

## EVALUATING FATIGUE LIFETIME OF POLYESTER-COTTON BLENDED YARNS

تقدير مدى عمر الخيوط المخلوطة من القطن والبولي استر  
By

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خلاصة - في هذا البحث اجريت محاولتين: لتبسيط طريقة تطبيق توزيع فييول وللتأكد من تبعية هذا التوزيع لمجموعه من النتائج العملية التي أمكن الحصول عليها بواسطة اجراء بعض اختبارات لكلل (التعب) على خيوط مخلوطة من القطن والبولي استر. ظاهرة الكلل (التعب) لهذه الخيوط بعد تعرضها لاستطاله متكرره وثني متكرر وتآكل مستمر وجد أنها تتبع جيداً توزيع فييول. احتمالية بقاء الخيوط بدون قطع بعد تعرضها لعدد معين من دورات الاجهاد أمكن حسابها بكل سهولة عطاياً ونظرياً وبالتالي أمكن تقدير مدى عمر الخيوط (عدد الدورات حتى القطع). النتائج أوضحت أن مدى العمر الاستهلاكي للخيوط يزداد بارتفاع نسبة البولي استر وأس البرم.

ABSTRACT- In this work two attempts are made to simplify the application method of Weibull's distribution and to check the fitting of this distribution to a set of experimental results obtained from the fatigue tests of cotton/polyester blended yarns. Fatigue phenomena of these yarns, which were exposed to repeated extension, repeated bending and abrasion, was found to conform well the Weibull distribution. The probability of survival of yarns, after a certain number of cyclic stresses, was calculated experimentally and theoretically and consequently the fatigue lifetime (number of cycles to rupture) of yarns could be evaluated easily. The plotted results showed that the fatigue lifetime of yarns increases when the polyester ratio and twist factor are raised.

### 1- INTRODUCTION

Fatigue properties of yarns are important not only in its applications, where the yarns are subjected in their end uses to cyclic stresses but also during weaving all types of fabric. The technological processes by which fabrics are produced impose repeated stresses on the yarns. The evaluation of fatigue performance of yarns requires a knowledge of the probability of survival or the probability of failure. Much work has been published concerning the fatigue of textile materials.

There are many problems for which the log-normal distribution and the first asymptotic are not satisfactory. For this, the only suitable distribution for fitting to experimental data is the third asymptotic or Weibull's distribution. But the mathematical treatment of Weibull's distribution is not simple, which prevents its application in generally to extend [1]. The application of Weibull's distribution has been assumed for studying fatigue phenomena of worsted yarns as a result of the different cyclic stresses [2]. The use of this distribution is discussed by several authors [3-8]. Prevorsek and Lyons [9-13] have discussed the use of the third asymptotic distribution to describe fatigue failure of single fibres. This distribution has several advantages in fatiguing processes. First, it describes the experimental data more accurately than any other distribution. Second, it gives a good representation of

phenomena that involve the fatigue of materials [7].

In the literatures [2, 14], there are some techniques that allow one to calculate the parameters of Weibull's distribution. Extreme value theory has been applied to the lifetime of cyclic loading of acrylic fibres and fabric and for worsted yarns [15]. But it has never before been applied to the blended yarns. In addition, it was very complicated

The object of this paper was to apply Weibull's distribution with a simple method for evaluating fatigue lifetime of the blended yarns and to check the fitting of this distribution to the experimental data of fatigue tests.

## 2. EXPERIMENTAL

### 2.1 Yarns Produced :

The tested blended yarns were single with a count of 15 tex ( $N_e = 40$  polyester/cotton blend of Giza 75 and polyester staple fibres. The polyester ratio in the blend was varied from 33% to 67% at a twist factor  $3.7 \alpha_e$ , but at 50% polyester/50% cotton, twist factor was varied from  $3.2 \alpha_e$  to  $4.2 \alpha_e$  with a twist direction Z. All blending operations have been carried out at the drawing stage. Combed cotton slivers (20% noil) were blended with polyester slivers at the first drawing frame and the required blend ratio was obtained by altering the sliver weight.

Given in Table 1 the main properties of the components used for producing the blended polyester/cotton yarns.

Table 1: Properties of cotton and polyester fibres

Property \ Type of fibre	Cotton	Polyester
Mean length (mm)	31	38.5
Fineness (millitex)	160	156
Tenacity (g/tex)	31.8	53.1
Extension (%)	5.62	22

To investigate fatigue problem, the blended yarns were fatigued to rupture in cyclic extension, cyclic bending and cyclic abrasion. The yarns were blended at three levels of polyester ratio (33%, 50% and 67% and at three levels of twist factor  $\alpha_e$  (3.2, 3.7 and 4.2).

### 2.2 Testing Methods of Yarns:

The yarns produced in Misr Spinning & Weaving Co. in El-Mehalla El-Kubra were fatigued under cyclic tension, bending and abrasion in the laboratories of Dept. of science of Textile materials in Moscow Textile Institute.

#### 2.2.1 Resistance of yarns to repeated extension:

The apparatus (PH-5) on which ten yarn specimens can be simultaneously fatigued in cyclic extension, at stroke of the upper jaw 4 mm, has been used. The frequency of the fatigue cycles could be set at 300 cycles/min. The apparatus subjects tested lengths of 500 mm to repeated

extension. The tested lengths are subjected to a static load (25% of the breaking load). 50 specimens of braided yarns were tested. The schematic diagram of this apparatus is shown in Fig. 1.

