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STRESS DISTRIBUTION OF DIFFERENT SHAPES OF BONDED JOINTS.

BY

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ABSTRACT :

Stress distribution in different bonded jionts are mathematically analysed. Three shaPes of joints; singlelaP, Butt joint and scarf joint are considered. The exPerimental was done on the above joints using two tyPes of adhesive materials. In additation to the effect of surface roughness degree (C.L.A.)on the different shaPes of the bonded joint strength was tested.Both theoretical and exPerimental results are comPared.

Keywords :
Bonded joint, stress analysis, dyamics of joint.

NOMENCLATURE :

α	:	Angle of contact in scarf joint,
E_3	:	Elastic modulus for adhesive layer,
G_3	:	Rigidity modulus for adhesive layer,
K_c, m(x),n(x)	:	Constants,
ℓ	:	Length of the bond line for single laP joint,
t_1, t_2	:	Thickness of Plates (1) and (2),
t_3	:	Thickness of adhesive layer,
T	:	Tensile force,
$\bar{T}$ (x), q (x)	:	Contact stress in scraf joint,
$\Delta\delta n$	:	Normal disPlacement,
$\Delta\delta t$	:	Tangential disPlacement,
$\Delta\delta y$	:	DisPlacement in (y) direction,
$\Delta\delta x$	:	DisPlacement in (x) direction,
ν_1	:	Possin's ratio for isotroPic medium,
ν_{2x}	:	Possin's ratio for orthotroPic medium in (x) direction,
ν_{2z}	:	Possin's ratio for orthotroPic medium in (z) direction,
ν_3	:	Possin's ratio for adhesive layer
$F_t(x)$	:	Total tensile load in material (2) for scarf joint.

1. INTRODUCTION :

Machine tools are not generally manufactured as a continuous casting of fabrication due to the difficulties in manufacture and for functional reasons such as the necessity to incorPorate guide

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ways. Most Practical designs for machine tool structures, therefore, incorPorate some form of connections between the basic elements, such as the connections can be classified as fixed joint (bonded, bolted, rivited and welded joint) and sliding joints. In both fixed and sliding connections, forces are transmitted across the joint interfaces and therefore , one could exPect that the overall static and dynamic characteristics of the machine tool is influenced by the comPliance at these individual connections.

A large number of investigations have been conducted on adhesive bonded joints. Many of them were concerned with one dimensional stress distribution and the mechanical ProPerties of the adhesive joints under static loading/1 . Several theoretical analysis of this comPlicated Problem have aPPeared in the Previous work since the early work by GOLAND & RESSINER/2 . Their study concerned only with two limiting cases, i.e.,where the adhesive layer is so thin and stiff(the thickness and modulus of elasticity ratio of the adhesive is much less than that of adherends). This means that its effect on the flexibility of the joint may neglected. Secondly, where the joint flexibility is mainly due to that of the adhesive layer.

The Present work Provides analysis and exPerimental comParision for different shaPes of bonded joints which are both simPle enough for design PurPose and yet include sufficient Parameters. So to Provide good correlation between test and theory. The durability of adhesive bonded joints dePends on the following factors; shaPe of the joint, tyPe of adhesive materials, tyPe of adherends and the surface PreParation for both metal and comPosite adherends/3/. In this work the effect of shaPe of the sPecimen, tyPe of adhesive materials and tyPe of adherends are discussed.

2. STRESS ANALYSIS IN SINGLE-LAP JOINT :

This joint consists of Plate (1), be bonded to Plate (2) by adhesive materials (3) as shown in Fig (1-a). From this sPecification the following relations may be assumed :

$$t_2 \ll t_1 \ll \ell \ll L_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and that the tensile force T is large enough, then,

$$\delta_A = \frac{T}{t_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where δ_A = average tensile stress in the Plates,

1.2- Determination Edge loads of joint :

The joint is devided into three Parts as shown in Fig. (1-b) considering the equilibrium of these Parts seParately, the edge tension force acting on the joint ($\bar{T}$) is given by:

$\bar{T}$ = T cos $\alpha \simeq$ T where angle α is very small .

$$\bar{T} = T = \delta_A . t_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The edge monent (M_e) is given by :

$$M_e = K . \frac{e}{2} . \delta_A . t_1^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

the edge transverse force (F_e)

$$F_e = \frac{1}{2\ell}\left[(t_1 + t_2^2)\, T - 2\, M_e\right]$$

$$= \frac{1}{2\ell}\left[(1-k)\, t_1^2 + t_1 . t_2\right] \delta_A \ldots\ldots (5)$$

2.2- Stresses in the joint :

Steress distribution in the joint must be determined in three directions (σ_{x1} , σ_{y1} and T_{xy1}), (σ_{x2} , σ_{y2} & T_{xy2}) and (σ_{x3} , σ_{y3} and T_{xy3}). From two dimensional theory of elasticity /4/.

$$\frac{\delta \sigma x_i}{\delta x} + \frac{\delta Txy_i}{\delta x} = 0 ,$$

$$\frac{\delta Txy_i}{\delta x} + \frac{\delta \sigma y_i}{\delta y} = 0 \qquad (i = 1,2,3....)$$

From the boundary conditions of this joint :

y_1	=	t_1	:	Txy_1	=	0	,	σy_1	=	0
y_1	=	0	:	Txy_1	=	Txy_3	,	σy_1	=	σy_3
y_2	=	0	:	Txy_2	=	Txy_3	,	σy_2	=	σy_3
Y_2	=	$-t_1$	:	Txy_2	=	0	,	σy_2	=	0
x	=	ℓ	:	σx_1	=	0	,	Txy_1	=	0
				σx_3	=	0	,	Txy_3	=	0

3.2- The Boundary conditions of the Stresses at the Ends of Single-LaP Joint :

$$x = \ell \; : \; \sigma x_1 = 0 , \quad Txy_1 = 0$$

$$\sigma x_3 = 0 , \quad Txy_3 = 0$$

$$\sigma x_2 = \left(\frac{\bar{T}}{t_1} + \frac{6M_e}{t_1^2}\right) + \frac{12}{t_1^2}\left(\frac{y_2}{t_1}\right)$$

$$= 1 + 3k + 6k\left(\frac{y_2}{t_1}\right) \delta_A \ldots\ldots\ldots\ldots (6)$$

$$txy_2 = \frac{6F_e}{t_1}\left(\frac{y_2}{t_1}\right)\left(1 + \frac{y_2}{t_1}\right)$$

$$= 3\left[(1-k)\frac{t_1}{\ell} + \frac{t_1}{\ell}\right]\left(\frac{y_2}{t_1}\right)\left(1 + \frac{y_2}{t_1}\right) \ldots.(7)$$

$$\text{At } x = -\ell$$

$$\sigma x_1 = \left(\frac{\bar{T}}{t_1} + \frac{6M_e}{t_1^2}\right) - \frac{12\, M_e}{t_1^2}\left(\frac{y_1}{t_1}\right)$$

$$= \left[1 + 3k - 6k\left(\frac{y_1}{t_1}\right)\right] \delta_A \ldots\ldots\ldots\ldots\ldots (8)$$

$$Txy_1 = - \frac{6F_e}{t_1} \left(\frac{y_1}{t_1} \right) \left(1 - \frac{y_1}{t_1} \right)$$

$$= -3 \left[(1-k) \frac{t_1}{\ell} + \frac{t_2}{\ell} \right] \left(\frac{y_1}{t_1} \right) \left(1 - \frac{y_1}{t_1} \right) \sigma_A \quad \ldots (9)$$

$$\sigma x_3 = 0 \quad , \quad Txy_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots (10)$$

$$\sigma x_2 = 0 \quad , \quad Txy_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots (11)$$

3. STRESS DISTRIBUTION IN BUTT JOINT :

The joint consists of two adherends, that is flat interference with adhesive layer as shown in Fig. (2). In this type of joints, the stress is simple tensile stress distribution , then and from Ref./4/. it can be shown for the joint of rectangular cross section, that;

$$\sigma = \sigma_0 \sqrt[4]{1 - \left(\frac{2y}{t_1} \right)^2}$$

where :

σ_0 = tensile stress at centriod of joint,

= $2.5\ \sigma_{average}$,

$\sigma_{ave.} = \frac{T}{t_1 . w}$

T = tensile force,

t_1 , t_2 = thickness of the adherends (1) and (2),

t_3 = thickness of the adhesive layer,

w = width of the joint.

Consequently, the shear component is not exists.

But for the joint of circular corss section, the following equation may be used :

$$\sigma = \sigma_0 \sqrt[4]{1 - \left(\frac{-2y}{d} \right)^2 - \left(\frac{2x}{d} \right)^2}$$

where, d = diameter of joint,

4. STRESS DISTRIBUTION IN SCARF JOINT :

The joint consists of two tapered plates (1) and (2), be bonded together using adhesive material (3) as shown in Fig.(3) The problem of this analysis will be solved approximately under the following assumptions; the thickness of the adherends(t_1,t_2) is larger than other dimensions of the joint, the stress field in the plates may be considered a plane stress problem($\sigma_{1y} = 0 = \sigma_{2y}$), the contact stress acts on the tapered surfaces of the plates as abody forces, the adhesive material acts as a combination of shear and tension spring and the stress through the thickness distribution are neglected.

