

Computing Efficient Trajectories for Unmanned Air Vehicles

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احتساب المسارات الأكفأ للطائرات بدون طيار

إن التطور السريع في تصميم واستخدام الطائرات بدون طيار سيجعل استخدامها في مهام هجومية أمراً واقعاً في المستقبل القريب. ومن التحديات الكبيرة التي يتعين حلها في هذا الصدد هو تخطيط المسار الأمثل لها أثناء الطيران. ولقد تم توظيف طرق مماثلة لتلك المستخدمة لتخطيط المسارات للروبوتات لحل هذه المشكلة، وعلى سبيل المثال تقسيم الخلايا و خريطة الطرق والقوى الافتراضية والبرمجة الخطية. وفي هذه الطرق يتم تكوين دالة التكلفة من كلاً من طول المسار و التهديد الواقع عليه مع قيمة الترجيح ومقدار الأولوية لكل منهما. ويكون المسار الأمثل هو الذي لدالة تكلفته أقل قيمة. وبالرغم من أن دالة التكلفة هذه تحتوي على كلاً من عنصري التكلفة (الطول والتهديد) إلا أنها تخفى مصدراً للخطأ مما يترتب عليه أن المسار الأمثل الناتج عنها ليس هو الأمثل فعلياً، كما أنه يحتوي على تكلفة تهديد كبيرة وقد يكون غير آمن بالمرة، وهذه النتيجة ترجع إلى ما يلي:

- أن تكلفة طول المسار هي السائدة على دالة التكلفة حتى مع إعطاء أولوية وقيمة ترجيح أكبر لتكلفة التهديد.
- أن التهديد المحسوب في دالة التكلفة هو الناجم عن مواقع الرادارات المعادية أي احتمال النقاط المركبة بواسطة هذه الرادارات بينما التهديد الناجم عن مواقع الصواريخ المضادة لها (SAMs) غير مدرج في دالة التكلفة.
- قيم الترجيح المعطاة قد تكون مناسبة لقيم الطول والتهديد لأحد الأرجل المكونة لمسار ما بينما لا تكون مناسبة لقيم باقي الأرجل بنفس المسار و المسارات الأخرى.

ويقدم هذا البحث خوارزم مقترحاً لاحتساب المسار الأمثل اعتماداً على دالة الملازمة، حيث تعطي هذه الدالة المقترحة تأثيراً متوازناً لكل من ملازمة طول المسار وملازمة قيمة التهديد، وبالإضافة إلى ذلك فإن قيمة التهديد تتكون من شقين هما: الرادارات وال SAMs وليس الرادارات فقط. ويقوم هذا الخوارزم بنفسه باحتساب قيم الترجيح المناسبة والتي توجه دالة الملازمة ناحية عنصر التكلفة ذو الأولوية الأعلى. كما أن هذا الخوارزم يوفر المرونة المناسبة للمركبة لتعديل القيم الأولية المعطاة لاحتساب مسارات مثلى أخرى من أجل تمكينها من الالتزام بتوقيت وصول متزامن مع بقية أعضاء الفريق كل إلى هدفه.

Abstract

Rapidly developments in the design and implementation of autonomous Unmanned Air Vehicles (UAVs) make the use of them in combat missions a real fact in the near future. One of the challenging problems yet to be solved is the real time planning of the optimal trajectory. Techniques used for robot motion planning are implemented for UAV e.g. cell decomposition, road maps and virtual forces. In these techniques a cost function consists of the threat cost and the length cost with priority factors and weighting quantities for both. The optimal trajectory (or timed path) is that producing the minimum cost function. Although this cost functional is composed of the two cost elements (length and threat) it conceals a source of error that produces optimal paths with high threat values i.e. not actually optimal and not safe. These results return to: a) the length cost dominates the effect on the cost function even with large weighting and priority for the threat cost. b) radar threats or probabilities of being detected by adverse radars are only considered, while threats of Surface to Air Missiles (SAMs) are not included in the threat function. c) weighting quantities given may be suitable for costs of some path edges and not suitable for other edges of the same path and other paths.

In this paper a proposed algorithm to compute the real efficient path depending upon a proposed fitness function. The function gives a balanced domination for both length fitness and threat fitness. Besides, the threat on it consists of the two components: radars and SAMs. The algorithm autonomously computes the proper values of the weighting quantities that guide the fitness function towards the higher priority cost element in case of time constraint. It also gives the vehicle a necessary flexibility to adjust the pre-given priority factors in order to compute other optimal paths that fulfills a rendezvous time with the other team members.