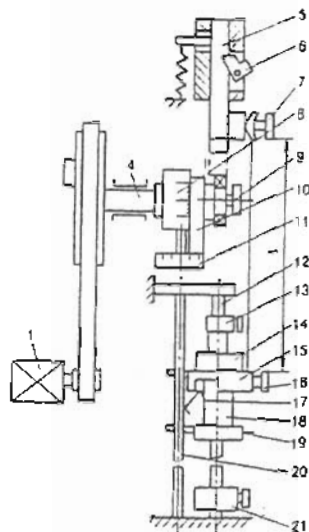


Fig. 1: Diagram of pulsator (NH-5)

The movement from the electric motor 1 is given by the belt drive to the shaft 4 with which the sinus head 8-11 is rigidly connected; this motion reaches to the pusher 5 and the upper jaw 7 reciprocates. The yarn is fixed in the upper jaw 7 and the lower jaw 16 located on the side of the lock 15. The latter can move only downwards together with the lower jaw 16 along the guiding rod 12 when the residual cyclic deformation of the yarn appears and can be smoothly fixed when the yarn is tensioned. The bottom lock 18 is used for reaching the lower jaw 16 to the initial position and for fixing it. This lock is shifted by pressing the tongue of the locator 19. The lower lock 18 has a contact 17 connected with the electric cycle counter. When the yarn breaks, the contact 17 coacting with the contact wire 20 opens and the tester stops. The number of cyclic extension is recorded by the counters disposed on the front panel of the control block adjoining the tester (not shown on the drawing). The number of counters corresponds to the number of simultaneously tested yarns, i.e. 10.

Test method: First the initial length between the upper jaw 7 and the lower jaw 16 is adjusted (see Fig. 1). For this, the upper jaw 7 is fixed by the catch 6 and the lower jaw 16 together with the lock 18 is shifted to a necessary distance along the guiding rod and fixed with the rest 13. The stroke of the upper jaw is adjusted by the micrometric screw 3 on the scale 1 (see Fig. 2) (with 2 mm divisions) of the sine head (Fig. 2) after slackening the stop screw 2. After adjusting the desired stroke of the upper jaw the top screw 2 is tightened as far as it reaches. Consequently, the static load on the yarn is applied. The yarn is fixed in the upper jaw 7 and a weight of preliminary tension is suspended from its free end. The mass of this weight is chosen depending on the linear density of the yarn. Consequently, the yarn is attached in the bottom jaw 16. The counters of the number of cycles 1 on

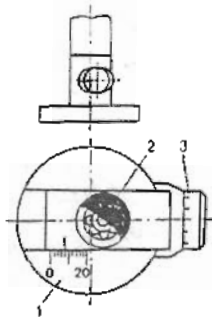


Fig. 2: Sine head

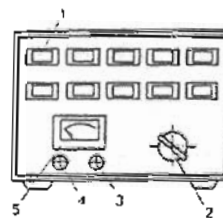


Fig. 3: Control block of pulsator (PH-5)

the control unit (Fig. 3) are set at zero. With the "start" push button 4 depressed first turn the handle 2 counterclockwise as far as it will go and then smoothly turn it clockwise for the necessary number of cycles (within 0 - 1000 cycles per unit). This number is controlled in the bottom zone of scale 5. The upper jaw 7 (see Fig. 1) is carefully removed from the catch 6 which ensures the contact of pusher 5 with the sine head. Having pressed the tongue of locator 19, the lock 18 is lowered; the yarn is now under tension. On lowering the lock 18 the counter of cycles switches on automatically. When the yarn breaks, the lock 18 with weight 14 falls on the lower rest 21 and breaks the contact 17 and the counter switches off. The tester is stopped by pressing the "stop" push button 3 (Fig. 3).

#### 2.2.2 Resistance of yarns to repeated bending:

The apparatus DP-5/3, on which three yarn specimens can be simultaneously fatigued in cyclic bending, was used. The angle of bending being  $180^\circ$  and the frequency 100 cycles/min. The applied load on yarn specimens was 25% of the breaking load. (50 specimens of blended yarn were tested). The diagram of the Flexometer (DP-5/3) is shown in Fig. 4.

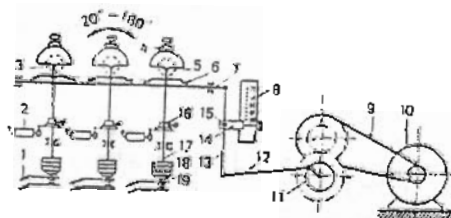


Fig. 4: Diagram of tester (DP-5/3)

Test method: The specimens are fixed in the clamps 4 and 16. The tester has three pairs of clamps can be used for testing simultaneously three specimens. The upper clamps having at their end a pair of jaws 3 with a radius of bending working surfaces ( $r = 0.2 \text{ mm}$ ). The lower clamps 16 freely move in their supports and may be slightly raised for fixing the specimens. The specimens are inserted first into the open top clamp 4 and then into the raised bottom clamp 16. The bottom clamp is closed after ensuring that the specimen has been fixed. After keeping the yarn specimen in straightened condition the top clamp is closed. The static load is applied to the yarns by means of the weights 18 which are carefully placed on the supporting disc 19 of the suspension 17 connected

with the bottom clamp. Further, the motor 10 is switched on which the shaft 11 is rotated at 100 rpm by one belt drive 9 and gears. The shaft 11 carries the crank 12 connected with the rod 13. The upper loose end of the connecting rod is articulated with slides 7 to which a rectilinear motion is imparted. The link 14 serves to change the position of the rotation centre 15 of the connecting rod for increasing or decreasing the reciprocatory motion of the slides. Thus, the bending angle is adjusted on the scale 8 within  $20^{\circ}$  -  $180^{\circ}$ . Three pairs of toothed racks 6 are attached to the top side of the slides. Each pair of racks meshes with a pair of gears 5 whose shafts ends protrude from the tester body at the face side. On each end of the gear shaft there is one top clamp. Each pair of clamps is provided with a separate counter 2 recording the endurance i.e. the number of cycles of double flexing of the yarn until it breaks.