From the equilibrium of plate (2) as shown in Fig.(3) the following can be seen ;

Total force/unit width acting in plate (2)

$$F_t(x) = \int_0^x \left[\bar{T}(c) \sin\alpha + q(c) \cos\alpha \right] \frac{dc}{\cos\alpha} \ldots\ldots$$

or

$$\int_0^x [\bar{T}(c)\cos\alpha + q(c)\sin\alpha]\frac{dc}{\cos\alpha} = 0 \quad \ldots\ldots\ldots\ldots (1)$$

Then, $\bar{T}(c)\cos\alpha = q(c)\sin\alpha$ .... (2)

$$F_t(x) = \int_0^x [q(c)\frac{\sin^2\alpha}{\cos\alpha} + q(c)\cos\alpha]\frac{dc}{\cos\alpha}$$

$$F_t(x) = \frac{\cos\alpha}{\cos\alpha}(\tan^2\alpha + 1)\int_0^x q(c)\,dc.$$

$$= (1 + \tan^2\alpha)\int_0^x q(c)\,dc \quad \ldots\ldots\ldots\ldots (3)$$

The following equation may be written from the equilibrium of the adhesive layer as shown in Fig. (3).

$$\Delta\delta_n = \frac{t_3}{E_3}\cdot\bar{T}(x) \quad \ldots\ldots\ldots\ldots (4)$$

$$\Delta\delta_t = \frac{t_3}{G_3}\cdot q(x)$$

$$d_{2x} - d_{1x} = \Delta\delta_x = t_3\left(\frac{1}{G_3} + \frac{\tan^2\alpha}{E_3}\right) q(x)\cos\alpha \quad \ldots\ldots (5)$$

In this case ; $E_z = E_{1z} = E_{2z} = 0$

Then, $$\sigma_{2x}(x) = \frac{F_t(x)}{x\tan\alpha}, \quad \sigma_{1x}(x) = \frac{T - F_t(x)}{t_1 - x\tan\alpha} \quad \ldots (6)$$

The following differential equation can be written from equations (3), (5) and (6) :

$$\frac{\delta^2 F_t}{\delta x^2} = m(x)\cdot F_t(x) = n(x)$$

where m (x) and n (x) are constants and from Ref./5/, the following equations give the values of the constants,

$$m(x) = \frac{1}{K_e}\left[\frac{1-\mu_{2x}\cdot\mu_{1x}}{E_{2x}\cdot x_2\tan\alpha} + \frac{1-\mu_1^2}{E_1(t_1 - x\tan\alpha)}\right] \quad \ldots\ldots (7)$$

$$n(x) = -\frac{(1^2-\mu_1^2)\,T}{K_e E_1(t_1 - x\tan\alpha)} \quad \ldots\ldots (8)$$

$$K_e = \frac{t_3\cos\alpha}{1+\tan^2\alpha}\left(\frac{1}{G_3} + \frac{\tan^2\alpha}{E_3}\right) \quad \ldots\ldots (9)$$

Equation (3) solved this subject using the following boundry conditions ;

$F_t(0) = 0$, $F_t(L) = T$

and from equation (6) it can be noticed that, at :

AT $x = 0$, σ_{2x} and if $t_1 = t_2$

$\sigma_{2x} = L\tan\alpha$ at $x = L$, σ_{1x} are not defined which may be exPressed as:

$$\sigma_{2x}(o) = \frac{F_t'(o)}{\tan\alpha} ,$$

$$\sigma_{1x}(L) = \frac{F_t'(L)}{\tan\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

5. EXPERIMENTAL WORK

1.5- TyPes of SPecimens

Four different shaPes of sPecimens were manufactured butt, scarf by 30°, scarf by 45° and single-laP joint .all these sPecimens are shown in Fig's (1,2,3) and they were manufactured from three materials (st/st , Al/Al and coP/coP). The two Parts of the bonded sPecimens are either made from the same material for a comParision between theoretical and exPerimental results or from the same material and combination for testing the effect of surface roughness on the strength of joint. All these sPecimens were subjected to Pure tension.

2.5- TyPes of adhesive Used :

Two tyPes of commerically available adhesives have been used in this investigations, its sPecifications are listed below.

a- Bison Bison Kombi-Kit

It is a two Part ePoxy-resin and hardener with mixing ratio 1:1 by weight and its setting time is 24 hours at room temPerature (20° C). The surfaces to be bonded have been wiPed by a cloth diPPed in trichloroethylene to remove excessive grease Prior to vaPour treatment.

b- Bison Kombi-Rabid :

It is a two Parts ePoxy-resin and hardener. The mixing ratio is 1:0.8 by weight at room temPerature (20° C) Bison kimb RaPid sets after (10) minutes, the bonded object resists handling at lower temPeratures . The setting time increases at 10°C and the resistance of handling is tained after about 20 minutes. It does not set below 5° C.

3.5- Testing Configurations :

In the tests, the sPecimens have the following sPecifications; the same degree of surface roughness, the adhesive thickness is the same for various joint (127 μm) the same surface PreParation and the curing time is constant for every tyPe of adhesive. It must be noticed that, each set of testing configuration have been rePeated four times and the loading on these sPecimens is Pure tension without generating any bending effects.

6- DISCUSSION AND CONCLUSIONS :

1.6- Effect of the Joint ShaPe

In Fig's (4,6,8 and 10) the curves are Plotted for the different shaPes of bonded joint, (Butt, scarf 30°, scarf 45° and single laP joint) when using adhesive No.I. A grouP of these curves for the theoretical results and the remainder for the exPerimental results. From these figures the theoretical results seem in good agreement.

From Fig's (10, 11) the same tyPes of adherends were used, the same degree of roughness (C.L.A.) and the same tyPe of adhesive, the trend of the curves in the figures is exactly the same, but the second gives a good results comPared to the joint of 45^0 face angle, that is also dePendent on the increase of the contact area. In Fig's (12, 13) the results of single-laP joints are Plotted. In these figures the curves take the same trend, but in the first when using adhesive (No.1) gives a good results comPared with the second, that is dePend uPon the Performance of the adhesive only as mentioned before, In this tyPe of joint the results is higher than any tyPe. We have two reasons for that. The first, is the increase of the contact area. The second is the increase surface grains of the Parts which affects the strength of this tyPe of joints.

2.6- Effect of Surface Roughness :

The exPerimental work were carried out to investigate the effect of surface roughness (C.L.A.) on the strength of bonded joints. The sPecimens used in these tests were single-laP, scarf, butt joint. The utilized joints were st/st, coP/coP and Al/Al or a combinations of those materials. The results of the tests are Plotted in Fig's (14-19) The tensile strength of the bonded joints was Plotted versus the surface roughness (C.L.A.) shaPe of joint, tyPe of adhesive (using two tyPes matintained in the Previous Part) and the thickness of adhesive was maintained constant and equal to 127 μm. From fig's (14-19) it is clear that. the surface roughness Plays an imPortant role and affects the joint strength. The oPtimum value of surface roughness is ranging between 20 μm and 35 μm These values give good results for all shaPes of joints and for all tyPes of adhesive material too. If are comPared Fig's (14-19) it can be found that the single laP joint gives higher strength with resPect to the others joints. This may be attributed to the increase of the contact area. In the second case when using adhesive (No 2) the results shown in Fig's (15,17,19) for different shaPes of bonded joints. It is clear that, the first tyPe of adhesive material gives good results with resPect to the second tyPe of adhesives. In Fig's (15,17,19) the curves take the same trend exactly but give less values comPared to the first tyPe of adhesive material. Now, it is clear that, the first tyPe is better than the second and the tensile strength of the single-laP joint is the highest one with resPect to the other different shaPes of joints.

Finally the following can be concluded; (1) the design of joint on the basis of tensile strength, a single-laP joint and steel/steel material is the oPtimum one.(2) The surface roughness (C.L.A.) values from 30 to 40 μm gives a good value of tensile strength. (3) The adhesive (No.1) Boison Kombi-Kit has the highest Performance.

