FORCED CONVECTION FILM CONDENSATION OF FLOWING VAPOUR ON AN INCLINED CIRCULAR TUBE

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التكثيف الجبري الرقائقي لبخار ينساب على أنبوبة مائلة ذات مقطع دائري

ملخص نموذج نظري تم تطويره لإجراء التكثيف الطبقي الرقائقي على أنبوبة مستديرة المقطع ومائلة على الأفقي تحت تأثير الانسياب الجبري للبخار، حيث يعتبر الاحتكاك الناتج عن سرعة البخار هو المتحكم الأساسي في حركة التكثيف و تعتبر قوى الجاذبية ذات تأثير مهمل نسبيا. هذا وباستخدام طريقة الخواص الرياضية (method of characteristics) أمكن إيجاد حل تحليلي دقيق للمسألة لحساب معامل انتقال الحرارة وسمك طبقة التكثيف. كما يبين النتائج أن زاوية ميل الأنبوبة التي عندها يكون معدل تكثيف وبالتالي معامل انتقال حرارة قيمة عظمى تكون دالة في النسبة للأنبوبة. هذا كما يعطي من العام الناتج نفس الحلول الخاصة للأنابيب الرأسية والأفقية وكذلك السطح المستوى المائل المسلطة في دراسات سابقة. مما يدعم صحة النموذج النظري

ABSTRACT

In this work, an analytical study is made on the process of vertical downflow forced-convection-laminar-film condensation of a pure saturated vapour on an inclined circular tube. The govening partial differential equation for the local condensat film thickness has been solved analytically by applying the method of characteristics. An explicit analytic closed-form solution; has been obtained fur calculating. The local and average Nusselt numbers. The general solution yields, the special known analytic solutions of condensation on the vertical and horizontal tubes. The results show that there is an optimum tube inclination, at which the maximum rate of condensation on the whole tube surface is a function of the ratio L/D.

INTRODUCTION

Since the pioneering work of Nusselt (1916), many studies have been made on the laminar film condensation of a quiescent vapour on the surfaces of various shapes Among those studies is that of Hassan and Jacob (1958) who adopted the Nusselt model to treat the problem of free convectiou laminar film condensation on an inclined circular tube.

However, recent engineering applications, such as oerospace planes, nuclear reactors, etc. have promoted research on the condensation processes under forced vapour flow. Therefore, in recent years, much work has been done concerned with forced convection film condensation on surfaces of various forms. Many studies have been made on flat plates, e.g., Cess (1960), Koh (1962). Other studies made on horizontal and vertical tubes with circular and non-circular cross sections, e.g., Shekriladze and Gomelauri (1966), Schnal (1972), Gaddis (1979), Rose (1984) and Yang and Chen (1993). Recently, Rose (1988) made a comprehensive review on the forced convection laminar-film condensation.

Shekriladze and Gomelauri (1966) neglected the effects of convection, inertia, subcooling and gravity forces in the condensate film for vapour flow along horizontal and vertical plates, and for vapour downward flow over a horizontal tube. In addition, they assumed that the shear stress at the condensate surface is due to the change in the momentum of vapour mass condensing onto the condensate surface. They estimated this interfacial shear by using an asymptotic infinite-condensation-rate expression. Thus, simple analytical solutions could be obtained..

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To the author's knowledge, there has been no study on the forced-convection - dominated laminar film condensation on an inclined circular tube, as that made by Hassan and Jacob on free convection-dominated film condensation. Therefore, the aim of the present work is to carry out this study. The assumptions of Shekriladze and Gomelauri. will be made as in the analysis

ANALYSIS

Physical Model and Derivation

Consider a circular tube of radius R and inclined with an angle φ to the horizontal, is situated in a stream of pure dry saturated vapour having vertical down uniform velocity V_∞. The wall temperature T_w is assumed uniform and below tbe vapour saturation temperature T_s. Thus a, condensation film will form and flow on the external surface of the tube in both the axial and peripheral directions. Figure 1 shows a sketch of the physical model and coordinate system. In the present analysis, the vapour velocity is assumed to be sufficiently high so that the film motion is mainly controlled by the vapour shear and the effect of gravity force is negligible. In comparison with the momentum transferred by tha condensing vapour mass, the effect of pressure gradient in the film may be neglected. In addition, tha condensate film flow is considered laminar, steady and with negligible viscous dissipation. The condensing vapour is considered to be of au ordinary liquid. Thus, the effects of inertia and convection in the film may also be neglected.