When the first and second yarn specimens break, only their counters are turned off. After the breakage of the third specimen, not only the counter but the motor is also switched off. Switching-on and switching-off of counters is effected by means of electric contacts 1 disposed under the weighting suspension 19.

### 2.2.3 Resistance of yarns to abrasion:

To measure the yarn abrasion resistance a universal tester (Fig. 5), on which ten yarn specimens can be simultaneously fatigued in cyclic abrasion, was used. Any angle of abrasion may be set in the range of  $0 - 120^{\circ}$  in this tester. In this tester, the plate 7 reciprocates and the rubbing of one yarn against another occurs in the zone of loop intersection, which is obtained by special threading of the yarn. In the present case, the yarn is not only a specimen but also an abrasive.

Start motor by pressing the "start" button. The carriage speed (80 cycles per minute) is set by turning the knob on the back wall of the tester (not shown in Fig. 5) in position corresponding to the required speed.

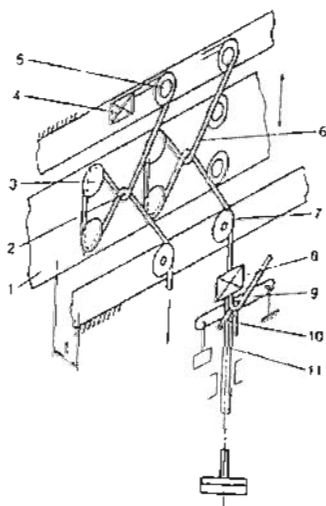


Fig. 5: Schematic diagram of the abrasion-resistance tester

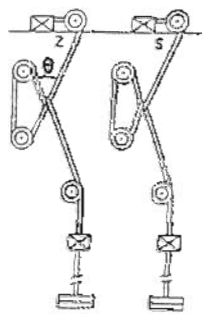


Fig. 6: Diagram of yarn threading into the tester clamps

The movable plate 1 is set for an abrasion angle  $\alpha = 90^\circ$  by means of a stop screw, while the counter is set to zero by means of clearing knobs. The suspensions 11 with bottom clamps 8 are set in threading position, for this the handle on the side face of the tester is turned clockwise until a click sounds.

Test method: The yarns 6 are inserted into the tester clamps with the account of the direction of final twist according to the diagram in Fig. 6.

A yarn 0.5 metre long is unwound from the package. One end of the yarn is threaded into the upper clamp 4, while the second end is run around the guiding bush 5 and bushes 3 mounted on the movable plate 1. As a result, on the bushes 3 loop 2 of two yarn portions is formed and the loop intersection is at one level with the central mark of plate 1. The loose end of yarn running around the guiding roller 7 is passed through the bottom clamp 8 and then is run around pin 10 on the preloading lever 9; at the right end of this lever there is a weight ensuring the application of a preliminary load on the yarn. When the pin 10 is raised into horizontal position, the jaws of the bottom clamp are closed. The weight for preloading is chosen, depending on the yarn linear density.

When threading, it is necessary to make a difference between the yarns of final (Z) and (S) twist (see Fig. 6).

The working load on the yarn is taken 1 cN/tex. The suspensions with weights are moved in working position by turning the handle, fixed on the side wall of the tester, counterclockwise which the suspensions remain hanging on the tested yarns.

The movable plate 1 is set for the working abrasion angle of  $90^\circ$ . When the plate 1 is set for the working abrasion angle the loop intersection must be in coincidence with the central mark on the plate. Then the motor is started by the "start" push button. The carriage starts its reciprocation and the yarn is abraded at the point of loop formation. After the breakage of one of the yarns, which is indicated by stoppage of the motor, the yarn lengths squeezed in the top and bottom clamps are removed from the bushes of movable plate 1 to give them free sagging in vertical position so that they cannot impede the abrasion of remaining yarns.

Then the counter readings, i.e. the number of cycles needed to break the given sample, are taken. The motor is started again by depressing the "start" push button. These operations are repeated until the testing of all ten yarns is completed. In total not less than 50 tests are carried out.

3- APPLICATION METHOD

According to the Weibull's distribution [2,4,8,11,12], the probability of survival to X cycles is given by:

$$Q(X) = \exp \left[ - \left( \frac{X-X_0}{V-X_0} \right)^K \right] \quad \text{at} \quad \begin{matrix} X > X_0 \\ K > 1 \end{matrix} \quad \dots\dots(1)$$

where X is the number of cycles to rupture, V is the characteristic extreme life, X<sub>0</sub> is the minimum life and K is a shape parameter.

