THE FINITE ELEMENT CONTRIBUTION TO ELECTRICAL MACHINE DESIGN

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اسدام العتاصر المحددة فى تصميم الآت الكهربيـــــــة

الخلاصة _ هذا البحث محاولة لشرح كيفة استخدام طرق نحلـل المدال بـالعتاصر المحددة فــــــــــى
نصميم المحركات الكهربية كما بناقش البحث النتائج والخبرات من نطبيقات تحليل العـاصر المحـــــددة
للمحركات الكهربية لمشاكل المدال المغناطسى الخطة والغير خطة حى يكسب معلومات هصـــــــــم
معدة من هذه الطريقة التحليلية العددية ، والبحث بستعرض نطبيقات مختلفة نغل الثوابـــــــت
المغناطبسة ذارجة المدال المغناطسى للشمرة الهوائية ، حباب المضاهدة المعتاطسة لعـــ بـار
التسرب ، ودرامات تفصيلة لتوزيع المدال وتأثيراته . وهذا العمل بعطى مدلولات أداء وتصمـــــم
وأدوات حباية بنداعد المعدمين لتصميم محركات كهربة بمعدلات عالية قادرة على أن نواحه طـــــروف
التشغيل العادية والغير عادية بطريقة اقتصادبة بركن البها .

ABSTRACT

This paper attempts to describe how finite element field analysis methods are being employed
in the design of electrical machines. It discusses results and experiences obtained from
finite element analysis application to electrical machnies-for linear and nonlinear electric
and magnetic field problems- in order to gain useful design information from this numerical
analysis tool. It presents various applications covering magnetic constants describing the
air gap flux, slot leakage, leakage path permeance evaluation, and the detailed studies of
field distribution and its effects. This work produces design and performance data and comp-
utation tools which will help engineers to design highly rated electrical machines capable
of meeting normal and abnormal operating conditions reliably and with economy.

1. INTRODUCTION

Designers face an increasingly difficult task: to design electrical machines to high perfo-
rmance specifications, being light and small, but at the same time having high efficiency
(low energy consumption), being highly reliable and having a long service life.

In order to evaluate the performance with precision, one must accurately and in detail pre-
dict the electromagnetic fields produced under various operating conditions. Classical anal-
ysis methods and analog techniques have proved unsatisfactory except for simplified geometr-
ical structures and boundary conditions. Therefore, the need for numerical solutions was
recognized even in the nearly stages of the design art. Nevertheless, only the advent of
large-scale digital computers has enabled the development and extensive use of such methods
for solving the field distribution in electrical machines with great detail and accuracy.

The principal numerical methods currently in use fall into three categories, namely :
(1)divided difference schemes, (2)integral equation techniques, and (3)finite element
iques. Finite element analysis is one of the modern analysis and design tools which
ractical without the availability of high speed computers. The computation speed a
and the computation cost reductions of the past dacade have increased the usage of
element analysis rather dramatically. This has been quite evident from the numbe
publications in the area of electromagnetic analysis by means of finite elemen'

The paper presented is an extension to the autor's work[9]. It will initial'
of the basic design considerations and there after present selected result?
as well as for nonlinear analysis cases. It will be shown that the lines'
of use when the classical analysis can not accurately describe the field
The nonlinear analysis will give detailed insight into the effects of F

be a basis for better understanding of the magnetic field in electrical machines.

The proposed method of analysis has proved to be sufficiently accurate for design and is believed to offer considerable advantages in convenience and simplicity of application over the existing methods.

2. GENERAL CONSIDERATIONS

Electrical machine design deals with the synthesis of a complex mechanical structure which contains a 3-dimensional nonlinear magnetic field. The classical design method synthesizes the total solution from a set of idealized subroutines. This process has the advantage that the effects of various design details can be traced through the computation and allows to adjust the details of the design appropriately. One of its main disadvantages is that the sub-problem solutions do not consider the total picture. In addition even the sub-problems had to be idealized to facilitate closed form numerical solutions.