List Of Symbols:

- $J_{total(p)}$ the total cost of a path p .
 $J_{L(p)}$ the length cost.
 $J_{r(p)}$ the radars threat cost.
 $J_{s(p)}$ the threat on the path due to a SAM site.
 $J_{th(p)}$ threat on path p due to radars and SAMs.
 $J_{L(av)}, J_{th(av)}$ the average length costs and threat costs of all feasible paths.
 K_L, K_r the priority factors for length cost and threat cost.
 k the number of the edges of the path.
 L_i the length of the edge i of a path.
 d_r the distance from the vehicle position to a radar site.
 N_r the number of engaged radar sites
 $d_{(1/6)_{Lj}}^4$ the distance from the $1/6^{th}$ point on i_{th} edge to the j_{th} radar.
 $d_{(1/2)_{Lj}}^4$ the distance from the $1/2^{th}$ point on i_{th} edge to the j_{th} radar.
 $d_{(5/6)_{Lj}}^4$ the distance from the $5/6^{th}$ point on i_{th} edge to the j_{th} radar.
 Q_L, Q_r weighting quantities for length and radar threat.
 Q_{th} Weighting quantities due to Radar and SAMS together.
 d_s the distance from UAV position and SAM site.
 r_s the minimum radius of the safe circle.
 n_p number of feasible paths.
 $F_{L(p)}, F_{th(p)}, F_{total(p)}$ the fitness of the length cost, threat cost and total cost.

1. Introduction

UAVs (sometimes called air robots) have evolving roles in the current time and the near future. They have civilian applications such as: disaster relief, environmental monitoring, weather reconnaissance, search then rescue during wildfire incidents, and planetary exploration. They have many useful military applications, including reconnaissance, search

then destroy and search then rescue missions in hazardous environments such as battlefields or disaster areas. Recently, there has been a considerable interest in the possibility of using large teams (swarms) of UAVs functioning cooperatively to accomplish a large number of tasks. This necessitates that these vehicles are behaving autonomously, giving rise to the search area of unmanned autonomous vehicles. These are usually seen as rather simple vehicles, acting cooperatively in teams to accomplish difficult missions in dynamics, known, hazardous environment.

The potential advantages of UAVs over manned aircrafts are significant and motivate the development of advanced UAV technologies. The important affecting factors are [1-3]:

1. *Altitudes*, greater than 60,000 ft increasing survivability.
2. *Endurance*, its value is primarily in the economics of fleet size necessary to accomplish a specific mission.
3. *Reliability*, flight management system (including onboard flight control, communication links, and ground station support) is the primary contributors in this factor.
4. *Storability*, this requires a vehicle for fast surge response.
5. *Maximum Maneuverability*, the maneuverability of a UAV could fast exceed that of a manned aircraft.
6. *Low Risk To Human Operators*, UAVs are suitable for missions where the risk to pilots would be deemed unacceptably high.
7. *Significant Weight Savings*, because there is no pilot or cockpit in a UAV. Superior *Coordination*, it is the cooperation in which the actions performed by each agent take into account the actions executed by the other agents, and UAVs are Cost Effective, there cost is significantly low compared to their manned counterpart.