The three parameters K, V and X<sub>0</sub> of this distribution can be computed, by a method developed by Freudenthal and Gumbel [17, 18], from the sample average  $\bar{X}$ , standard deviation  $\sigma$  and skewness  $\mu_3/\sigma^3$ . These values can be obtained from the experimental results by familiar methods. The skewness can be used to calculate K as follows:

$$\frac{\mu_3}{\sigma^3} = \left[ \Gamma \left( 1 + \frac{3}{K} \right) - 3 \Gamma \left( 1 + \frac{2}{K} \right) \cdot \Gamma \left( 1 + \frac{1}{K} \right) + 2 \Gamma^3 \left( 1 + \frac{1}{K} \right) \right] \cdot \left[ \Gamma \left( 1 + \frac{2}{K} \right) - \Gamma^2 \left( 1 + \frac{1}{K} \right) \right]^{-\frac{3}{2}} \dots(2)$$

where  $\mu_3$  = the third moment of sample, and  $\Gamma$  = complete gamma function.

The parameters V and X<sub>0</sub> can be obtained by solution of the following equations:

$$\frac{V - X_0}{\sigma} = \left[ \Gamma \left( 1 + \frac{2}{K} \right) - \Gamma^2 \left( 1 + \frac{1}{K} \right) \right]^{-\frac{1}{2}} \quad \dots\dots(3)$$

and 
$$\frac{V - \bar{X}}{\sigma} = \left[ 1 - \Gamma \left( 1 + \frac{1}{K} \right) \right] \cdot \left[ \Gamma \left( 1 + \frac{2}{K} \right) - \Gamma^2 \left( 1 + \frac{1}{K} \right) \right]^{-\frac{1}{2}} \dots(4)$$

The average of Weibull's distribution is dependent on the parameters K, V and X<sub>0</sub> according to the following relationships: The average is given by

$$\bar{X} = X_0 + (V - X_0) \Gamma \left( 1 + \frac{1}{K} \right) \quad \dots\dots(5)$$

By means of knowing the parameters of K, V and X<sub>0</sub> the distribution can be plotted on the Weibull probability paper which was constructed by Gumbel[17], and a straight line can be fitted to satisfy the parameters of K, V and X<sub>0</sub> for all the experimental points.

This common method of application involves complicated calculations for facilitating the calculations in this method an attempt has been made[12,16,19] already to simplify the application method by assuming that the minimum life is equal to zero and thus the expression for the probability of survival reduces to

$$Q(X) = \exp - \left( \frac{X}{V} \right)^K \quad \dots\dots(6)$$

Therefore, it is necessary to establish another method not complicated and contains the minimum life (X<sub>0</sub>).

On the other hand the mathematical expression of Weibull's distribution [5] being as follows:

$$Q(X) = \begin{cases} \exp \left[ - \left( \frac{X - X_0}{V} \right)^K \right], & \text{at } X \geq X_0 \\ 1 & \text{at } X < X_0 \end{cases} \quad \dots\dots(7)$$

This particular distribution has several advantages. First, it describes the experimental data more accurately than any another distribution. Second, it was felt to be the most accurate representation of the phenomena being examined.

The use of this distribution for analysis is discussed by Bolyvin [5]. In which the author calculated the parameters of distribution ( $K$ ,  $V$ ,  $X_0$ ) from the experimental data with a more complicated method. Then the mathematical expression in Eq. 7 was taken into consideration in order to simplify its calculations.

In our method it could be suggested that the minimum life,  $X_0$ , can be obtained [2] by the following equation:

$$X_0 = V - \sigma \cdot B \quad \dots\dots(8)$$

$$\text{where } B = \frac{V - X_0}{\sigma} = \left[ \Gamma^2 \left( 1 + \frac{2}{K} \right) - \Gamma^2 \left( 1 + \frac{1}{K} \right) \right]^{-\frac{1}{2}}$$

But the parameters of distribution ( $K$ ,  $V$ ) could be calculated [16] from the experimental data as shown in the following example. Also the average is given [8] by Eq. 5. And when the minimum life  $X_0$  is known, the calculations of the probability of survival are simplified.

In this case, the following method is intended for application Weibull's distribution when the sample size ( $n$ ) is not less than 50. The successive steps which can be followed are:

- i) determine the average of class,  $X_i$ ;
- ii) determine the absolute frequency,  $f_i$ ;
- iii) determine the accumulative absolute frequencies,  $\sum f_i$ ;
- iv) determine the accumulative frequency,  $\omega_i$ ;

$$\text{where } \omega_1 = \frac{\sum f_i + \sum f_{i-1}}{2n}$$

- v) find the value of recurrence numbers (deviations from mode)  $y_i$ , corresponded to  $\omega_i$ , from Table [38 Ref. 16] (see Table I in Appendix);
- vi) plot the values of  $y_i$  against  $\log X_i$  on Gumbel probability paper as constructed by Salaveev and Kereokhin [16];
- vii) calculate the average of the sample,  $\bar{X}$ , and the standard deviation,  $\sigma$ .

An example of the application method will be given as follows:



Table 1: Values of experimental and theoretical recurrence numbers.