Numerical analysis techniques like finite elements carry the subdivision of the problem significantly further but solve all the sub-problems simultaneously. In case of first order subdivisions, the solution of the magnetic field in each element is constant throughout the element. However, carrying the level of subdivisions into sufficient detail will make the total solution of the field problem nearly an exact one. Hence, the accuracy of the finite element solution is directly related to the level of subdivision, especially in those areas where high field gradients are expected.

Finite element analysis gives the complete field solution at one instant in time and space. That also implies that for example the fundamental flux linked with the stator windings of an induction machine from a single solution plot is generally not the average a.c flux measured at the air gap with search coils. Rotor/stator position effects like slotting and slotting related saturation can only be found from comparing solutions for different rotor/ stator positions. In addition, if one wants to utilize finite element analysis results to improve the magnetic design constants used in the classical design method the problem arises how to reduce the complete solution to the specific design parameters. This work at times necessitates a review of the original parameter definition and a possible redfinition.

3. PRACTICAL ASPECTS

There are numerous difficulties which can arise when first applying finite element methods to rotating machine problems. It is frequently necessary to compromise, while maintaining an acceptable level of accuracy. The following points, some of which are obvious, might serve as a useful guide:

(1) Model planning, before building the model, careful thought should be given to all the alternative geometries, excitations and materials, to be studied. Some simplifying dec- isions have to be made to ignore unimportant details such as the insulation between iron and copper, in most of the electromagnetic studies. This obviously dose not apply to ther- mal problems. It is at this stage that high field strength regions should be anticipated and finely discretised accordingly. A special study has shown that a crude discretisation in the air gap and tooth tip region, can lead to overestimation of the gap mmf. This is mainly due to oversimplification of the tooth fringing field.

(2) Representation of thin layers, such as unwanted gaps at joins, insulation in thermal problems, etc. In order to avoid discretisation of a large number of elements, having bad aspect ratios, it is often possible, with care, to arbitrarily increase the thickness and the permeability, in proportion. This need not affect the field in the regions of interest. This is only permissible where the flux lines are mainly prependicular to the thin layer.

(3) Theoretically, slotted regions can frequently be modelled as homogeneous, anisotropic regions. In a tooth/slot region, where the flux lines are mainly radial, an equivalent magnetisation curve may be defined, where:
$$B_{eq} = B.(t-s)/t + \mu H.(s/t)$$

where B_{eq} : equivalent flux density over slot pitch

B : real flux density in the tooth   H : magnetic field intensity

s : slot width   t : slot pitch μ : permeability

If the slot sides are parallel, a good approximation can be obtained by using 't' at 1/3 of the slot depth from the narrowest end of the tooth. The above approximation can only be used where the main point of interest is not in the slotted region.

(4) Adjustment of axial length. The various parts of the machine have to be referred to a common reference length, since this assumption is inherent in any 2-D flux plot. The equivalent magnetisation curve is simply:

$$B_{eq} = B.(\text{ iron length/reference length })$$

This adjustment ensures that the mmf along the region is exact, but the B values have to be rescaled. The most convenient choice of reference length is the axial length of the air gap modified for ducts and fringing. This avoids having dummy air permeabilities different from unity.

(5) Boundaries. Many problems in the x-y plane have readily identifiable boundaries of flux lines or symmetry lines. Where this is difficult to achieve, within a pole pitch, because of a non-integral (no. of slots/no. of poles) value, it is possible to replace the actual slotting with an equivalent slotting having approximately the same number of slots as the original, but with an integral number per pole. The radial dimensions of the replacement should be the same as those of the real slotting, but the circumferential dimensions scaled to give the same proportions of iron to conductor at any radius(θ).

(6) Equivalent excitation. Equivalent slotting leads naturally to the idea of equivalent excitation. For example, for an a.c machine the analysis need only be conducted for the fundamental current loading.

(7) Leakage in a direction normal to the plane of interest can be allowed for by artificially increasing the permeability of the 2-D leakage path. This has to be done carefully, but experience has shown that it can be very successful.