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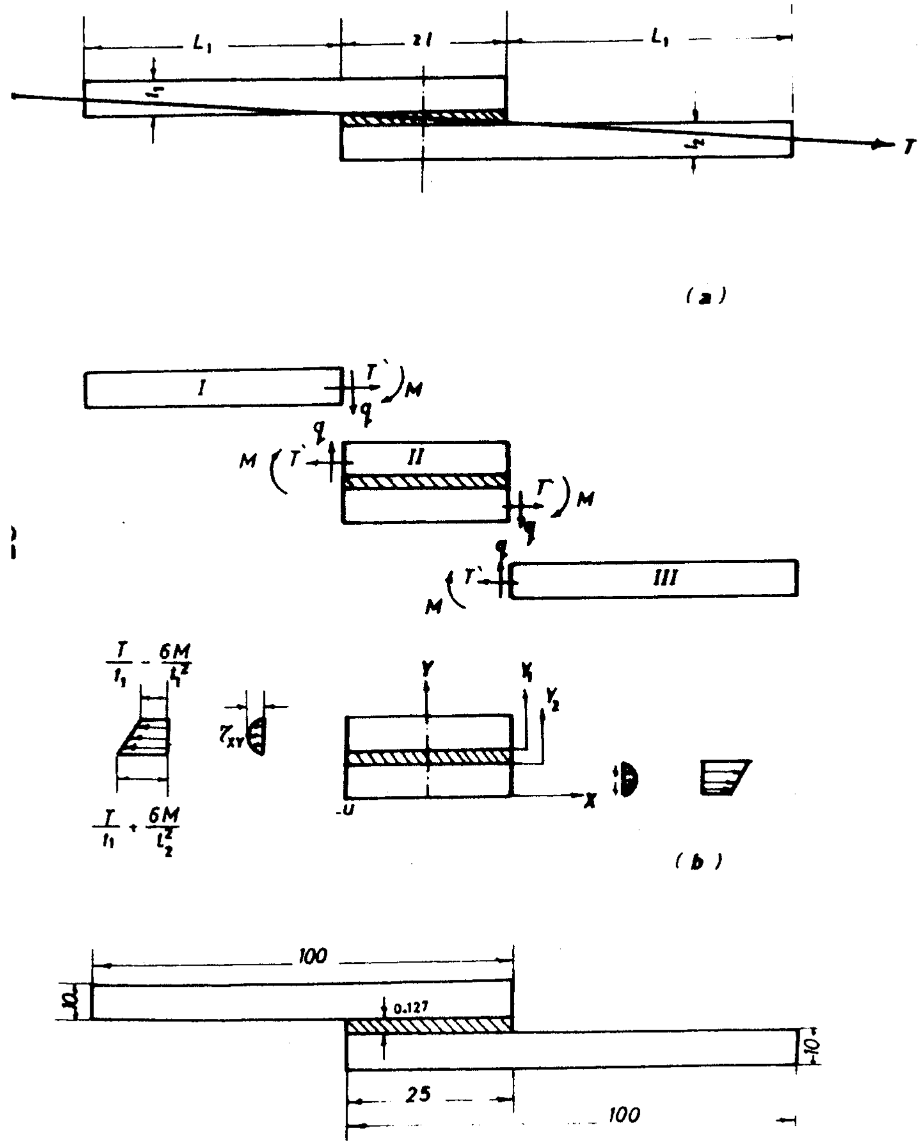


FIG.(1) SINGLE LAP JOINT

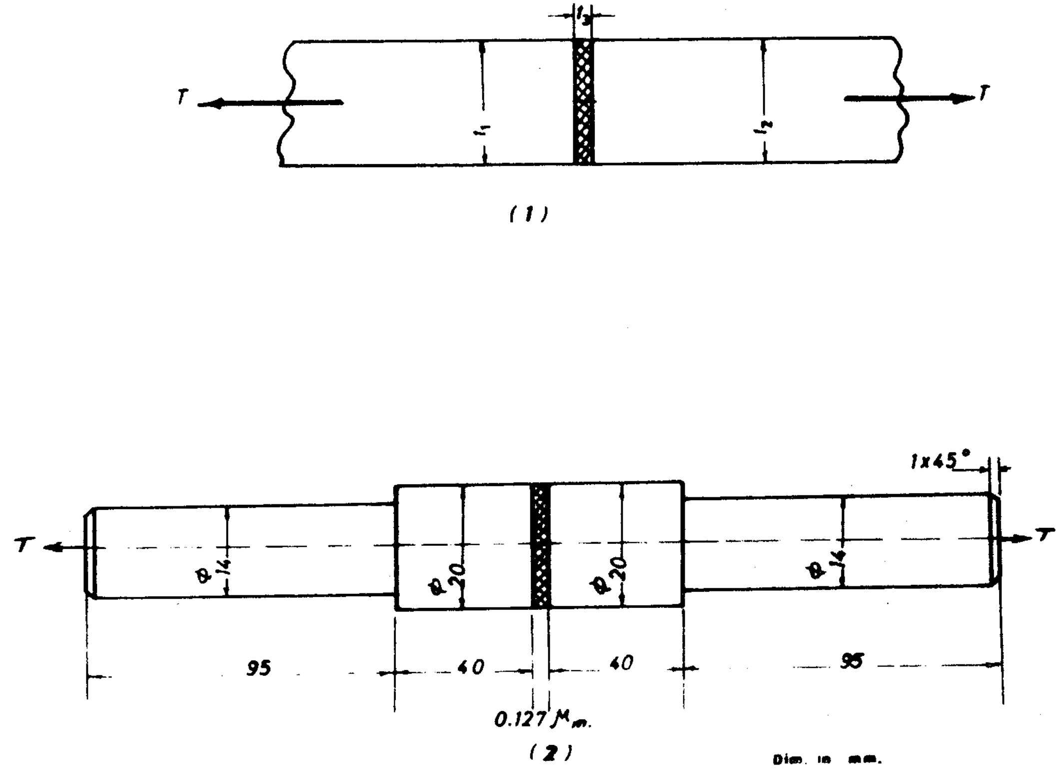
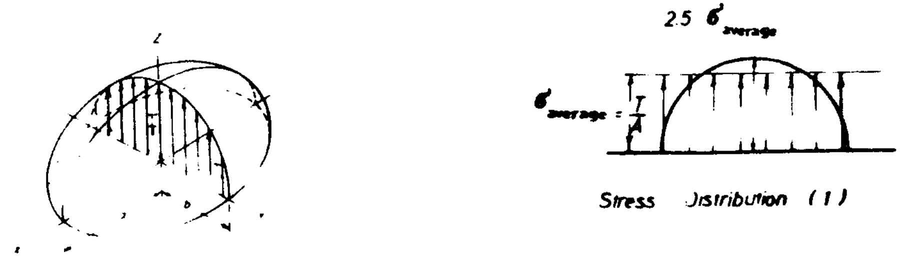


FIG.(2) BUTT JOINT



Stress Distribution (2)

Stress Distribution (1)