Resistance to repeated extension	Average (cycles), $\bar{X}_i$	Absolute frequency $f_i$	Accumulative absolute frequency $\sum f_i$	Accumulative frequency $W_i$	Log $X_i$	Experimental recurrence number $y_i$	Theoretical recurrence number $Y_i$
1600-1799	1699.5	1	1	0.01	3.23	-1.53	-1.20
1400-1599	1499.5	4	5	0.06	3.18	-1.04	-1.00
1200-1399	1299.5	6	11	0.16	3.11	-0.605	-0.73
1000-1199	1099.5	3	14	0.25	3.04	-0.33	-0.455
800-999	899.5	8	22	0.36	2.95	-0.025	-0.102
600-799	699.5	5	27	0.49	2.85	0.332	0.288
400-599	499.5	8	35	0.62	2.70	0.783	0.874
200-399	299.5	12	47	0.82	2.48	1.62	1.734
0-199	99.5	3	50	0.97	2.00	3.49	3.613

$$\bar{X} = 748, \quad \sigma = 441.02$$

These results were obtained for the resistance to repeated extension of 15 tex (40 Ne) blended yarns of sample No.1 (33% polyester/67% cotton at twist factor 3.7  $\alpha_e$ ). The data are shown in Table 1 and the plot of the distribution on a Gumbel's probability paper is given in Fig. 7.

From a knowledge of  $\bar{X}$  and  $\sigma$ , it is possible to find the shape parameter,  $K$ , and the characteristic extreme life,  $V$ , by means of Table II in Appendix [Ref. 16] and the following equations:

$$\bar{X} = L_1 \cdot V \quad \dots\dots(9)$$

$$\sigma = L_2 \cdot V \quad \dots\dots(10)$$

where  $L_1 = \Gamma(1 + \frac{1}{K})$ ,  $L_2 = \Gamma(1 + \frac{2}{K}) - L_1^2$

In this case, it was found that  $K = 1.7$  and  $V = 838.57$ . Then the theoretical recurrence numbers,  $Y_i$ , of the distribution can be calculated [16] by the following equation:

$$Y_i = 2.3 K (\log V - \log X_i) \quad \dots\dots(11)$$

If a straight line can be fitted to the experimental data, it will be an indication of the correctness of Weibull's distribution.

But the minimum life,  $X_0$ , can be obtained [17, 18] from Eq. 8. It was found that  $X_0$  is equal to 19.08 cycles. The calculated average of sample,  $\bar{X}$ , can be obtained by substituting in Equation 5. It was found that  $\bar{X} = 750.4$  cycles.

From a knowledge of the parameters  $K$ ,  $V$  and  $X_0$ , the calculations of the probability of survival are simplified compared to the previous mentioned method.

In the previous example, the estimated parameters could be calculated by the two methods. These parameters are given in Table 2.

Table 2: Estimated parameters of Weibull's distribution for resistance to repeated extension of blended yarns (Sample No. 1) by the two mentioned methods.

Estimated parameters	Method I	Method II
	$Q(X) = \exp \left[ -\left( \frac{X-X_0}{V-X_0} \right)^K \right]$	$Q(X) = \exp \left[ -\left( \frac{X-X_0}{V} \right)^K \right]$
Measured average number of cycles to rupture, $\bar{x}$	748.00	748.00
Standard deviation, $\sigma$	441.02	441.02
Sample skewness, $\mu_3/\sigma^3$	0.3997	0.3997
Shape parameter, $K$	2.058	1.70
Characteristic extreme life, $V$	859.5	838.57
Minimum life, $X_0$	-117.298	19.08
Reduced average for characteristics extreme, $(V-\bar{x})/\sigma$	0.2528	0.205
Reduced average for minimum life, $(V-X_0)/\sigma$	2.2149	1.096
Calculated average number of cycles to rupture	748.00	750.06

Table 3: Values of  $Q(X)$  calculated by the two methods at different values of number of cycles to rupture.

Number of cycles to rupture $X_i$	Method I	Method II
	$Q(X) = \exp \left[ -\left( \frac{X-X_0}{V-X_0} \right)^K \right]$	$Q(X) = \exp \left[ -\left( \frac{X-X_0}{V} \right)^K \right]$
100	0.9558	0.9814
300	0.8408	0.8557
500	0.6781	0.6779
700	0.5003	0.4956
900	0.33716	0.33710
1100	0.2072	0.2144
1300	0.1162	0.1281
1500	0.0592	0.0721
1700	0.0275	0.0383

To check the fitting of Weibull's distribution to the experimental results, the deviation from the mean  $x = (X_i - \bar{X})$  must be found. On the basis of these values the theoretical absolute frequency,  $F_i$ , can be calculated by the following equation:

$$F_i = \frac{I \cdot n}{\sigma} \cdot \frac{e^{-\frac{1}{2} \left( \frac{x}{\sigma} \right)^2}}{\sqrt{2\pi}} \dots\dots(12)$$

where  $I$  = interval of class; and  
 $n$  = total number of measurements.

Then the differences between the absolute frequencies of the experimental distribution and the values of the theoretical frequencies have also been calculated. Hence, by means of the value of  $\chi^2$  test (Table 4) the goodness of fitting of the distribution can be checked.

Table 4:  $\chi^2$  value and the differences between the experimental and theoretical absolute frequencies.