(8) Checking of results. A visual inspection of a simple flux plot, can be an invaluable indicator of the correctness of material and excitation settings. Another obvious method is to check the mmf/(integral H dl) ratio. If the highly saturated regions are not properly discretised, this ratio can be far from unity. A plot of H along a critical path of high saturation, or small gaps, can be highly revealing. Flux density distribution curves can be useful in showing if field concentrations appear where expected. Their absence could be due again to poor discretisation. Care should always be taken to ensure that contour paths lie in a definite region, rather than on a boundary, where ambiguities arise. It is also useful to be able to check material properties and excitation levels.

4. STATEMENT OF PROBLEM

The basic problem of the flux distribution in the cross section of an electrical machine is solved by the following elliptic equation:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right) = -J_z$$

where
A_z : magnetic vector potential normal to the section

J_z : current density vector normal to the section

This equation is reformulated by Galerkin weighted residual procedure, and first-order triangular elements are used to discretize the field region. Applying the Galerkin procedure over each element in turn yields a set of linear simultaneous equations, which is solved using a preconditioned conjugate gradient method. The resulting magnetic vector potential are used to determine the slot leakage inductance, as explained in the Appendix.

5. APPLICATION EXAMPLES

To illustrate some of the points mentioned above, and to show the types of problem which may now be easily tackled, examples are given below.

5.1 LINEAR ANALYSIS EXAMPLES

It has been mentioned before that most emphasis is placed upon nonlinear finite element analysis. This of course is fully justified since nonlinear field distribution is most difficult to analyze using classical methods. However, it is felt that linear finite element analysis is quite useful also since it allows us to separate out the geometry effects quite

well. A simple example is the semiclosed slot shown in fig.(1). The classical analysis as
found in textbooks[10,11] defines the geometric boundary lines for the reactance calculation
as shown on the left. Area 5 is only considered for large air gaps. Finite element anal-
ysis can not handle the geometric boundary lines very well and the flux line boundaries
shown on the right of fig.(1) are the ones to be used for its analysis[2].

The finite element flux distribution plots for the single slot model, at different levels .
of excitations, are shown in fig.(2a&b). The study of flux plots for the single slot ana-
lysis, shows that, if one follows classical analyses methods, some of the flux entering the
tooth top in area 4 in case of small air gaps does not really represent leakage flux.
Finite element analysis results based upon the flux line boundary area subdivision will
show negative values for the area 5 for small air gap to slot pitch ratios reflecting the
above mentioned fact. Fig.(3) presents the total slot leakage inductance for a semiclosed
slot as a function of the slot opening to slot pitch ratio calculated from finite element
analysis and according to the textbook formulas. It is concluded that the difference
between the classical analysis and finite element analysis is mainly found in the flux
distribution in the slot opening area.

A second machine area to which finite elements can easily be applied relates to the slotting
effects. In the classical analysis and for the same excitation, the reduction of average
flux density in the air gap because of slotting is taken into consideration by the Carter's
coefficient. Carter's[12] analysis is based upon an infinite slot pitch and only considers
the average flux density. Finite elementanalysis has been shown to produce the same result
as Carter's analysis for the same cases. Fig.(4) compares the coefficient relating the max-
imum air gap flux density to the average value for single sided slotted air gap as calculated
from Carter's[12] and finite element analysis. Carter considers basically a very deep slot.
Finite element analysis can take any slot depth. The finite element analysis has been util-
ized to study the effects of the slot opening to slot pitch ratio and the air gap length to
slot pitch ratio upon the average air gap flux density coefficient and the fundamental flux
density coefficient (base=maximum flux density in gap) as shown in figures (5) and (6).
At large relative slot openings and air gaps one has to define what constitutes the maximum
flux density in the gap since the radial flux density component changes appreciably as one
traverses from the rotor to the stator snrface.

5.2 NONLINEAR MACHINE ANALYSIS

As has been mentioned before the greatest value of the finite element method lies in its
capability to accurately predict magnetic fields in a structure which contains nonlinear
magnetic materials. This is an area in which both the classical analysis and the imagin-
ative faculty of the mind easily lead to misconceptions. At first, the single slot anal-
ysis shall be discussed again.