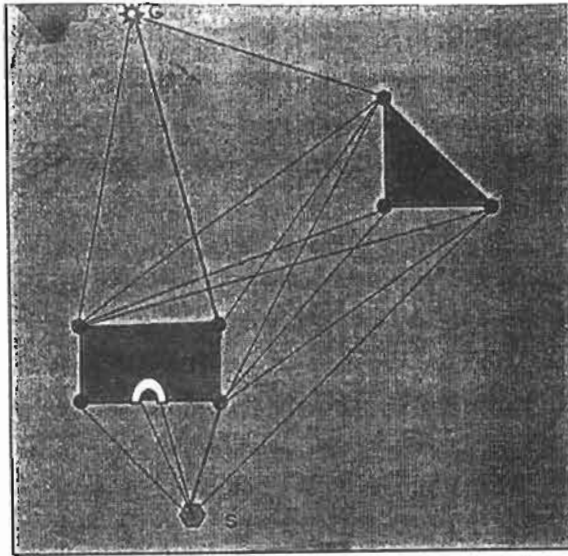


Fig. 1 Road Map Paths

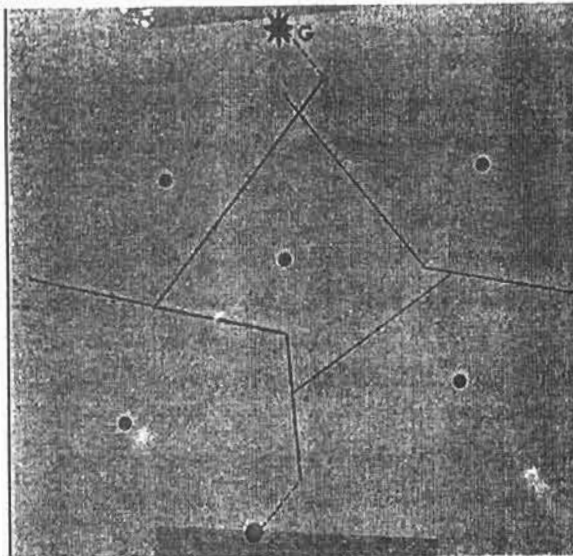


Fig.2 Paths by Cell Decomposition

One of the most challenging problems is the Trajectory Planning (TP) for a single and multiple vehicles. There are some proposed methodologies reported for TP. They can be categorized as : cell decomposition [4-11], road map [12-16], virtual forces [17-19] and linear programming techniques (integer and mixed integer) [20-21]. Figs. 1 and 2 show the first two of these techniques. A suitable algorithm (e.g. Dijkstra, A*, breads first, depth first, etc.) is used to search for the optimal path from all feasible paths. This is the path which has the minimum cost.

The cost function used in road maps and cell decomposition consists of the two elements: Length cost and radar threat cost [23, 24] is given as:

$$J_{total(p)} = K_L J_{L(p)} + K_r J_{r(p)} \quad (1)$$

Where $K_r = 1 - K_L$

This cost function conceals a source of error that gives continuous domination for the length cost regardless what the threat value is. Besides, the only threat considered is the possibility of being detected by the adverse radars. Consequently, the optimal path designed using this cost function is actually not the optimal one, and may be not safe at all.

In this paper, a proposed algorithm to compute an efficient path for UAVs. This path is actually the most efficient one in the sense of cost elements and priorities. A second advantage is that it takes the threat of the SAM sites and radar sites into consideration. Third, it gives the mission planner (and the autonomous vehicle) a chance to assign more priority for trajectory threat cost over its length or vice versa. This enables the vehicle to cooperate with the other team members autonomously by choosing a path that satisfies the rendezvous time. Forth, the algorithm computes the suitable values for the weighting factors (Q_L , Q_{th}) depending upon the priority factors.

2. Conventional Optimal Trajectory

Each path from the vehicle starting position to the target position consists of a number of path edges connected together to form a path (Fig. 3). The path length cost is the sum of its edge lengths i.e.

$$J_L(p) = \sum_{i=1}^K L_i \quad (2)$$

The threat on a vehicle at a specific point from a radar site is the probability of being detected by that radar. It is inversely proportional to the forth order of the space between them [20, 21].

$$J_r(p) = 1/d_r^4 \quad (3)$$

To compute the threat from a radar site on a path Eqn. 3 has to be integrated along the path length. This integration costs heavily, computation wise and is not feasible for a UAV, because this process has to be done in real time while the vehicle is flying.

To avoid integration in real time, a simplified method is widely used by computing the threat at three points only on the path. i.e. (1/6, 1/2, 5/6) of the path length, [19,23-26]. Threat costs are based on a UAV's exposure to adversary radars. Since the strength of a UAV's radar signature is proportional to $1/d_r^4$. The threat cost for traveling along an edge of path is proportional to the inverse of the distance to the fourth power as explained in Eqn.2.