Resistance to repeated extension	$x_i$	$x = x_i - \bar{x}$	$\frac{x}{\sigma}$	Frequencies		$\frac{(f_i - F_i)^2}{F_i}$
				Experimental $f_i$	Theoretical $F_i$	
1600-1799	1699.5	951.5	2.1586	1	0.882	0.0158
1400-1599	1499.5	751.5	1.7051	4	2.120	1.6672
1200-1399	1299.5	551.5	1.2516	6	4.144	0.8313
1000-1199	1099.5	351.5	0.9815	3	5.603	1.2093
800-999	899.5	151.5	0.3447	8	8.547	0.0350
600-799	699.5	-48.5	-0.1088	5	9.016	1.7888
400-599	499.5	-248.5	-0.5623	8	7.744	0.0084
200-399	299.5	-448.5	-1.0158	12	5.415	8.0078
0-199	99.5	-648.5	-1.4693	3	3.082	0.0022

Value of  $\chi^2 = (13.566)$

Whereas the frequencies are grouped into nine cells, the degree of freedom,  $S$ , will be  $9-4 = 5$ , where  $4 = 1 + \text{number of parameters of the used Weibull distribution } (V, K, X_0)$ . For the five degrees of freedom, the tables gives  $\chi^2 = 15.1$  at the 0.01 probability level. Since the calculated value of  $\chi^2$  is not significant. Also person's criteria  $K_r$  is less than three (constant value for all the distributions) as shown from the following expression:

$$K_r = \frac{|\chi^2 - S|}{(2S)^{\frac{1}{2}}} = \frac{13.566 - 5}{(2 \times 5)^{\frac{1}{2}}} = 2.7 < 3 \quad \dots \dots (13)$$

Therefore, it can be concluded that the experimental results are found to conform the Weibull distribution.

Also it is possible to check the goodness of fitting of the distribution by the Kolmogorov - Smirnov test ( $\lambda$ ). From Table 5, the maximum observed difference  $D_m$  for  $|\sum W_i - \sum W|$  is 0.1724,

$$\text{then } \lambda = D_m \sqrt{n} = 0.1724 \sqrt{50} = 1.219 \quad \dots \dots (14)$$

Consequently, the value of  $P(\lambda)$  can be determined from Table [6 Ref. 16] (see Table IV in Appendix) and it is equal to 0.112. This value is more than the 0.05 probability level; then the Kolmogorov - Smirnov test, will lead to the same conclusion in the case of  $\chi^2$  test.

Table 5: Differences between the experimental and theoretical accumulative relative frequencies.

Class	Absolute frequency $f_i$	Relative frequency $W_i$	Accumulative relative frequency $\sum W_i$	$t = \frac{x}{\sigma}$	Theoretical accumulative frequency $\sum W^*$	$ \sum W_i - \sum W^* $
0-199	3	0.06	0.06	-1.469	0.0712	0.0112
200-399	12	0.24	0.30	-1.015	0.1544	0.1456
400-599	8	0.16	0.46	-0.562	0.2876	0.1724 = $D_m$
600-799	5	0.10	0.56	-0.109	0.4561	0.1039
800-999	8	0.16	0.72	0.344	0.6320	0.088
1000-1199	3	0.06	0.78	0.982	0.836	0.056
1200-1399	6	0.12	0.90	1.252	0.894	0.006
1400-1599	4	0.08	0.98	1.705	0.955	0.025
1600-1799	1	0.02	1.00	2.159	0.984	0.016

50

\*  $\sum W$  is determined from Table III in Appendix.

## 4- RESULTS

The results have been graphically plotted on Gumbel probability paper. Figures 7 and 8 show the distributions of the yarns tested for resistance to repeated extension.

Similarly, Figures 9 and 10 show the distributions of the resistance to repeated bending presented on a probability paper. And finally, Figures 11 and 12 show similar plots for the tests of the resistance to abrasion of yarns. It can be seen that most of the points fall on a straight line.

In order to estimate the probability of survival to a given lifetime or conversly, to determine fatigue lifetime (number of cycles to rupture) at which a specimen has a selected probability of survival, it is necessary to calculate the parameters of distribution  $K$ ,  $V$  and  $X_0$ .

Various estimated parameters have been calculated for the lifetime results plotted on Figures 7 - 12. The results are presented in Table 6. These parameters were calculated from the experimental data as indicated in Section 3.

From a knowledge of the minimum average number of cycles to rupture,  $\bar{X}$  min, for the yarns plotted on each figure, the corresponded probability of survival can be determined easily and compared in order to evaluate the quality of different yarns. For example, for the sample No. 1 in Fig. 7 at  $X_i = 748$  cycles,  $Q(X)$  is equal to 0.45 or 45%, it means that the probability of breakage is 55%, i.e. at 748 cycles of repeated extension for all specimens, 55% of the tested specimens will be broken.

It could be remembered that the value of the probability of survival is dependent upon many factors such as yarn characteristics, used apparatus and testing conditions.

Conversly, the fatigue lifetime for the resistance to repeated extension, bending and abrasion of the yarns can be evaluated with a higher accuracy, especially, at 0.9 probability of survival as shown

