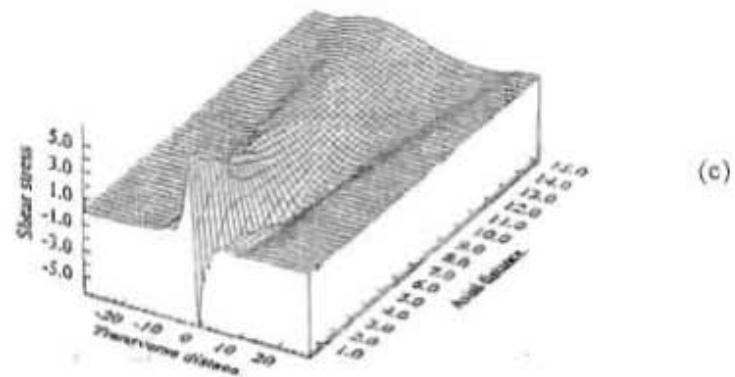
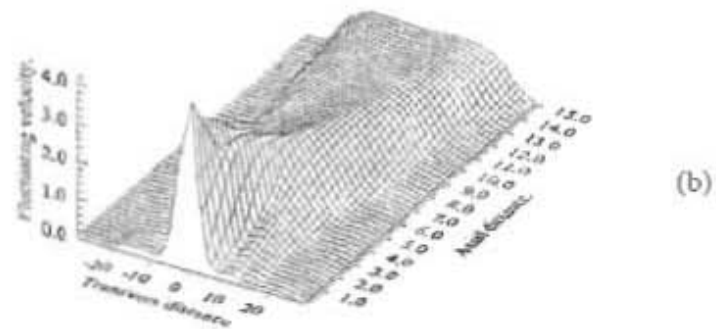
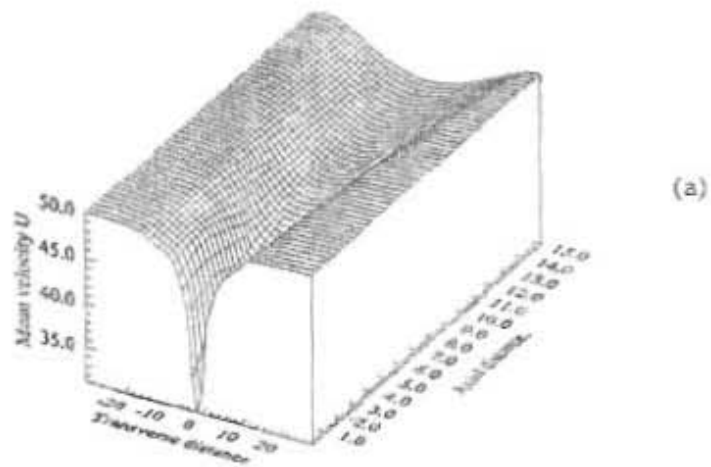


Figure (9) Response surface for the predicted wake characteristics for airfoil section NACA 0012 at $U = 50$ (m/s) and angle of attack = 2° .
 a) longitudinal mean velocity. b) longitudinal fluctuating velocity.
 c) shear stress.