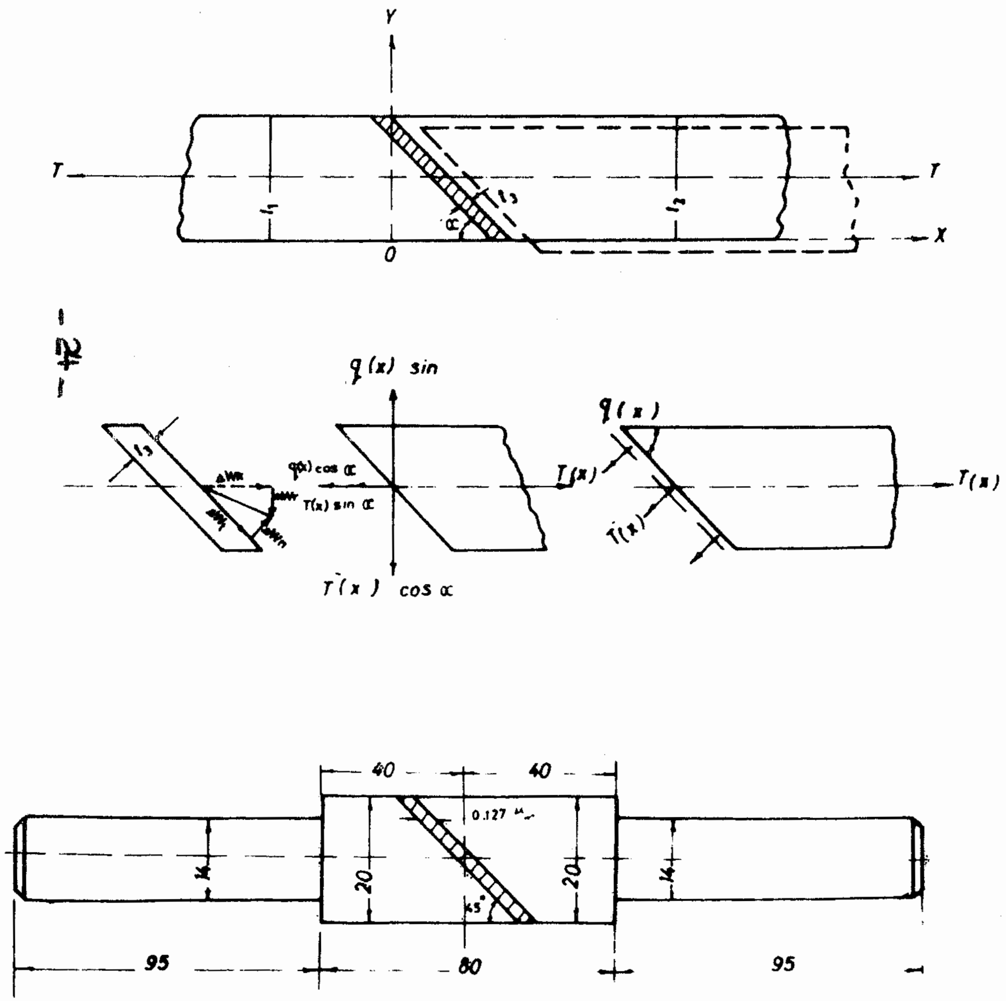


FIG.(3) SCARF JOINT

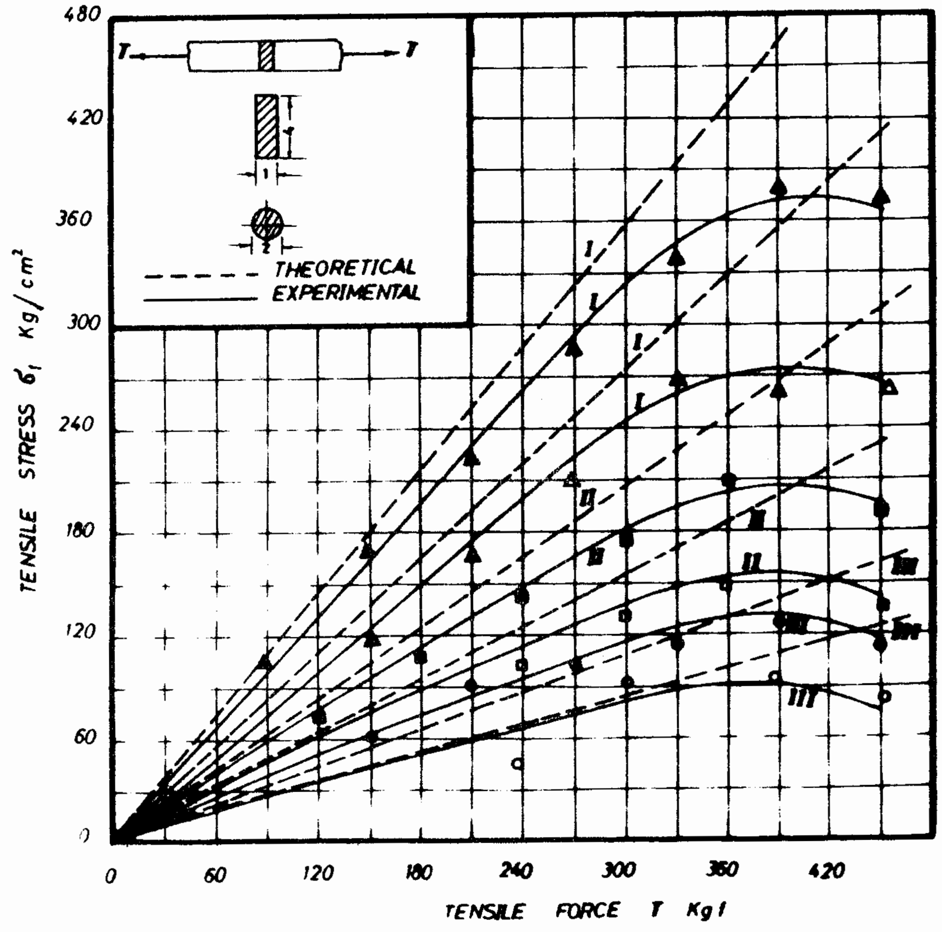
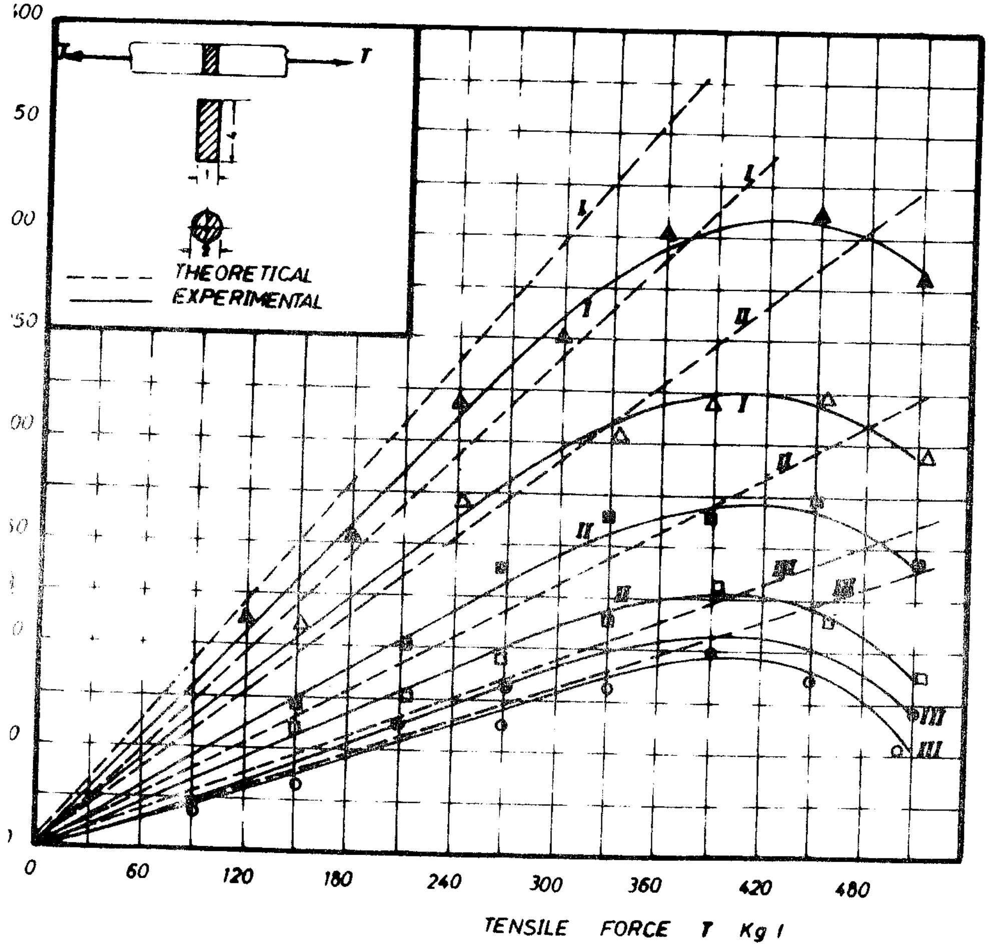


FIG. 4 (σ_1, T) THEORETICAL AND EXPERIMENTAL RESULTS FOR DIFFERENT SHAPES OF BONDED BUTT JOINT USING ADHESIVE NO 1 (I _ St ; St & II _ Cop ; Cop & III _ Al ; Al)



(5) (σ_1 - T) THEORETICAL AND EXPERIMENTAL RESULTS FOR DIFFERENT SHAPES OF BONDED BUTT JOINT USING ADHESIVE NO.2 (I - St/St & II - Cop/Cop AND III - Al/Al)

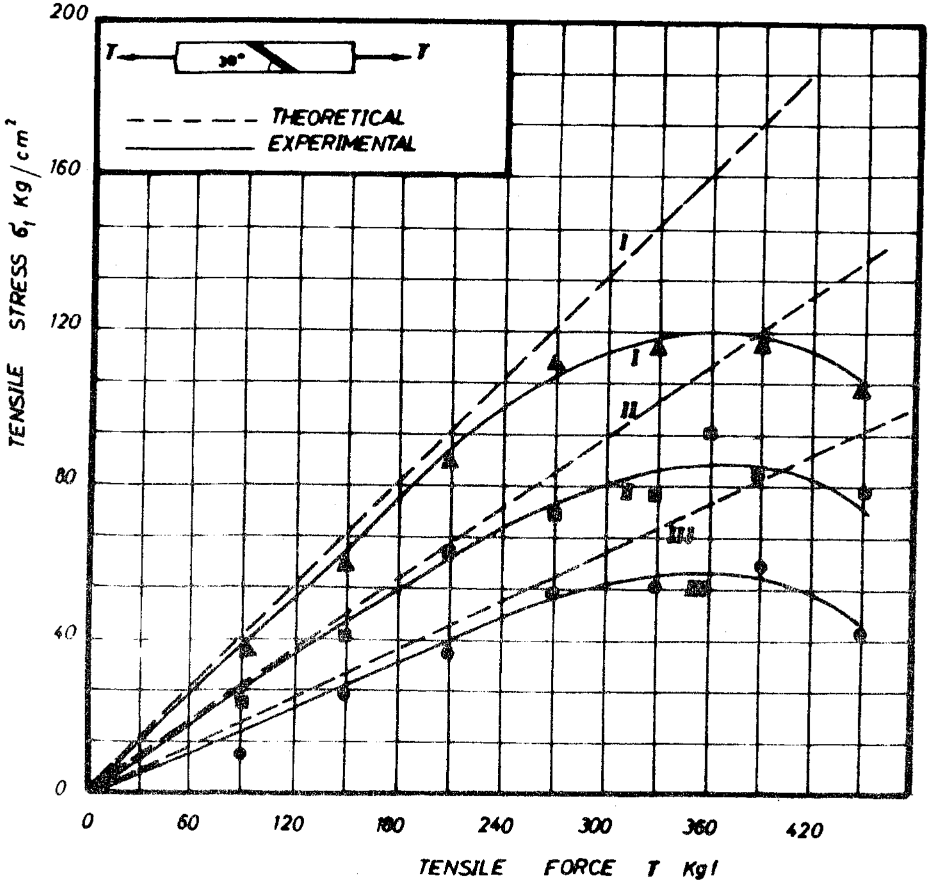


FIG (6) (σ_1 - T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SCARF JOINT (ANGLE 30°) USING THREE TYPES OF MATERIALS AND FOR ADHESIVE NO. 1 (I - St/St & II - Cop/Cop AND III - Al/Al)