The same slot as shown in fig.(1) has been analyzed with saturable iron of finite initial
permeability. The first thing to be noted is a reduction in leakage inductances as compared
to the previous analysis with constant infinite permeability even at unsaturated conditions.
This is obviously due to the fact that a certain amount of magnetisation is necessary to
drive the flux through the iron. In other words the slot walls are not equipotential lines
anymore. This effect becomes stronger as saturation increases. The inductances calculated
as before are shown in fig.(7) as a function of the excitation in the slot. It is easily
found that the main reason for the decrease of the leakage inductance is the saturation in
the tooth.

This behavior suggests that the single slot analysis presented only models the conditions
for a machine at no load/or light load at the point of maximum main flux density. It also
points right at the dilemma with this analysis. If we want to model the slot flux distri-
bution correctly, we have to do that for several operating conditions and with boundary
conditions which introduce the correct saturation distribution.

Under idealized short circuit conditions a different set of boundary conditions is necessary
since practically no flux crosses the air gap. Unlike Amin[9] this problem is solved by
adjusting the boundary conditions such that only cross flux exists. This is the situation
at the center of the phase belt. For the studies reported on here a slightly different set
of boundary conditions has been investigated which closely represents the situation at the
phase belt edge, where again no flux crosses the air gap but all flux has to come down the

Unslotted stator / rotor
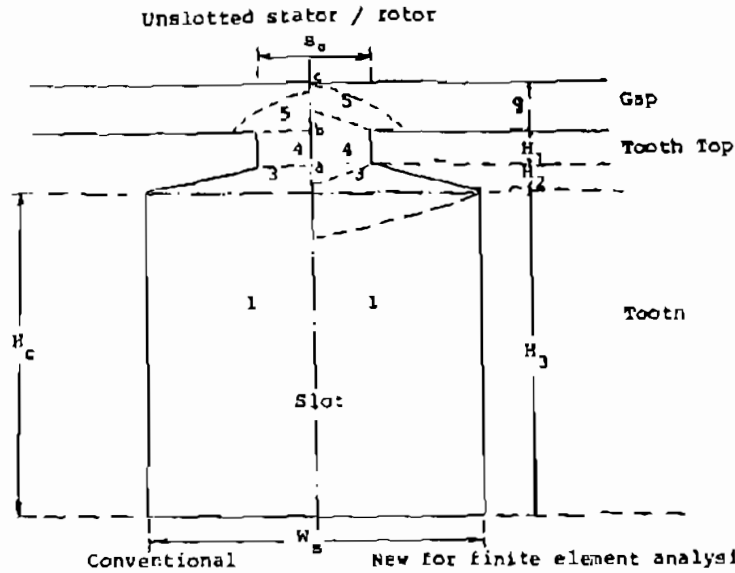
Definition of inductances left hand
areas for geometric boundaries:

1 conductor space
3 overhang space

4 slot opening space
5 tooth top space

Right hand areas for flux line
boundaries:

1&3 conductor space & overhang space
4   slot opening space
5   tooth top leakage flux space