The above-mentioned assumptions simplify the x- and z-momentum equations; describing the condensate film motion, as follows:

x-momentum conservation:

$$\mu \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

z-momentum conservation:

$$\mu \frac{\partial^2 w}{\partial y^2} = 0 \quad (2)$$

where u and w are the velocity components in the x- and z-directions, respectively.

The above equations are subject to the boundary conditions:

$$u = w = 0, \qquad y = 0 \quad (3)$$

$$\frac{\partial u}{\partial y} = \frac{\tau_{\delta x}}{\mu}, \quad \frac{\partial w}{\partial y} = \frac{\tau_{\delta z}}{\mu}, \qquad y = \delta \quad (4)$$

where $\tau_{\delta x}$ and $\tau_{\delta z}$ are respectively the x- and z- components of the interfacial vapour shear.

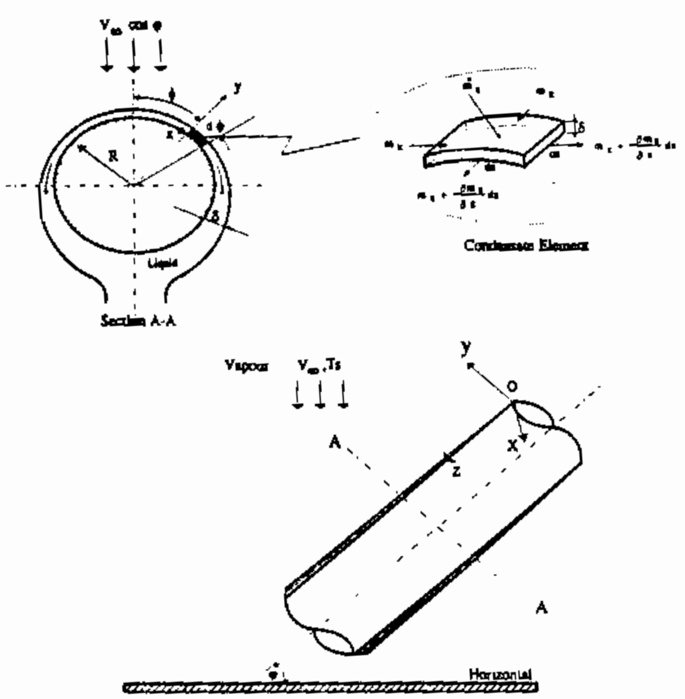

Fig. 1 Physical model and coordinate system

By assuming potential flow outside the vapour boundary layer, together with using the infinite condensation rate approximation of Shekriladze and Gomelauri (1966), the interfacial shear components τδx and τδz may be approximated by

$$\tau_{\delta x} = m_c''\left(2V_\infty \cos\varphi \, \sin\phi\right) \tag{5}$$

$$\tau_{\delta x} = m_c'' \, V_\infty \sin\varphi \tag{6}$$

where m''c is the local condensation mass flux.

Additionally, a heat balance at the liquid-vapour interface, as in the analysis by Nusselt, gives

$$h_{fg}'' \, m_c'' = k\frac{\Delta T}{\delta} \tag{7}$$

where h_{fg}'' = (hfg + 3CpΔT/8), is the modified latent heat of condensation proposed by Rohsenow (1965) to account for condensate subcooling.

Equation (7) implies that the rate of heat transfer by conduction across the condenset film is equal to the rate of heat liberation by due to the vapour condensation at the liquid-vapour interface.