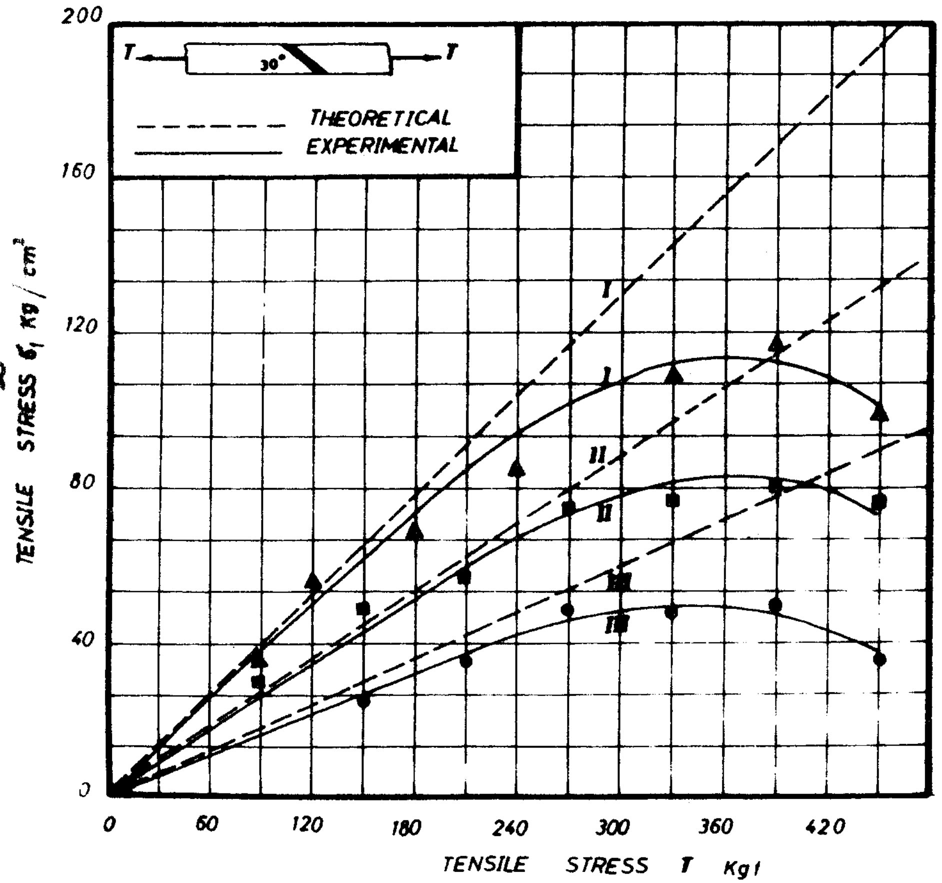


FIG (7) (σ_1' _ T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SCARF JOINT (ANGLE 30°) USING THREE TYPES OF MATERIALS AND FOR ADHESIVE NO. 2 (I _ St / St & II _ Cop/Cop AND III _ Al / Al)

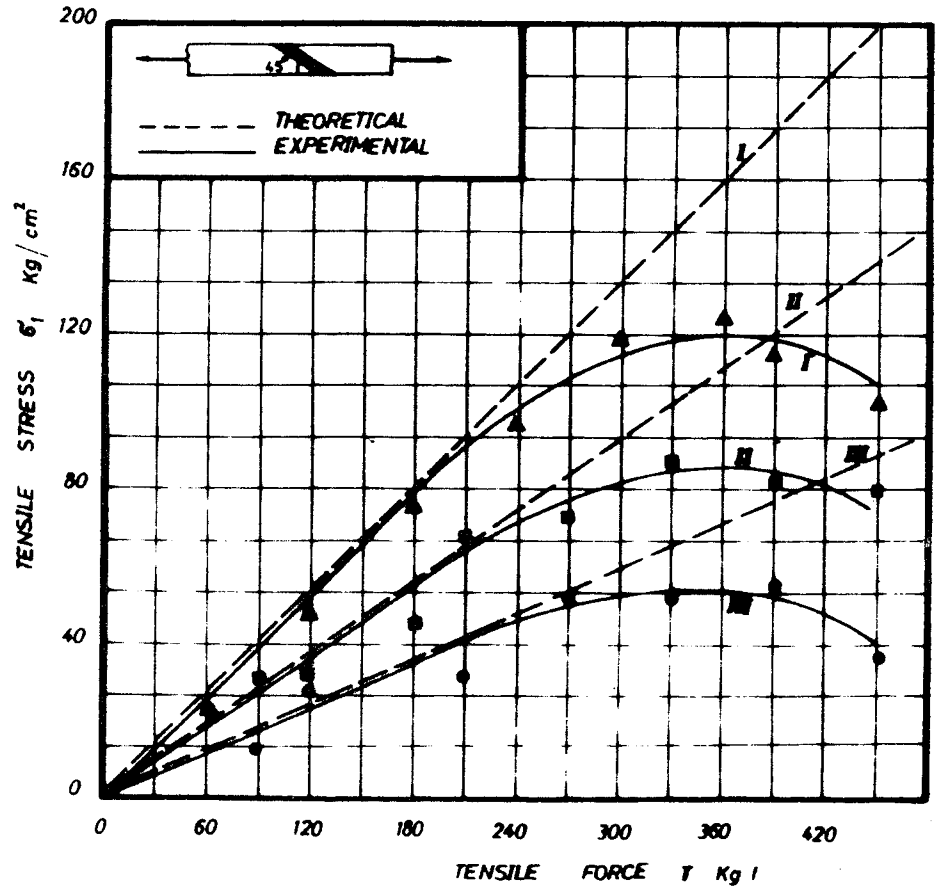


FIG (8) (σ_1' _ T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SCARF JOINT (ANGLE 45°) USING THREE TYPES OF MATERIALS AND FOR ADHESIVE NO 1 (I _ St , St & II _ Cop/ Cop AND III _ Al , Al)

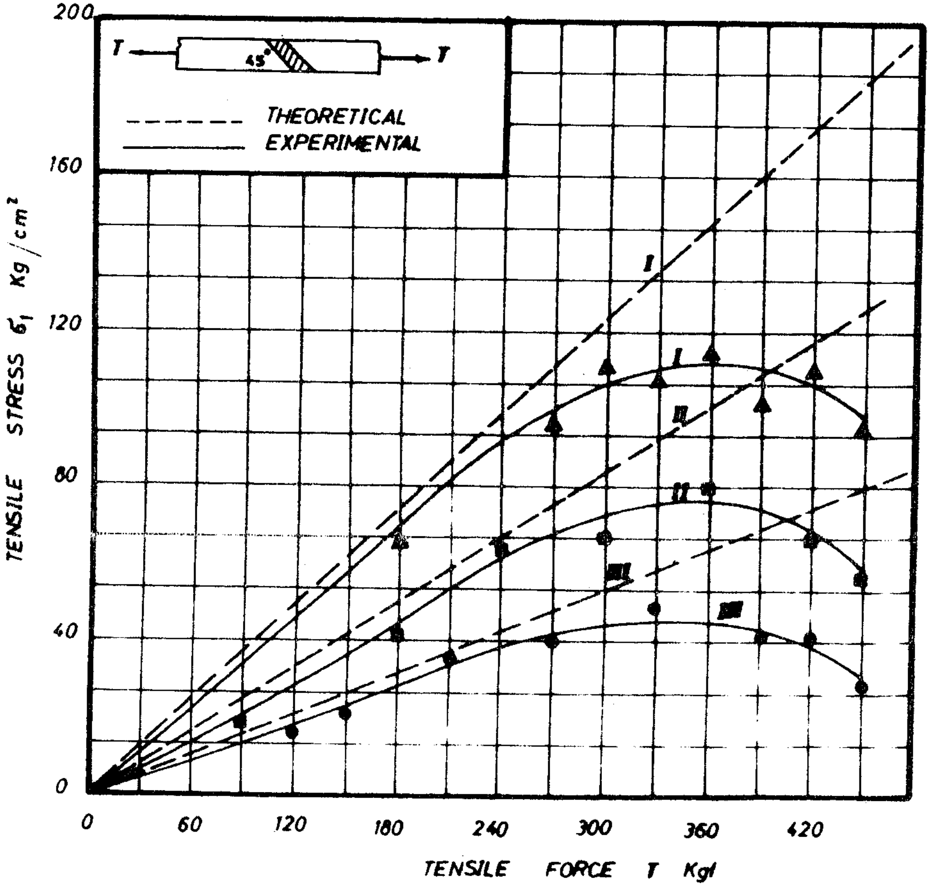


FIG (9) (σ_1 - T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SCARF JOINT (ANGLE 45°) USING THREE TYPES OF MATERIALS AND FOR ADHESIVE NO 2 (I _ St/St & II _ Cop/Cop AND III _ Al / Al).