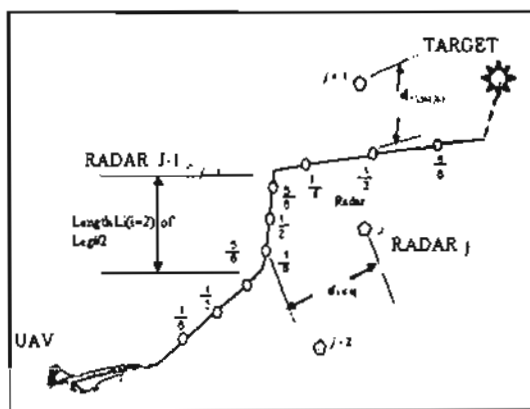


Fig. 3 Radar Threat Cost

The threat cost associated with the i^{th} edge is given by the expression:

$$J_{th}(p) = \left(L_i \sum_{j=1}^{N_r} \frac{1}{d_{(i/r),j}^4} + \frac{1}{d_{(i/r),j}^4} + \frac{1}{d_{(s/r),j}^4} \right) \quad (4)$$

A cost function is formed from these two crucial elements: the path length J_L and the threat J_{th} that a vehicle is subjected to. The two elements have the same priority and same weight []. The mission planner may give more priority and more weight for one element over the other. Minimizing the threat cost increases the safety of the vehicle and the possibility of mission success, while minimizing the path length, minimizes the fuel consumption and the flying time.

$$J_{total}(p) = Q_L K_L J_L(p) + Q_r K_r J_r(p) \quad (5)$$

The planning algorithm computes the cost functions and searches for the minimum one. Its path is the optimal path for vehicle U to its target T.

A major problem here arises when adding the large values of lengths to the fraction numbers of threats value (the threat is $\propto 1/d^4$). Eqn. 5 eliminates the effect of the threat value, and the length factor is dominating. ($Q_{th} = Q_r$ because the SAMS Are not considered in this conventional technique) Using a weighting factor Q_r and Q_L does not solve the problem because it may fit for some values of threats but not fit for other values. The net result is that the optimal path computed is not the optimal one. It may not be the safest one even with priority and weighting are given to the threat cost. this is detected numerically in the following example.

Example 1: A vehicle U has three feasible paths (path P_1 , path P_2 , and path P_3) to its target T Fig. 4. The length values of these paths are 400,200,300 (nautical miles) respectively, while the values of the radar threats are 0.001,40, and 2 as shown in table 1. From the given values it is clear that the path with *minimum threat cost* is P_1 , while the path with *minimum length cost* is P_3 . Using Eqn. 5 the results are summarized in table 1.

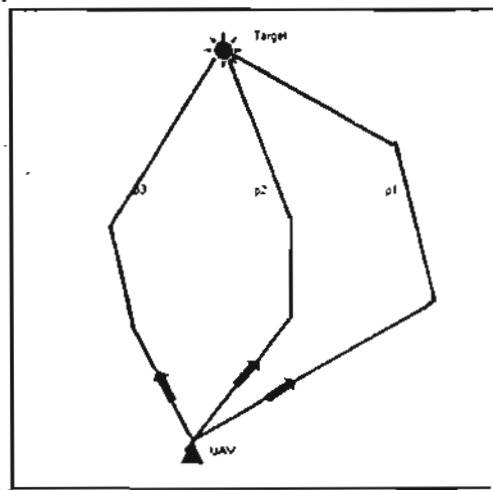


Fig. 5 Three Different Paths with Different Costs

Case A: Both length and threat costs have the same priority factors ($k_L = k_{th} = 1$), and the same weighting quantities ($Q_L = Q_{th} = 1$). The optimal path which has the minimum total cost function is P_2 ($J_{total} = 240$).

Case B: Giving more priority to the threat cost over the length cost ($k_r = 0.8, K_L = 0.2$), with the same weight ($Q_r = Q_L = 1$), The minimum cost function is path P_3 ($J_{total} = 616$).

Case C: Although the threat cost are given more priority than the length cost ($k_r = 0.8, K_L = 0.2$), the threat weighting is 10 times the length weighting. The minimum cost is also path P_3 .

Table 1 Values of the Cost Functions

	$L_1 = 400$ $J_{r1} = 0.01$	$L_2 = 200$ $J_{r2} = 40$	$L_3 = 300$ $J_{r3} = 2$	Notes
Case A J_{total}	400.01	240	302	$K_r = 1, Q_r = 1$ $K_L = 1, Q_L = 1$
Case B J_{total}	80.008	72.0	616	$K_r = 0.8, Q_r = 1$ $K_L = 0.2, Q_L = 1$
Case C J_{total}	3200	1608	2400.2	$K_r = 0.8, Q_r = 10$ $K_L = 0.2, Q_L = 1$

It is clear that all the three results are not producing the least threat path which is p_1 (its radar threat is 0.01).