In the above interfacial shear modeling, the shear force is assumed to be equal to the change in the momentum of vapour mass condensing at the condensate surface. The interfacial condensate velocity is considered negligible when compared to the vapour velocity, as postulated in the analysis of by Shekriladze and Gomelauri (1966).

Solving Eqs. (1) and (2) under the boundary conditions (3) and (4) in view of Eqs. (5) and (6) yields,

$$u = \frac{2\dot{m}_c}{\mu} V_\infty \; y \; \cos\varphi \; \sin\phi = \frac{2K\Delta T}{h''_{fg}\mu\delta} V_\infty y \cos\varphi \sin\phi \tag{8}$$

$$w = \frac{\dot{m}_c}{\mu} V_\infty \; y \; \sin\varphi = \frac{2k\Delta T}{h''_{fg}\mu\delta} V_\infty y \sin\phi \tag{9}$$

For constant condensate physical properties, a mass balance for the element, shown in Fig. 1, yields:

$$\begin{aligned} \dot{m}_c &= \frac{1}{\delta}\left(\frac{\partial \dot{m}_x}{\partial x} + \frac{\partial \dot{m}_z}{\partial z}\right) = \frac{K\Delta T}{h''_{fg}\delta} \\ &= \rho \frac{\partial}{\partial x}\left(\int_0^\delta u\, dy\right) + \rho \frac{\partial}{\partial z}\left(\int_0^\delta w\, dy\right) \end{aligned} \tag{10}$$

Combining Eqs. (8) through (10) and substituting $dx = Rd\phi$, one gets for the local film thickness the differential equation:

$$\cos\varphi\left(4\delta^2 \cos\phi + 2\sin\phi \frac{\partial \delta^2}{\partial \phi}\right) + R\sin\varphi \frac{\partial \delta^2}{\partial z} = \frac{2\nu D}{V_\infty} \tag{11}$$

Or in the dimensionless form ,

$$\frac{\partial \Delta^2}{\partial Z^+} + 2\sin\phi \frac{\partial \Delta^2}{\partial \phi} = \frac{1}{2} - 4\cos\phi\, \Delta^2 \tag{12}$$

for the dimensionless local film thickness ,

$$\Delta = \frac{\delta}{2D}\sqrt{\widetilde{Re}_D \cos\varphi} \tag{13}$$

as function of the dimensionless axial length ,

$$Z^+ = z/(R\tan\varphi) \tag{14}$$

and the peripheral angle ϕ.

The symbol $\widetilde{Re}_D = (V_\infty D/\nu)$ in Eq. (13) denotes the two-phase Reynolds number.

Equation (12) is subjected to the boundary conditions:

$$\Delta = 0 \quad \text{at} \quad Z^{*} = 0; \quad 0 \le \phi \le \pi \tag{15a}$$

$$\frac{\partial \Delta}{\partial \phi} = 0 \quad \text{at} \quad \phi = 0 \text{ or } \phi = \pi; \quad 0 \le Z^{*} \le \pi \tag{15b}$$

The first boundary condition implies that the condensate film starts with zero thickness around the circular periphery of the tube at its upper end. The second condition expresses the symmetry of the condensate film and its smoothness at the top ($\phi=0$) and bottom ($\phi=\pi$) along the tube.

SOLUTION

Special Solutions

An inclined tube is the general case of the following special cases:

Vertical tube. Setting $\varphi = \pi/2$ in Eq. (11) reduces it to one which leads finally to the relation:

$$Nu_z / \sqrt{\tilde{R}e_z} = 1/2 \tag{16}$$

for the local Nusselt number Nuz ($=hz/k$) as function of the two-phase Reynolds number $\tilde{R}e_z (= zV_\infty / \nu)$. Hayhew and Aggarwal (1973) obtained the same result for the vertical plate and recommended it for the vertical tube provided that the condensate film thickness is much less than the tube diameter.