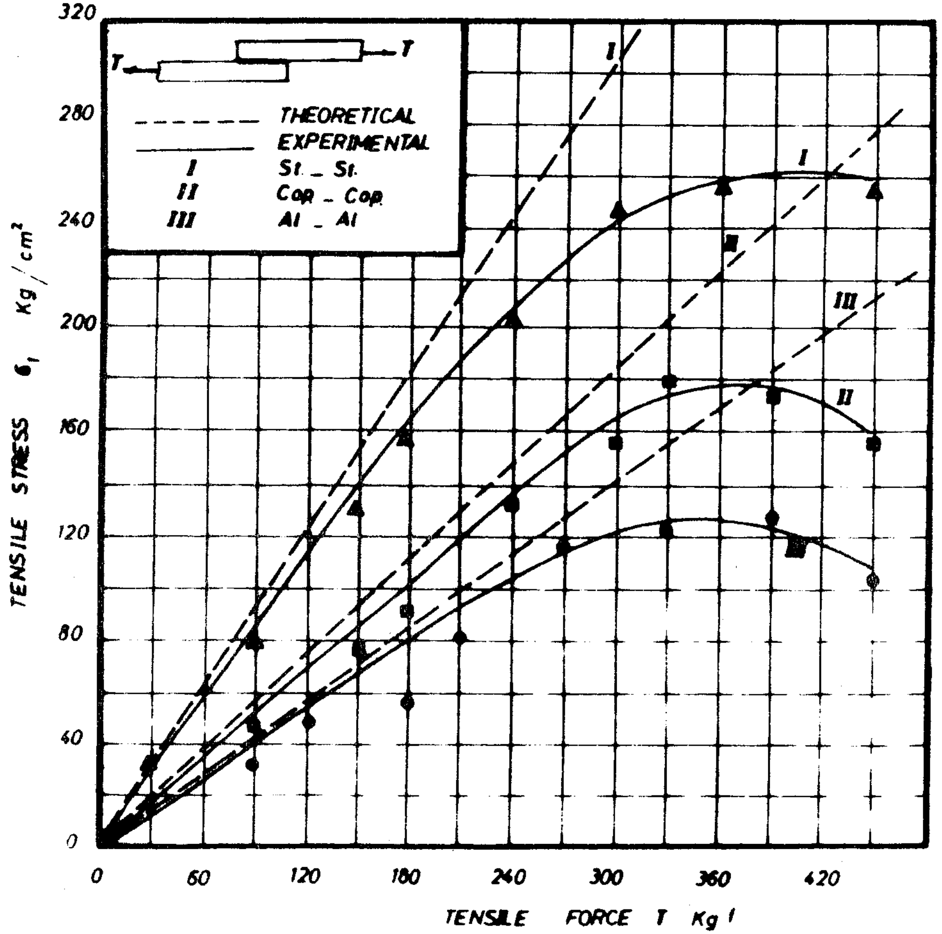


FIG (10) (σ_1 - T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SINGLE-LAP JOINT USING THREE TYPES OF MATERIALS AND FOR ADHESIVE NO 1

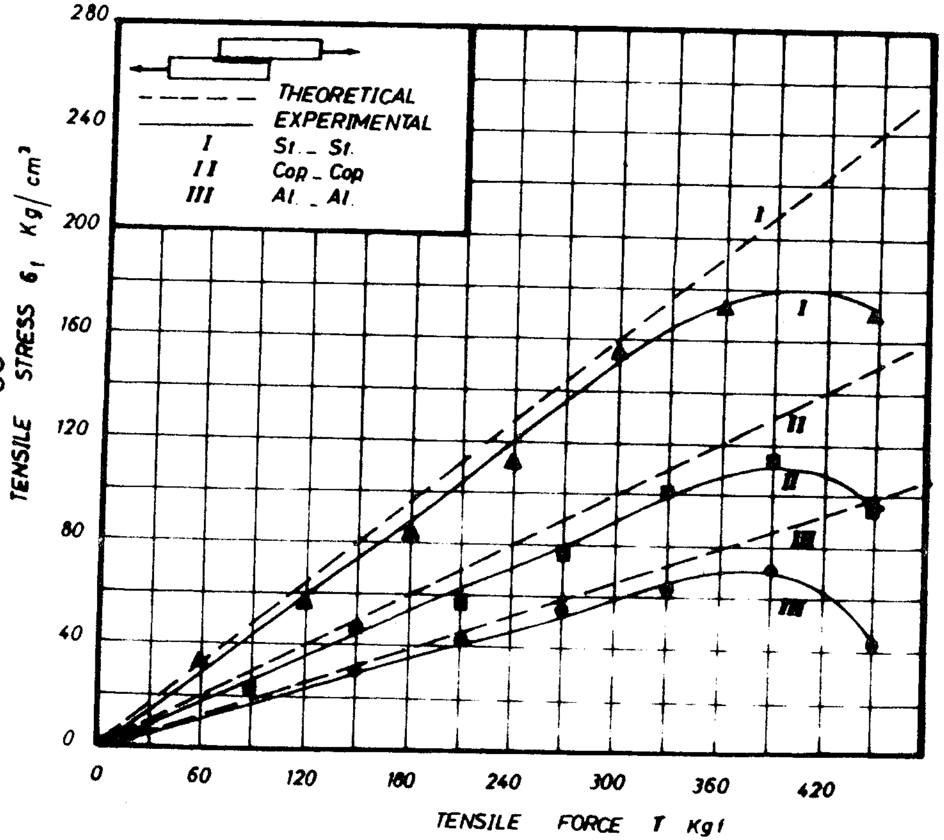


FIG (11) ($б_t$-T) THEORETICAL AND EXPERIMENTAL RESULTS FOR SINGLE LAP JOINT USING THREE TYPES OF MATERIALS AND ADHESIVE NO. 2.

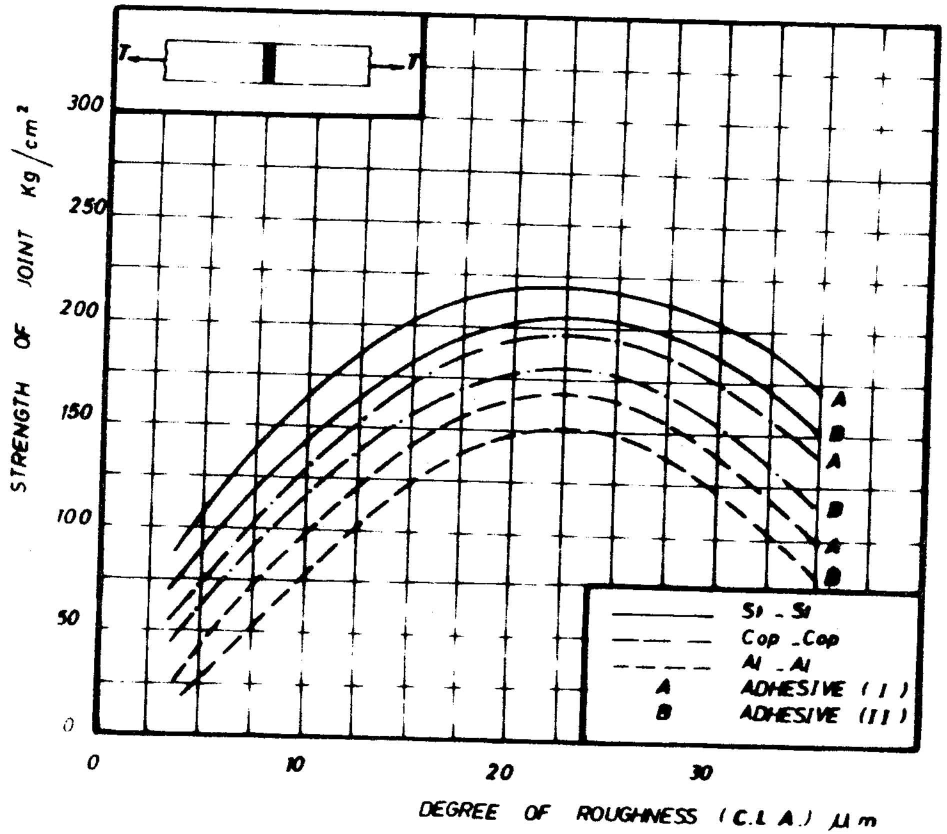


FIG (12) ($б_t$ - ROUGHNESS) RELATIONS FOR THREE TYPES OF MATERIALS AND FOR TWO TYPES OF ADHESIVES (BUTT JOINT)