3. The Proposed Algorithm

A UAV is subjected to different types of threats from soft weapons like jammers and decoys or hard weapons such as: shoulder launched homing weapons, radar directed guns, SAMs and early warning radars. The UAV is equipped with anti-jamming and anti-decoying sets. Flying at altitude above 5000 ft defeats most radar directed guns and above 15000 ft defeats most shoulder launched homing weapons. SAMs are of different types of equipment, namely small range medium range, large range and long-range fire control sensors. Fire control sensors work as tracking and sensing tools, it doesn't have any destructive capability [27].

It is clear that using the conventional methodology to compute the optimal path results in paths that are not optimal and do not

have the minimum threat regardless of the weighting factors or priority factors. Also, the only threat considered is that of radar sites. In the following an explanation of a proposed algorithm to compute a real cost function that considers for other threats.

- 1) The radar threat J_r is computed by Eqns.3 and 4 for every path.
- 2) Every SAM site has a specific range circle. A safe circle with a diameter greater by a safety factor than that of the range circle is formed as shown in Fig. 6. The path must be away from the circumference of a safe circle. It will be beyond the SAMs range. If a path is passing over a safe circle of any SAM site, this path is checked and dropped by the trajectory planning algorithm, before searching the optimal path. In this case this unsafe path is no longer considered.

These safe circles have different diameters depending on SAMs range, however the paths close to it is still subjected to threats from these SAMs due to sources of errors (e.g. sensors error, time delays, position estimation errors, etc). This threat is computed as [27,28]:

$$J_s = \begin{cases} \frac{1}{(1+d_s^2)} & \text{if } d_s \leq r_s \\ = 0 & \text{else where} \end{cases} \quad (6)$$

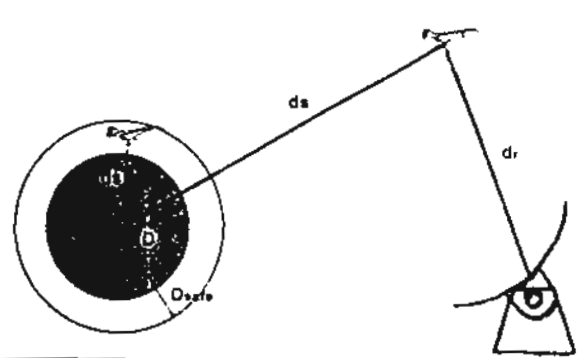


Fig. 6 Considered Distances from SAMs And Radar Sites

- 3) The threat cost of a path p is composed of two parts: radar threat J_r of all radar sites N_r and the other one is SAMs threat J_s of the

specific site the vehicle is turning around or tangent to it.

$$J_{th(p)} = J_r(p) + J_s(p)$$

$$J_{th(p)} = L_1 \sum_{j=1}^{n_p} \left(\frac{1}{r_{(U, \theta_{ij})}} + \frac{1}{r_{(U, \theta_{2j})}} + \frac{1}{r_{(U, \theta_{3j})}} \right) + \left(\frac{1}{1+d} \right) \quad (7)$$

4) From all feasible paths for vehicle U to target T the average path length is:

$$J_{L(av)} = \left(\sum_{p=1}^{n_p} J_{L(p)} \right) / n_p \quad (8)$$

5) The average threat subjected on all feasible paths from U to T is:

$$J_{th(av)} = \left(\sum_{p=1}^{n_p} J_{th(p)} \right) / n_p \quad (9)$$

6) Every considered path from U to T has a fitness value. For the length cost this fitness is computed as:

$$F_{L(p)} = J_{L(av)} / J_{L(p)} \quad (10)$$

The shortest path will have the highest length fitness $F_{L(p)}$. Similarly, the fitness of the threat cost of each path is:

$$F_{th(p)} = J_{th(av)} / J_{th(p)} \quad (11)$$

7) The weighting values Q_{th} and Q_L are neither given by the mission planner nor fixed for all paths. If priority is given to a certain cost element its corresponding weighting has a considerably high value. The algorithm computes the suitable weighting quantities for each path as:

$$Q_{av} = \frac{1}{2} \left(\sum_{p=1}^{n_p} J_{L(p)} + \sum_{p=1}^{n_p} J_{th(p)} \right) \quad (12)$$