Inclined plate. Setting $R = \infty$ in Eq. (11) reduces it to one for condensatioo on an inclined flat plate. This reduced equation leads finally to the expression:

$$Nu_z / \sqrt{\tilde{R}e_z} = \frac{1}{2}\sqrt{\sin\varphi} \tag{17}$$

for the local Nusselt Number as function of $\tilde{R}e_z$ and the inclination angle φ.

<u>Horizontal tube.</u> Substituting $\varphi=0$ in Eq. (11) reduces it to the ordinary differential equation

$$2\delta^2 \cos\phi + \sin\phi \frac{\partial \delta^3}{\partial \phi} = \nu D / V_\infty \tag{18}$$

whose solution subject to the boundary condition: $d\delta/d\phi = 0$ at $\phi = 0$, is

$$\delta = \sqrt{\frac{1-\cos\phi}{\sin\phi}}\sqrt{\frac{D\nu}{V_\infty}} \tag{19}$$

Frnm tbe above results, one gets the expression:

$$Nu / \sqrt{\tilde{R}e_D} = \sin\phi / \sqrt{1-\cos\phi}, \quad \pi \le \phi \le 0 \tag{20}$$

for the local Nusselt number $Nu\phi = (hD/k)$.

Taking the limit of the fraction ot $\phi= 0$, tbe value of $Nu / \sqrt{\tilde{R}e_D}$ equal $\sqrt{2}$

Next, integrating Eq. (20) yields ,

$$Nu_D / \sqrt{\widetilde{Re}_D} = 0.9 \qquad (21)$$

for the mean Nusselt number $Nu_D = (\bar{h}D/k)$.

The same solutions were obtained previously by Shekriladze and Gomelauri (1966) for a horizontal tube in a downward vapour flow in the absence of gravity.

Infinite-length inclined tube. For large distance from the starting edge of an inclined tube, the film becomes fully developed and the local film thickness is only function of the peripheral angle ϕ. For this case, Eq. (12) reduces to

$$\sin\phi \frac{d\Delta^2}{d\phi} + 2\cos\phi\, \Delta^2 = 1/4 \qquad (22)$$

This equation, except for the definition of Δ, has the same forms as Eq. (18) for condensation on a horizontal tube.

Solution of Eq. (22) subject to the boundary condition (15 b) yields

$$\Delta = \sqrt{1-\cos\phi}\,/\left(2\sin\phi\right) \qquad (23)$$

for the dimensionless local film thickness Δ.

Combining Eqs. (13) and (23) and using the classical relation $h = k/\delta$, gives for the local Nusselt number

$$Nu / \sqrt{\widetilde{Re}_D \cos\varphi} = \frac{\sin\phi}{\sqrt{1-\cos\phi}};\ 0 \le \phi \le \pi, t \qquad (24)$$

and at the upper point $\phi = 0$, the values equels $\sqrt{2}$

By integration Eq. (24), one gets for the mean Nusselt number

$$Nu_D / \sqrt{\widetilde{Re}_D \cos\varphi} = 0.9 \qquad (25)$$

The above expressions (24) and (25), except the normalization of NoD by the term $\sqrt{\cos\varphi}$, are the same as for the horizontal tube given by Eqs. (20) & (21).

General Solution

An inclined tube of finite length represents the most general case. In this case, Eq. (12) has to be solved subjected to the boundary conditions given by Eqs. (15a, b). An analytical solution is possible using the method of characteristics. The subsidiary equations read:

$$dZ^+ = \frac{d\phi}{2\sin\phi} = \frac{d\Delta^2}{\left(1/2 - 4\Delta^2\cos\phi\right)} \qquad (26)$$

Rearranging and integration yields

$$Z^+ = \frac{1}{2}\ln\left[\tan(\phi/2)\right] + C_1 \tag{27}$$

$$\Delta = \frac{1}{2\sin\phi}\left[\int_0^{\phi}\sin\phi \, d\phi + C_2\right]^{1/2} \tag{28}$$