Fig. (1) Subdivision of slot permeance calculations
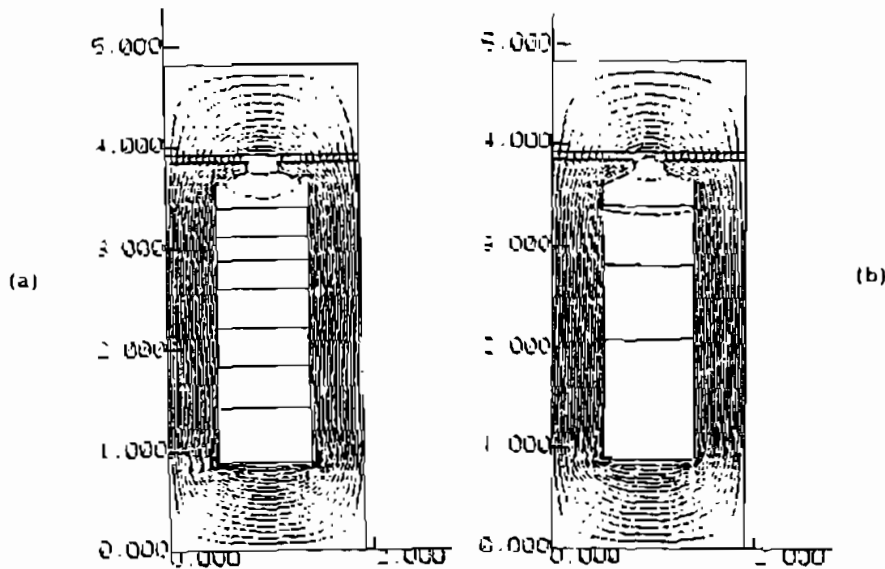

(a)

(b)

Fig. (2) Flux distributions for single slot model at
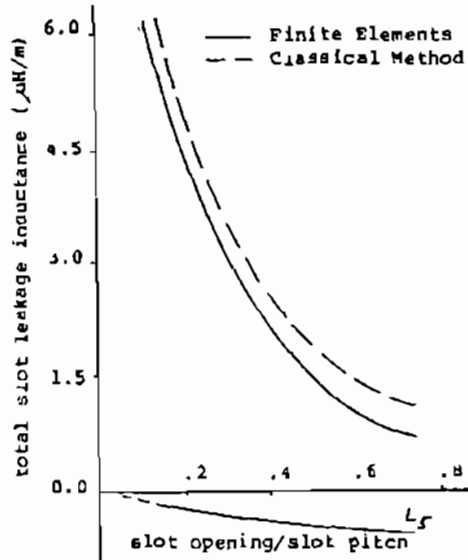different excitations a. at 20 A. b. at 5 A.

Fig.(3) Total slot leakage inductance versus slot opening to slot pitch ratio
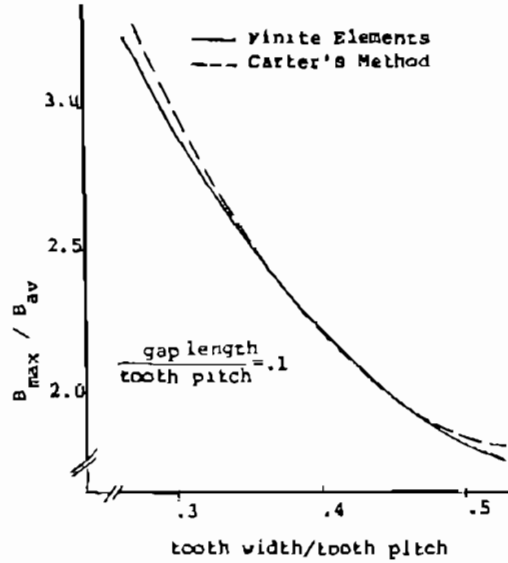
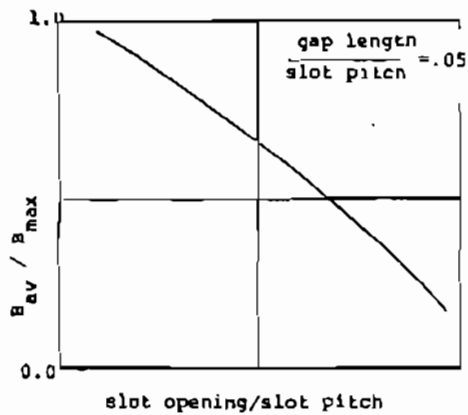Fig.(4) Ratio of maximum to average flux density for single sided slotted air gap