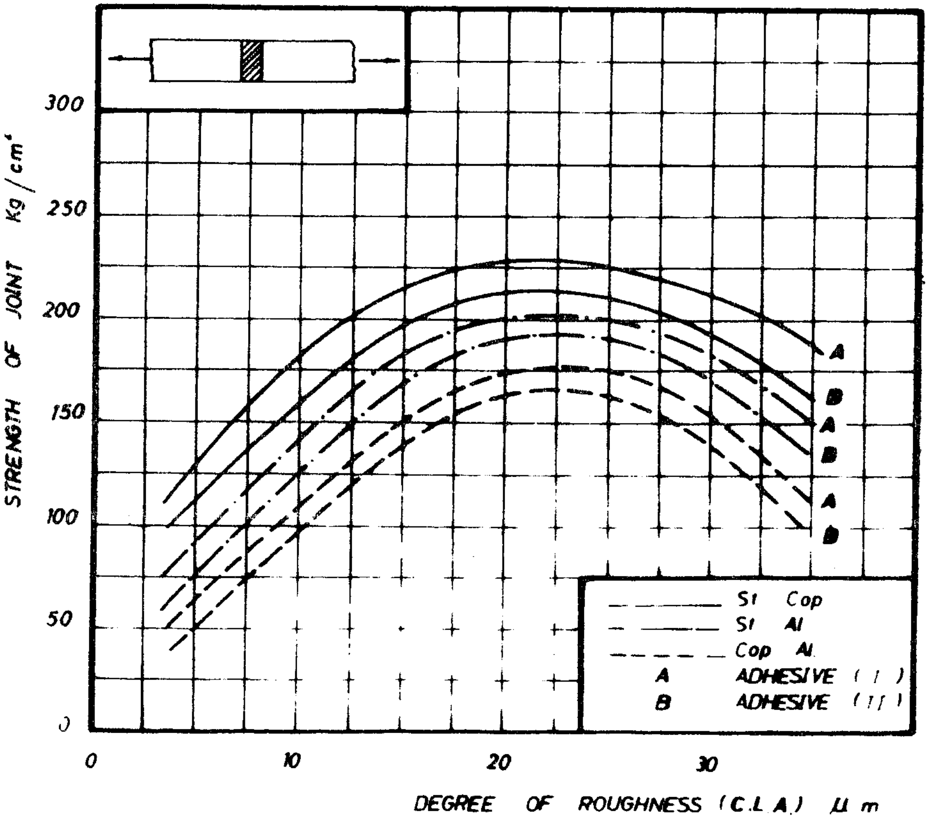


FIG.(13) (σ_t - ROUGHNESS) RELATIONS FOR A COMBINATION OF THREE TYPES OF ADHERENTS AND TOW TYPES OF ADHESIVES (BUTT JOINT)

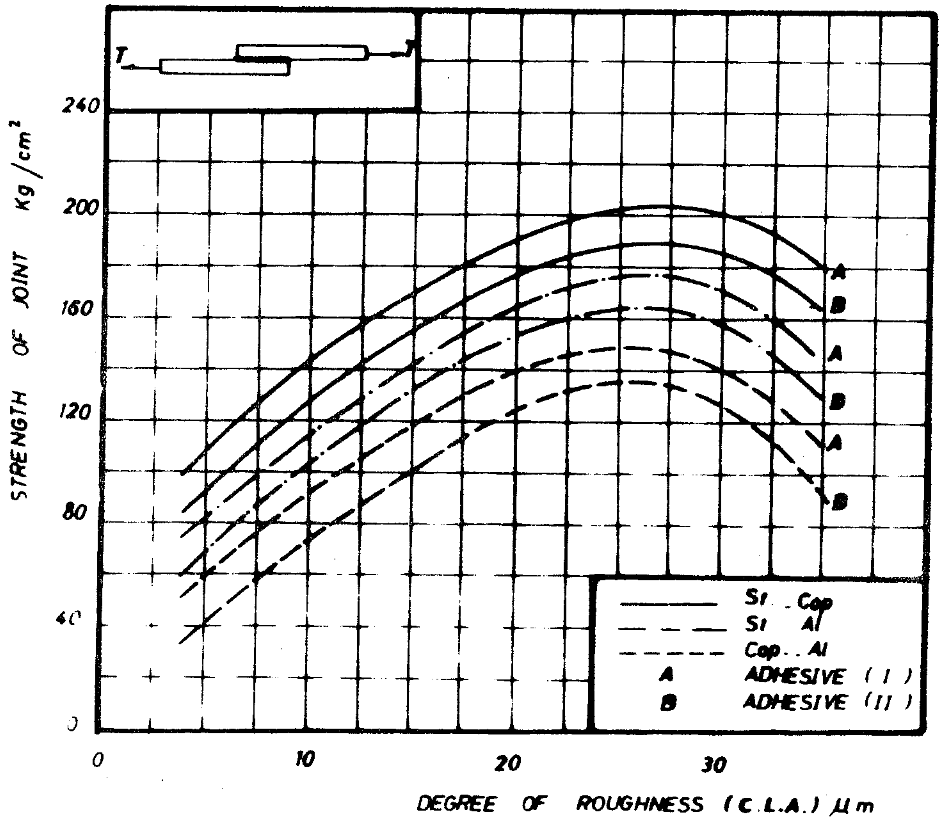


FIG.(14) (σ_t - ROUGHNESS) RELATIONS FOR A COMBINATION OF THREE TYPES OF ADHERENTS AND FOR TWO TYPES OF ADHESIVE (SINGLE - LAP - JOINT)

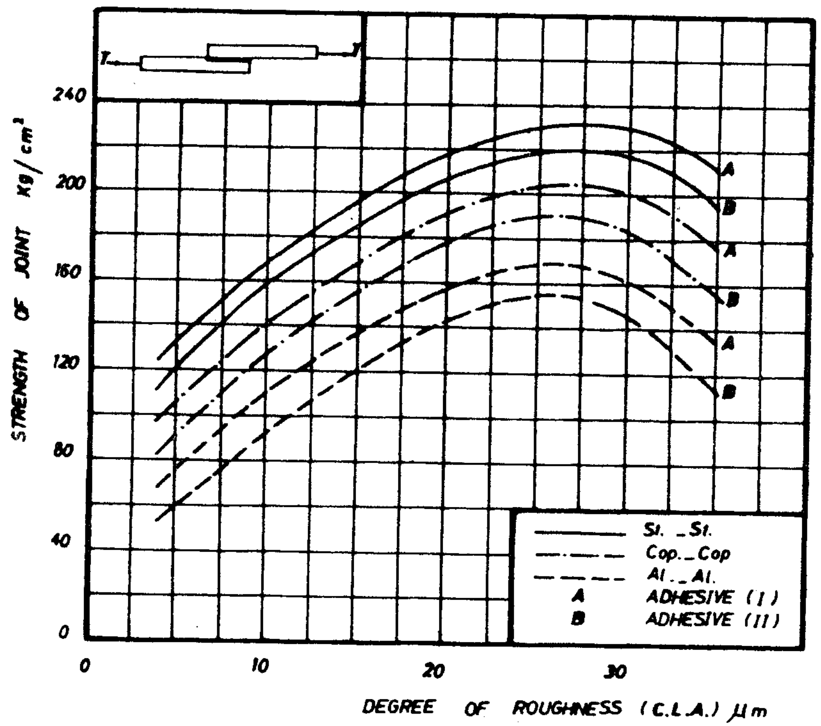


FIG.(15) (σ_1 - ROUGHNESS) RELATIONS FOR THREE TYPES OF MATERIALS AND FOR TWO TYPES OF ADHESIVE (SINGLE LAP JOINT)

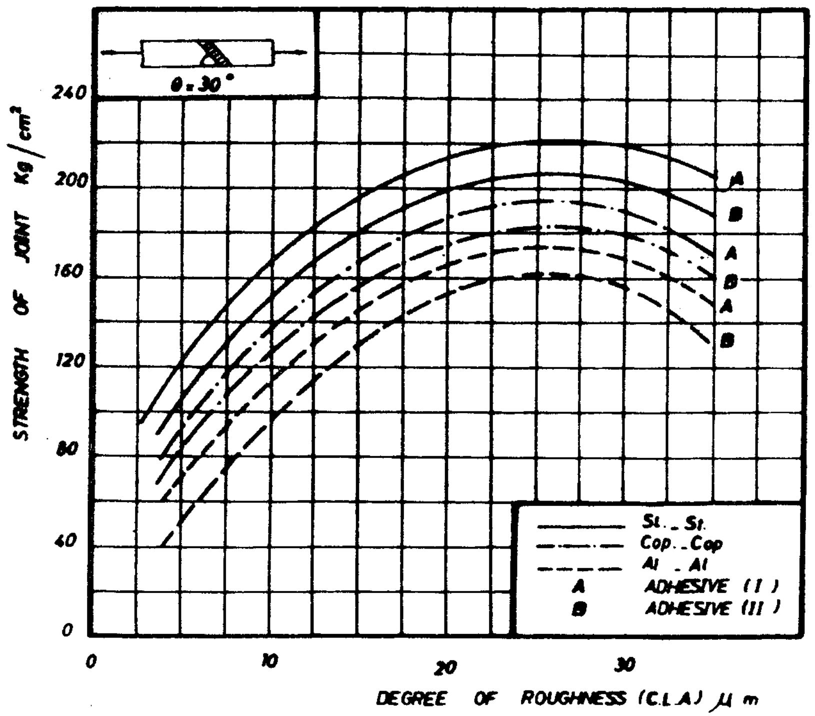


FIG.(16) (σ_1 - ROUGHNESS) RELATIONS FOR THREE TYPES OF MATERIALS AND FOR TWO TYPES OF ADHESIVES (SCARF JOINT)