The weighting quantities for that path Q_{th} and Q_L are computed as:

$$\left. \begin{array}{l} - \text{ If } K_L = k_{th} \text{ then } Q_{th} = 1, \quad Q_L = 1. \\ - \text{ If } K_{th} > K_L \text{ then } Q_{th} = Q_{av}, \quad Q_L = 1. \\ - \text{ If } K_{th} < K_L \text{ then } Q_{th} = 1, \quad Q_L = Q_{av}. \end{array} \right\} \quad (13)$$

8) The Fitness of path p is determined by the fitness of its total costs according to its priority values and consequently its weighting. Getting the results of Eqns. 10 to 13 the total fitness of pass p is:

$$F_{total(p)} = Q_L K_L F_{L(p)} + Q_{th} K_{th} F_{th(p)} \quad (14)$$

$F_{total(p)}$ is Computed for all feasible paths from U to T. The algorithm searches for the optimal path P which has the maximum total fitness $F_{total(p)}$.

$F_{total(p)}$ of Eqn. 14 provides a balanced effect of the threat component regardless their values. The mission planner has a chance to give priority for one component over the other depending upon the situation. The vehicle can compute autonomously the correct weighting quantities. It also can shift the given priority factors if necessary to search other near optimal path to achieve arrival synchronization with the other team members. The algorithm is explained in Fig. 6.

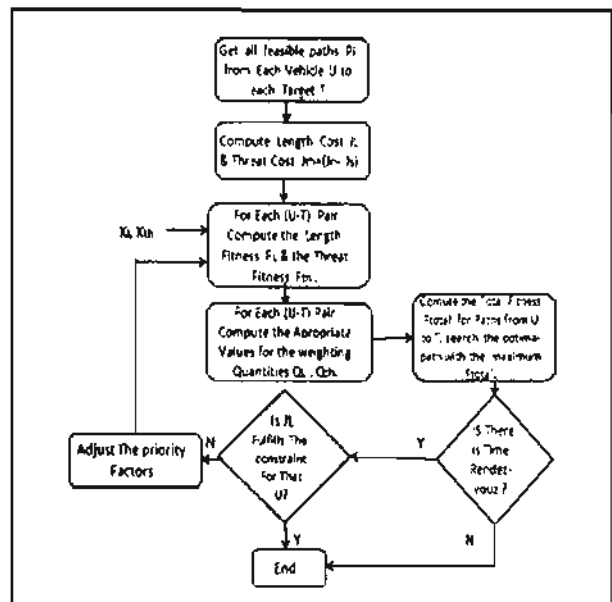


Fig. 6 Computing the Efficient Path

4. Simulation Results

In the following examples, the efficient path designed by the proposed method is compared to the optimal path designed by the conventional method. The threat value (of radars and SAMs) is assumed the same values given in example 1 (for radars only). The resulted optimal paths are apparently different.

Example2: Considering the same values given in example 1 (Fig. 4) the results of the proposed techniques is shown in table 2.

$$J_{L(av)} = 900/3 = 300, \quad J_{th(av)} = 42.01/3 = 14.003$$

$$Q_{av} = (900+42.01)/2=471.005$$

Table 2 Computation of the Fitness Function

	P ₁	P ₂	P ₃	
$F_L(p)$	300/400 = 0.75	300/200 = 1.50	300/300 = 1	
$F_{th}(p)$	14.003/0.01=1400	14.003/40 = 0.35	14.003/2 = 7.00	
Case A $F_{total}(p)$	1401.05	1.85	8.0015	$K_L=1, Q_{th}=1$ $K_L=1, Q_L=1$
Case B $F_{total}(p)$	527525.75	132.1	2637.8	$K_L=0.8, Q_{th}=Q_L$ $K_L=0.2, Q_L=1$
Case C $F_{total}(p)$	562.6	565.2	378.2	$K_L=0.2, Q_{th}=1$ $K_L=0.8, Q_L=Q_{th}$

Case A: Length and threat cost are balanced and have the same weight, the maximum fit is path P₁, which has the minimum threat value (0.01).

Case B: Threat cost are given more priority than the length cost ($k_r=0.8, K_L=0.2$). The maximum fit path is also P₁.

Case C: The length cost is given more priority than the threat cost ($k_r=0.2, K_L=0.8$). The most fit path is path P₂. This path has the minimum length cost.

So, the resulted efficient path is the one with minimum threat cost if priority is given to threat cost over length cost or if both priorities are equal. While it is the one with minimum length cost if the priority is given to the length cost.