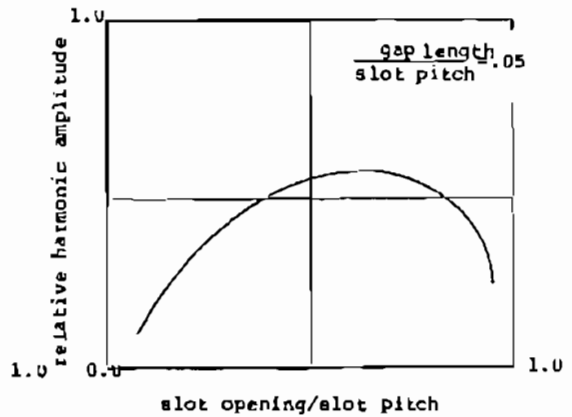Fig.(5) Average to maximum flux density ratio for single sided slotted air gap

Fig.(6) First slotting permeance harmonic versus slot opening to slot pitch ratio

shalt of the tooth. Fig.(8) shows the slot leakage inductance under these conditions as a
function of slot excitation. Also shown is the slot leakage inductance for the same slot
under no load conditions. The fact that the inductance at no load even under unsaturated
conditions is significantly smaller than at ideal short circuit can only be explained by
the fact that because of the small air gap a significant portion of the flux from the tooth
top crosses to the other side of the gap, while under short circuit conditions it is not
permitted to do so. The same reason also holds for the saturation effect: because of the
small air gap the tooth starts to saturate at much lower slot excitation levels than those
necessary to drive the tooth overhang and tooth proper into saturation at the short circuit
case.

Design of electrical machines has to consider over-excitation conditions. Any good text-
book will point out that once the teeth start saturating part of the main flux travels
through the slot space rather than through the saturated teeth or in other words, the total
main flux is larger than the sum of the tooth fluxes under one pole pitch. This case can
readily be investigated with finite element analysis of the single slot. Fig.(9) illustr-
ates how the flux in the slot space, ϕ_a, varies with the average flux density in the slot
pitch. Again this is a highly nonlinear relationship and additional eddy current losses
have to be expected in the conductors in the slot. The results shown in fig.(9) can be
translated into a flux versus ampere turns curve for one tooth pitch as shown in fig.(10).
These particular results are for a parallel sided tooth with a tooth width to tooth pitch
ratio of 0.33 . If narrower teeth are used, the contribution of the air space becomes
more significant.

6. DISCUSSION AND CONCLUSION

Several different applications and results of finite element analysis have been discussed
in the previous two sections. As one has to expect, the linear analysis has yielded more
immediately useful results since the superposition principle is still valid. As has been
shown, linear finite element analysis is quite useful in cases where the boundary condition
are too complex to be handled by closed form mathematical solutions. In the past, methods
of estimating the parameters and graphical methods have been employed to obtain useful
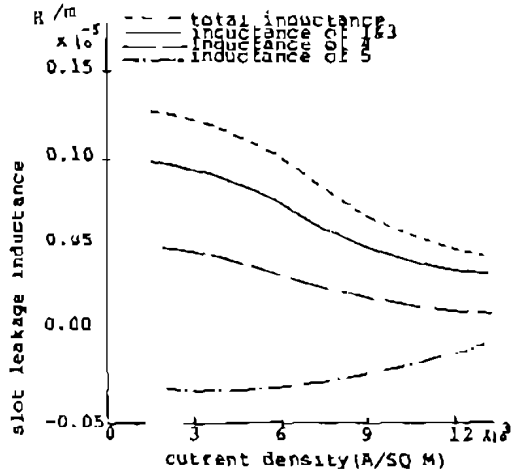values.

Fig.(7) Slot leakage inductance versus
        current density

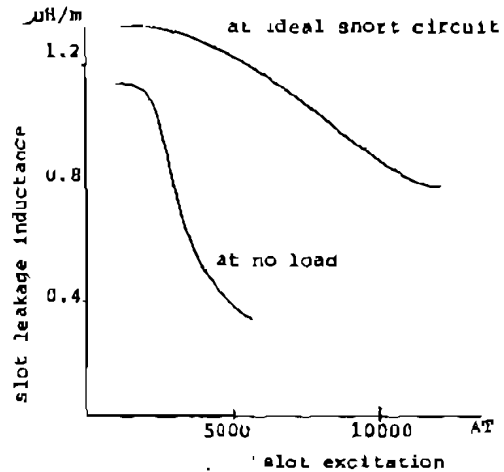Fig.(8) Slot leakage inductance versus
        ampere turns

However, as the level information to be obtained becomes more sophisticated, e.g., the distribution of the flux, determines the value of the parameter, the classical methods tend, to become less reliable.