Equation (27) describes the flow path of the condensate on the external tube surface.
Applying the boundary condition (15a) on Eqs. (27) and (28) yields respectively,

$$Z^+ = \frac{1}{2}\ln\left[\tan(\phi/2)/\tan(\phi^*/2)\right] \tag{29}$$

$$\Delta = \frac{1}{2\sin\phi}\sqrt{\cos\phi^* - \cos\phi} \qquad 0 < \phi < \pi \tag{30}$$

Equation (12) with the boundary condition (15b) give the dimensionless local film thickness at the top ($\phi = 0$) and the bottom ($\phi = \pi$) of the inclined tube as follows:

$$\Delta_{\phi=0} = \sqrt{\left(1 - \exp(-4Z^+)\right)/8} \tag{31}$$

$$\Delta_{\phi=\pi} = \sqrt{\left(\exp(4Z^+) - 1\right)/8} \tag{32}$$

Combining Eq. (13) with Eqs. (30) to (32) and using the basic relation $h = k/\delta$, leads finally to the local Nusselt number expression:

$$\begin{aligned} Nu/\sqrt{Re_D\cos\varphi} &= \sin\phi/\sqrt{\cos\phi^* - \cos\phi} && 0 < \phi < \pi \\ &= \sqrt{2/\left[1 - \exp(-4Z^+)\right]} && \phi = 0 \\ &= \sqrt{2/\left[\exp(4Z^+) - 1\right]} && \phi = \pi \end{aligned} \tag{33}$$

The angle ϕ^* is defined from Eq. (29) by

$$\phi^* = 2\arctan\left[\tan(\phi/2)/\exp(2Z^+)\right] \qquad 0 < \phi < \pi \tag{34}$$

Equations (33) and (34) represent the general solution of the problem. For $Z^+ \to \infty$, Eq. (34) yields $\phi^* = 0$. Accordingly, Eq. (33) reduces to Eq. (24) of the infinite-tube-length case. Comparing Eqs. (33) and (24) reveals that the term $\cos\phi^*$ in Eq. (33) accounts for the finite length effect. Next, the local peripherally averaged Nusselt number can be calculated from ,

$$Nu\Big/\sqrt{\bar{R}e_D \cos\varphi} = \frac{1}{\pi}\int_0^{\pi}\frac{1}{2\Delta}\,d\phi \tag{35}$$

Finally, the mean Nusselt number $\bar{N}u_D\ (=\bar{h}D/k)$ for the entire surface of tube with dimensionless total length L+, can be estimated from

$$\bar{N}u\Big/\sqrt{\bar{R}e_D \cos\varphi} = \frac{1}{\pi L^+}\int_0^{L^+}\int_0^{\pi}\frac{1}{2\Delta}\,d\phi\ dZ^+ \tag{36}$$

The numerical evaluation of Eqs. (35) and (36) is not fully determined because at $\phi=\pi$ and Z+ $\to\infty$, the value of Δ, calculated from Eq. (32), goes to infinity. This problem can be overcome by substituting $\phi=\pi$ by values very close to π. Numerical calculations have shown that with $\phi=3.115$ reasonable accuracy could be achieved. Simpson's rule has been used for carrying out this numerical task. The simple analytic solution of the infinitely long tube, given by Eqs. (24) and (25), has been used as a reference to assess the accuracy of the numerical results. The relative absolute errors in Nusselt numbers were less than 1%.

RESULTS AND DISCUSSION

Figure 2 shows the variation of the dimensionless local film thickness around the tube periphary; at different axial locations, calculated using Eqs. (29) to (32). It can be seen that downstream from the starting "upper" edge of the tube, the film thickness increases in both the axial and peripheral directions. Beyond a short length of Z+ $\approx$2.6, the peripheral variation of the dimensionless local film-thickness takes a fixed profile as that at infinity.