Table 3 summarizes the comparison between the resulted optimal path in the two techniques.

Table 3 Results of the Proposed Technique Vs. the Conventional one

CASE	Path Computed		Correct Optimal Path	NOTES
	Conventional	Proposed		
Equal Priority	P2	P1	P1	P1 has the minimum threat value(0.01)
Priority for threat cost	P3	P1	P1	P1 has the minimum threat value(0.01)
Priority for length cost	P2	P2	P2	P2 has the minimum Length path(200)

Example3: to plan a trajectory for a UAV from its starting position to it's target through six SAMs and five radar sites, all possible path are generated by the planning algorithm explained in [25]. The optimal path is computed by the proposed algorithm (path A), and the conventional method (Path B) Fig. 7. Their costs are:

$$J_{L(A)} = 1340, J_{th(A)} = 8.4, J_{L(B)} = 1230, J_{th(B)} = 20.5.$$

The priority of the two costs are equal in the two cases, $K_L = k_{th} = 0.5$. The weighting values : $Q_L = Q_{th} = 1.0$ for the two cases.

It is clear that the optimal path designed by the proposed algorithm is safer and a little bit longer. The probability of detection of the vehicle on path B is higher because some part of it is closer to radar site # 2.

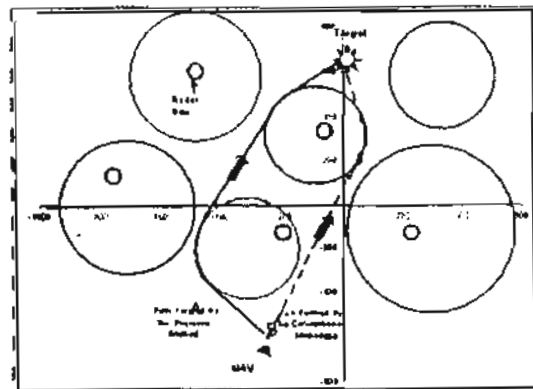


Fig. 7 Proposed Optimal Path vs. Conventional One

Example 4: this example explained in Fig. 8 shows one advantage of the proposed algorithm that is: giving different priorities for one cost element guides the algorithm to choose the path which satisfies this condition in an optimal manner. In path A the length cost is given more priority over the threat cost i.e. $k_{th} = 2, k_L = 8$. While in Path B the threat cost is given higher priorities $k_{th}=8, k_L=2$, the algorithm searches for the most safe path with the minimum length. This minimum length is clearly longer than that of case A.

This property manages the vehicle to cooperate with other team members for synchronized arrival to targets. But it must be noted here that not every change in the priority values produces different paths. This is restricted by the feasible paths available.

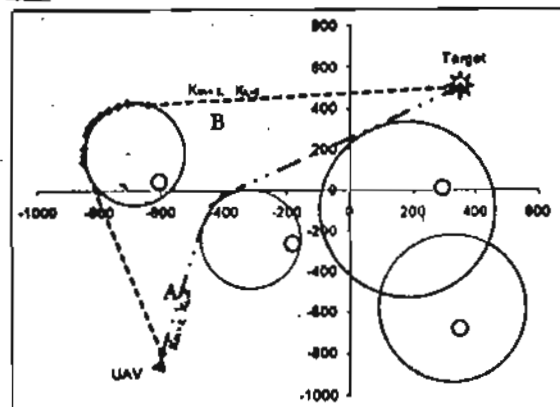


Fig. 8 Different priority Produces Different Optimal Paths

Conclusion:

The conventional cost function produces an optimal path which does not have the minimum cost but the minimum length path (regardless the threat cost). This is due to the domination of length over the threat in the cost function. Another disadvantage of the conventional method is that: it considers only for the radar sites i.e. the probability of being detected by the adverse radars. There is an important source of threat not considered that is the probability of passing over a SAM site. This paper introduces a proposed algorithm to compute an efficient path. It has the following strengths:

- 1) The resultant path has the maximum fitness (consequently, the minimum cost) with a balanced effects of the cost elements (length and threat) on the cost function.
- 2) It considers for the real threats on the field i.e. threats of the SAMs and radars.
- 3) The algorithm computes the proper weighting values for each path. These

values preserves the priorities of cost elements.

- 4) It Provides a sort of flexibility for the vehicles to fulfill rendezvous time by adjusting autonomously the priority factors.

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