The first message from the nonlinear analysis is that for a real machine most of the design constants will be more difficult to establish. The obvious reason is that events which the designer needs to look at in a decoupled fashion are often closely coupled through the saturation in the machine. This is quite evident from the different results for the slot leakage inductances. It is clear from these results that more work is necessary to obtain meaningful design data. The results also make it clear that any method to separate the various effects chosen in an approximation of the real case and as such has to be verified by tests as to its range of validity.

On the other hand, the field investigation has been proven quite useful and interesting. It has become clear that any effort to identify and avoid parasitic losses in any electrical machine will greatly benefit from the nonlinear finite element analysis of the magnetic field distribution, and even though no final numbers have been obtained for the slot leakage permeances, the studies have indicated where tooth saturation will occur and how it will effect the slot leakage permeance.

The work reported in this paper is part of an ongoing project to extract useful design data from finite element analysis. Although only two-dimensional finite element fields has been gained and some useful data obtained. The work has also shown what approaches have to be taken to make this numerical analysis tool more useful for electrical machine design.
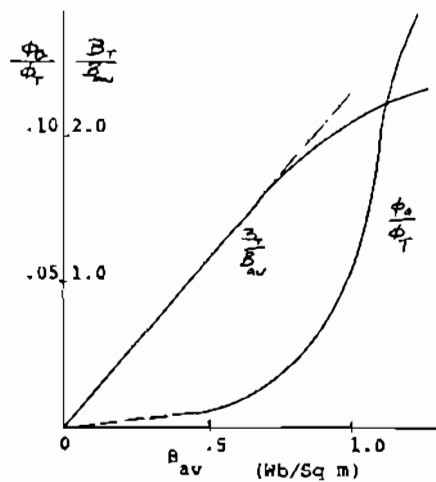

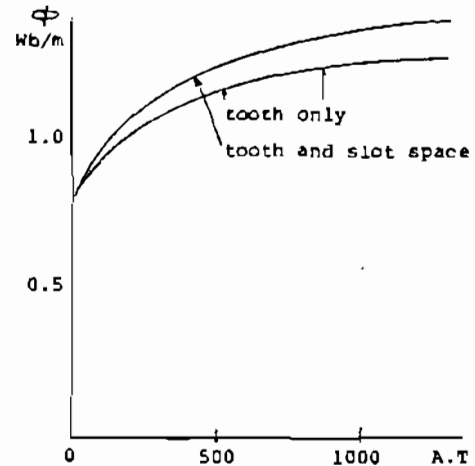Fig.(9) Flux density and flux distribution in a slot pitch versus average slot pitch flux density

Fig.(10) Magnetization curves(flux & ampere turns) for one tooth pitch(tooth width=½ tooth pitch)

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Appendix : Leakage inductance calculation

As in traditional flux leakage calculation methods, it is convenient to partition the leakage flux into separate components as follows:

a)Slot leakage inductance: It may be evaluated by calculating the energy stored in the slot and equating it to the energy stored in an equivalent inductance. Total magnetic energy stored

$$W = \frac{1}{2} \int J \, A \, dv$$

Then, the slot leakage reactance

$$X = 8\pi \cdot frequency \cdot W/I^2$$

b)Slot top leakage inductance: It is evaluated from the flux that passes over the top of th conductor, through the slot opening, i.e that passes between a and b(fig.(1))

$$L_{top} = Real(A_a - A_b) / I$$

c)Air-gap leakage inductance: It is determined from the flux that passes over the top of the slot, through the air gap, i.e that passes between b and c(fig.(1))

$$L_{gap} = Real(A_b - A_c) / I$$