Variation of the local, peripherally averaged Nusselt number, estimated numerically by Eq. (35), along the tube, is shown in Fig. 3; in terms of $Nu\Big/\sqrt{\bar{R}e_D \cos\varphi}$ versus Z+. It decreases with increasing Z+ ; from an infinite value at the start point (Z+ = 0) to assume a constant value equals 0.9 as Z+ $\geq$ 2.6. This value is the constant value of the infinite-length solution. Thus, the condensation process beyond a short length from the upper tube edge becomes as that on an infinitely long tube.

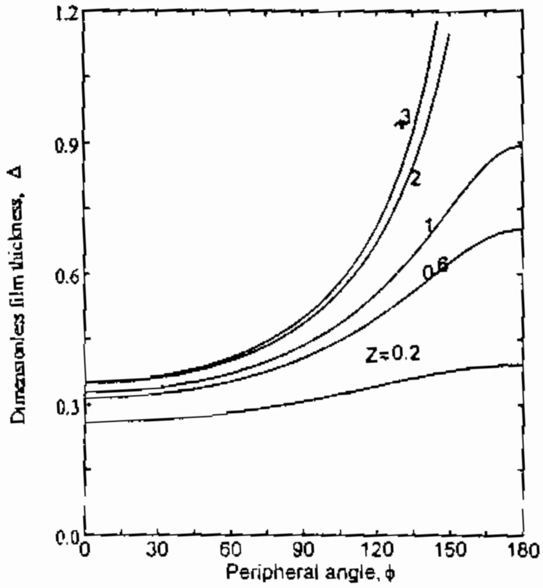

Fig. 2 Dimensionless film thickness distribution around the tube periphery; at different axial locations

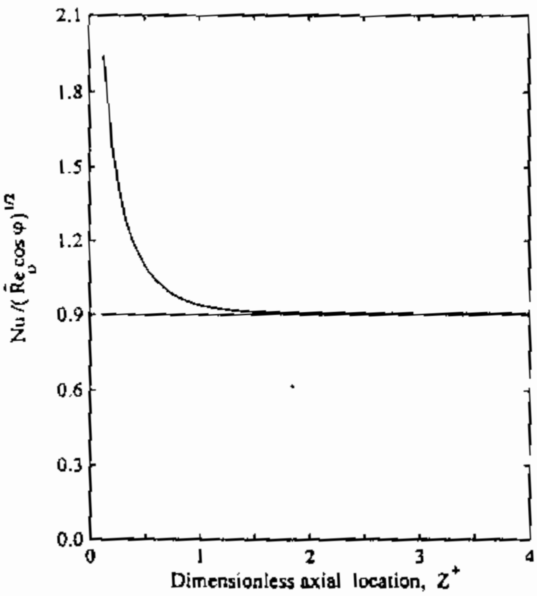

Fig. 3 Variation of peripherally-averaged Nusselt number along tube

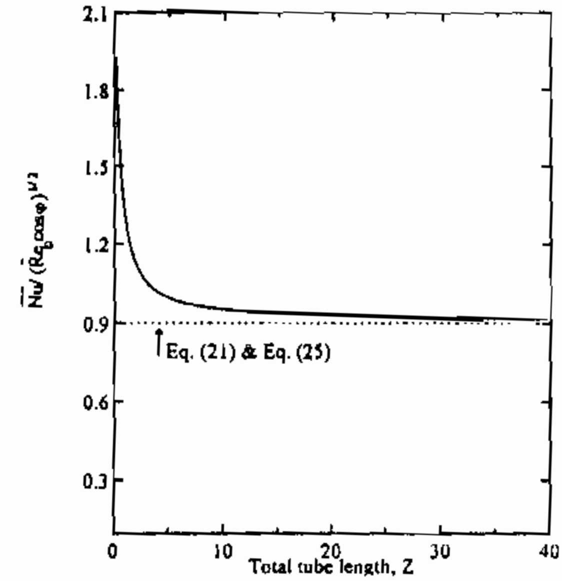

Fig. 4 Mean Nusselt number as function of dimensionless tube length, $Z^{+}=Z/(R\ \tan\varphi)$.