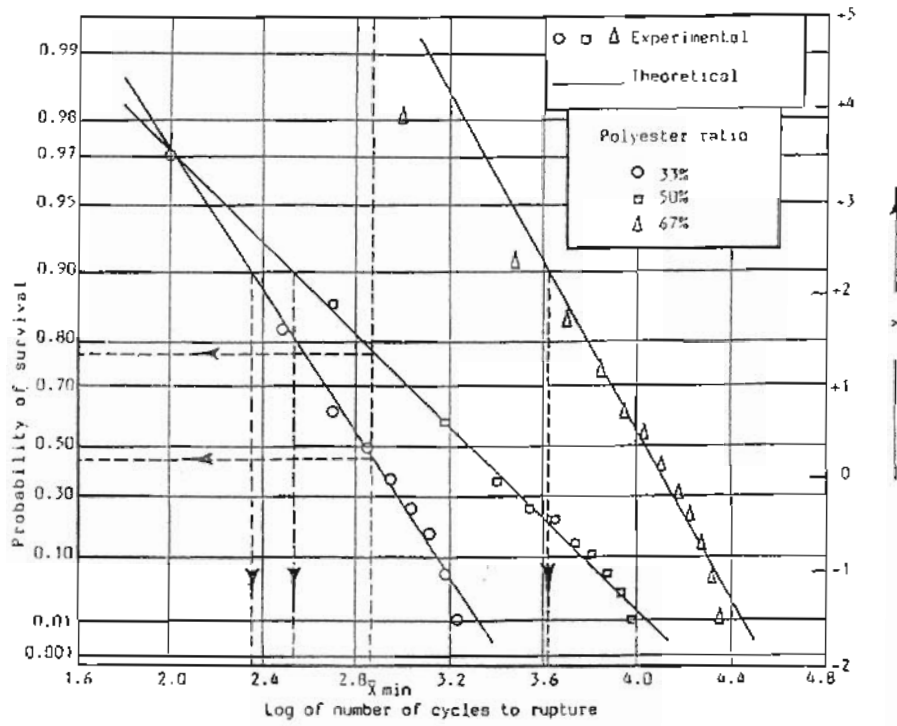


Fig. 7: Probability of survival plotted on Gumbel probability paper for blended yarns at various blend ratios fatigued in cyclic extension.

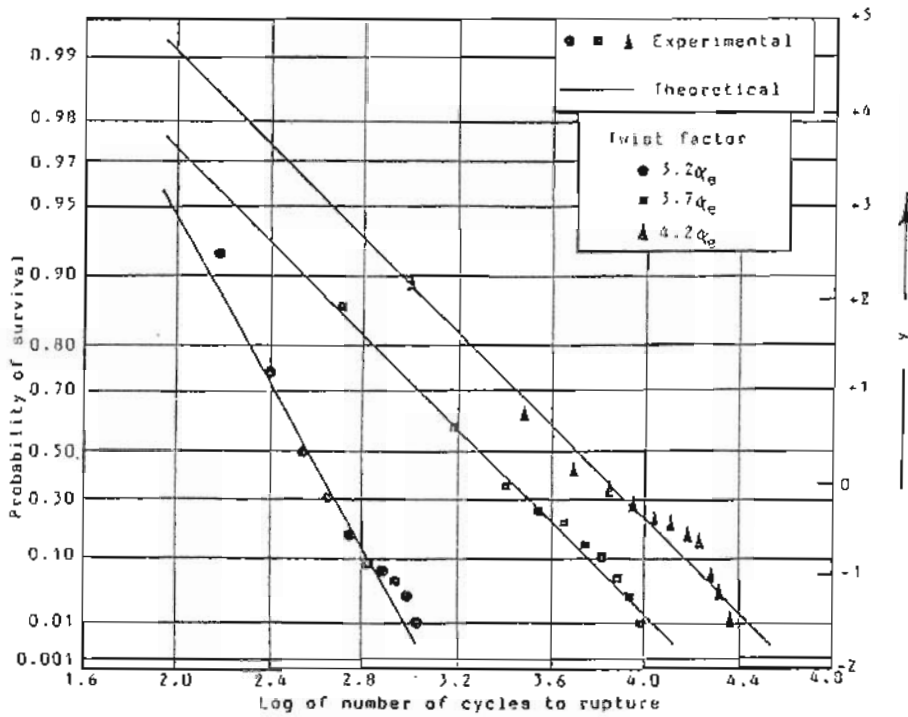


Fig. 8: Probability of survival plotted on Gumbel probability paper for blended yarns at various twist factors fatigued in cyclic extension.