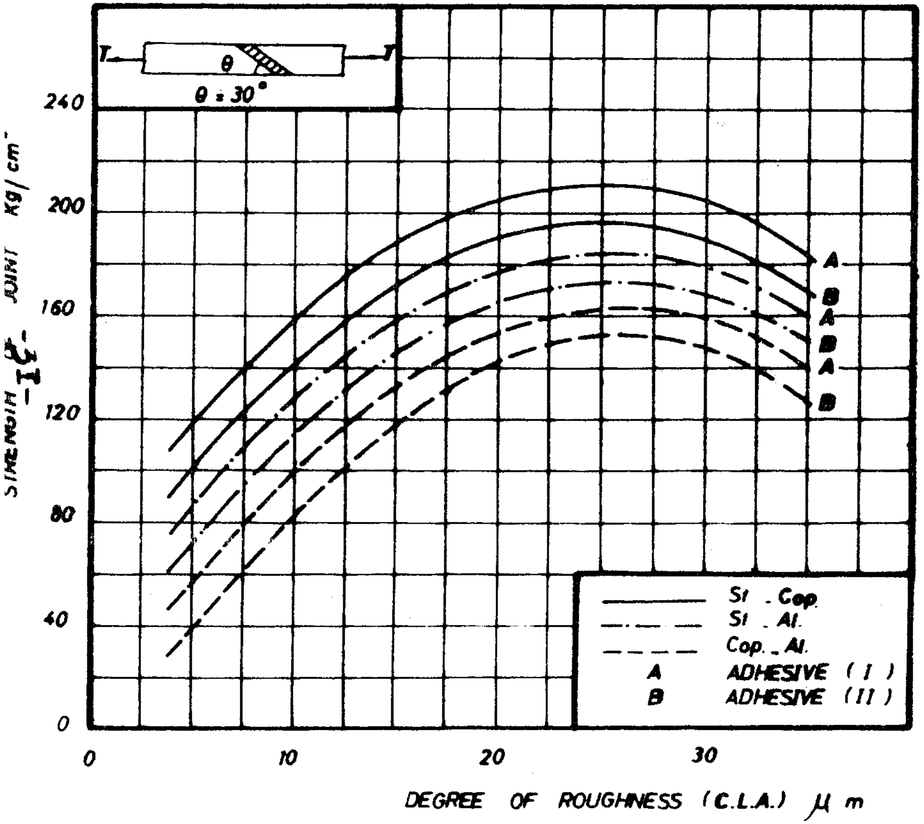


FIG.(17) (σ_1 - ROUGHNESS) RELATIONS FOR A COMBINATION OF THREE TYPES OF ADHERENTS AND FOR TWO TYPES OF ADHESIVE (SCARF JOINT)

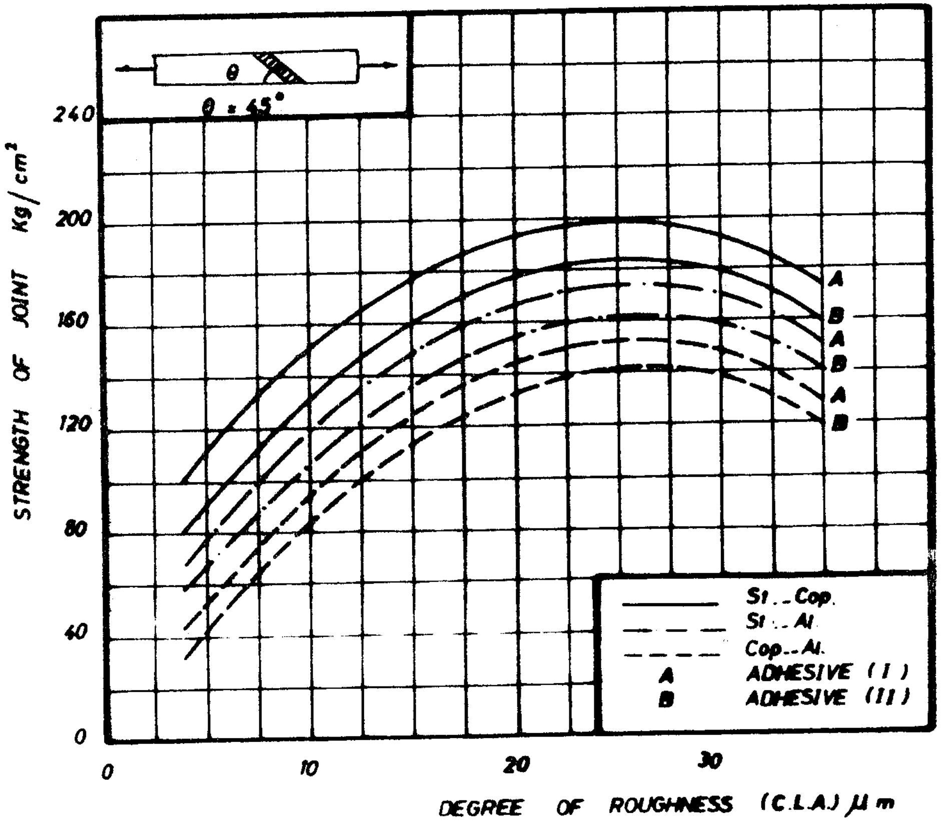


FIG.(18) (σ_1 - ROUGHNESS) RELATIONS FOR A COMBINATION OF THREE TYPES OF ADHERENTS AND FOR TWO TYPES OF ADHESIVE (SCARF JOINT).

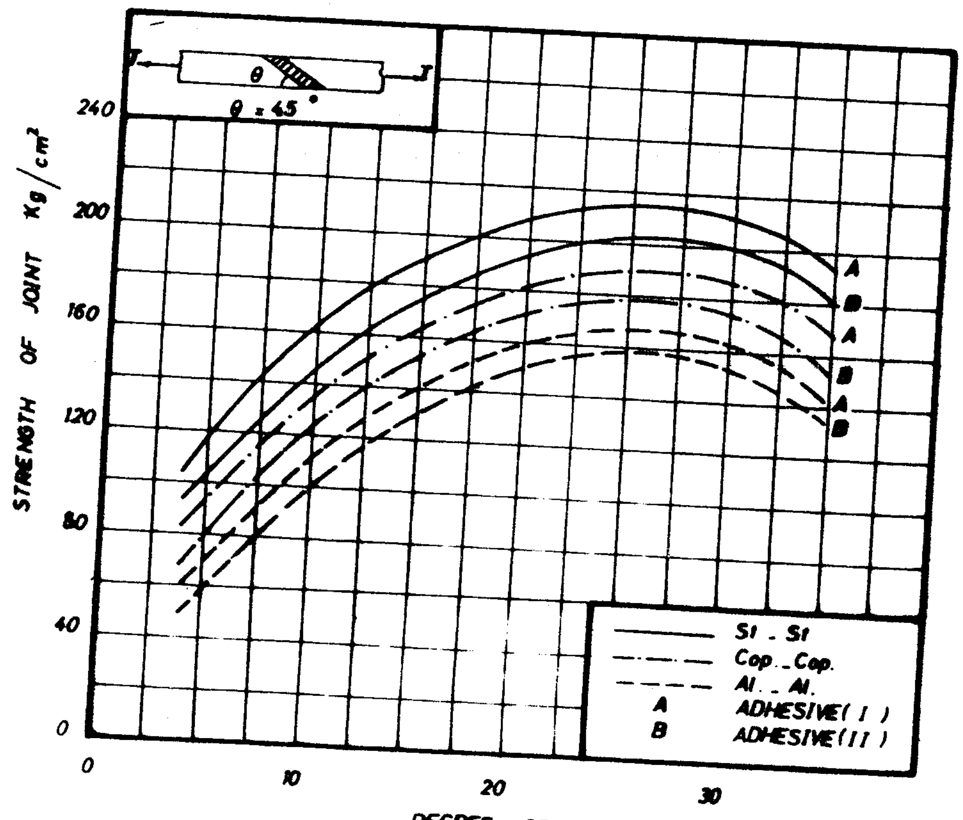


FIG.(19) (σ_t _ ROUGHNESS) RELATIONS FOR THREE TYPES OF MATERIALS AND FOR TWO TYPES OF ADHESIVES (SCARF JOINT)