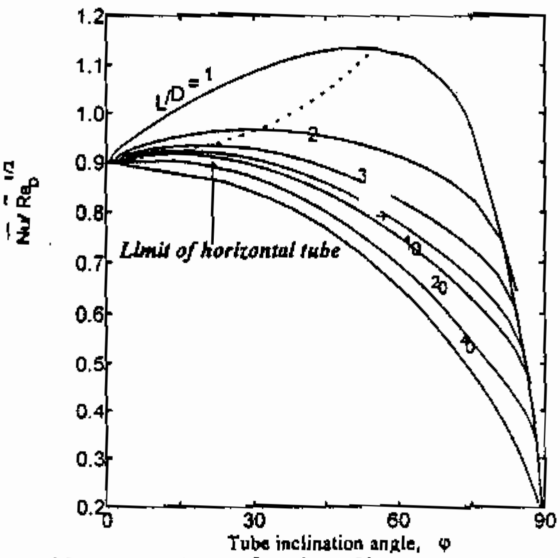

Fig. 5 Mean Nusselt number as function of inclination angle

The mean Nusselt number for the whole tube surface is plotted in Fig. 4; in the form of $\overline{Nu} / \sqrt{\cos\varphi \, \widetilde{Re}_0}$ versus the dimensionless tube length Z+=Z/(R tanφ). The results indicate that for L+ ≥ 40, the mean Nusselt number is the same as that of an infinitely long tube, given by Eq. (21).

Dependence of the mean Nusselt number on the tube inclination angle is shown in Fig. 5. Clearly, an optimum angle, at which $\overline{Nu}$ is maximum, decreases as the ratio L/D increases, as shown in Fig. 5.

CONCLUSIONS

This is the first analytical approach for solving the problem of forced-couvection laminar film condensation on an inclined circular tube. The main points that can be drawn from this study are:

1. An explicit general solution, in closed-form given by Eq. (33), has been obtained analytically for calculating the local Nusselt number.

2. Closed-form explicit expressious (24) and (25) have been obtained analytically for calculating local and mean Nusselt numbers in the case of an infinite-tube-length. These expressions can be applied with good accuracy for the inclined tubes of L/D≥40, encountered in most practical purposes.

NOMENCLATURE

C_1, C_2 integration constants

C_p specific heat of condensate at constant pressure

h_{fg} latent heat of condensation

h''_{fg} modified latent heat of condensation

D tuber diameter

k thermal conductivity of condensate

L^+ dimensionless tube length

$\dot{m}$ condensate mass flow rate per unite peripheral length

m''_c condensation mass flux

Nu $= \alpha D/k$, local Nusselt number

$\overline{Nu}_o$ $= \bar{\alpha} D/k$, mean Nusselt number

R tube radius

$\widetilde{Re}_0$ $= V_\infty D/\nu$, two-phase Reynolds number for horizontal tube

$\widetilde{Re}_z$ $= V_\infty z/\nu$, two-phase Reynolds number for vertieal tube

T temperature

ΔT $=(T_s - T_w)$, temperature drop across the condensate film

u, w condensate velocity components in x- and z- directions, respectively

V_∞ free-stream vapour velocity

x peripheral coordinate

z axial coordinate

Z^+ dimensionless axial coordinate.

Greek symbols

α $= k/\delta$, local heat transfer coefficient

δ local film thickness

Δ dimensionless local film thickness

ρ density of condensate

ϕ peripheral angle from top point of tube

φ angle of inclination of tube with horizontal

ν kinematic viscosity of condensate

$\tau_{\delta x}$ interfacial vapour shear in x-direction

$\tau_{\delta z}$ interfacial vapour shear in z-direction.

Subscripts

— mean value

Z tube length

w tube wall

s saturation

x x-direction

z z-direction

ϕ location corresponding to ϕ-angle.

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