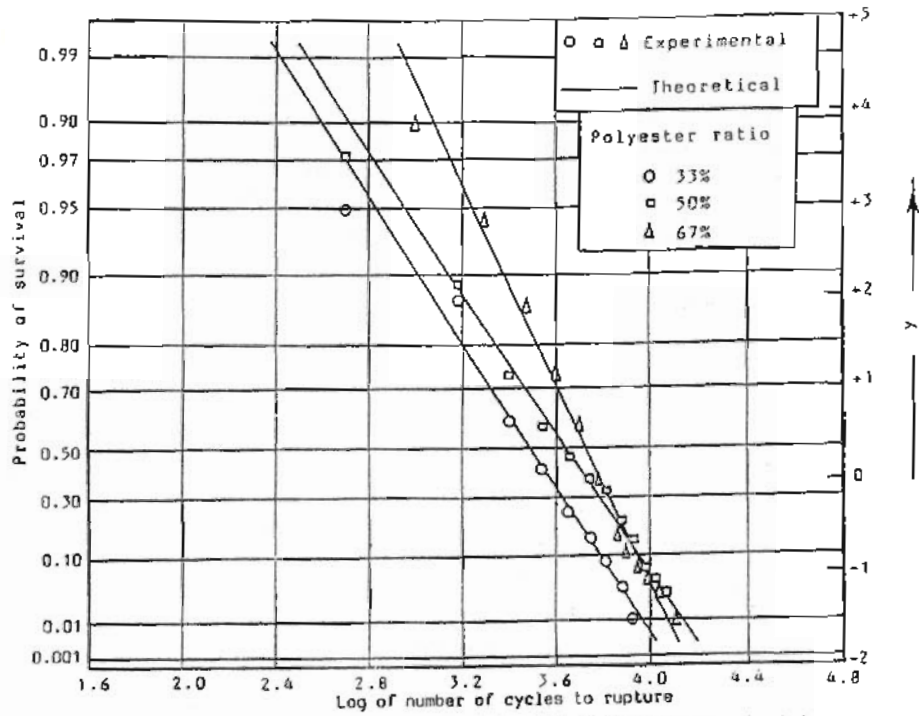


Fig. 9: Probability of survival plotted on Gumbel probability paper for blended yarns at various blend ratios fatigued in cyclic bending.

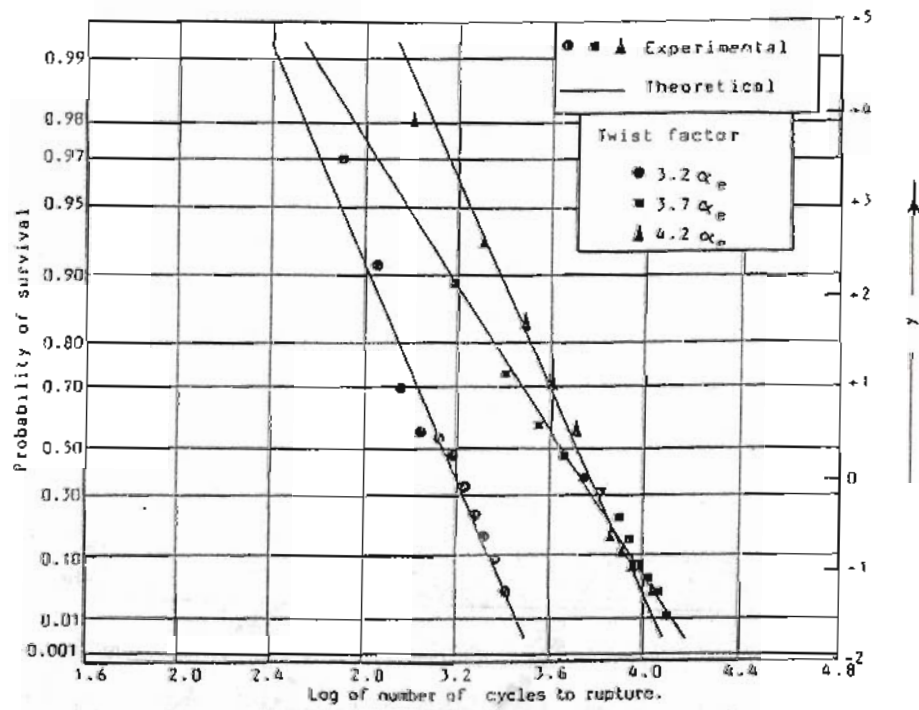


Fig. 10: Probability of survival plotted on Gumbel probability paper for blended yarns at various twist factors fatigued in cyclic bending.

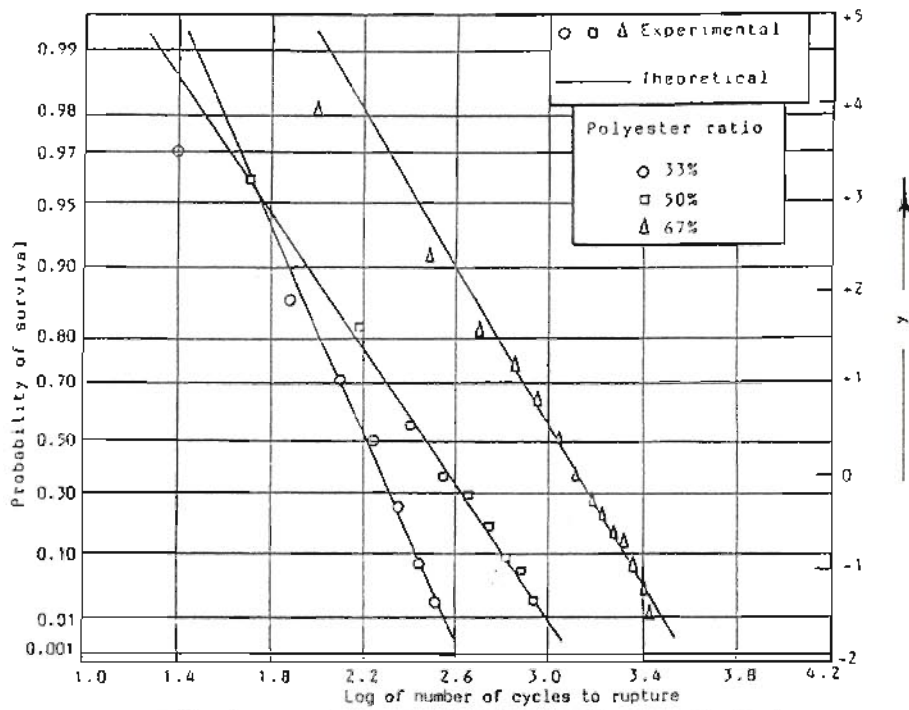


Fig. 11: Probability of survival plotted on Gumbel probability paper for blended yarns at various blend ratios fatigued in cyclic abrasion.

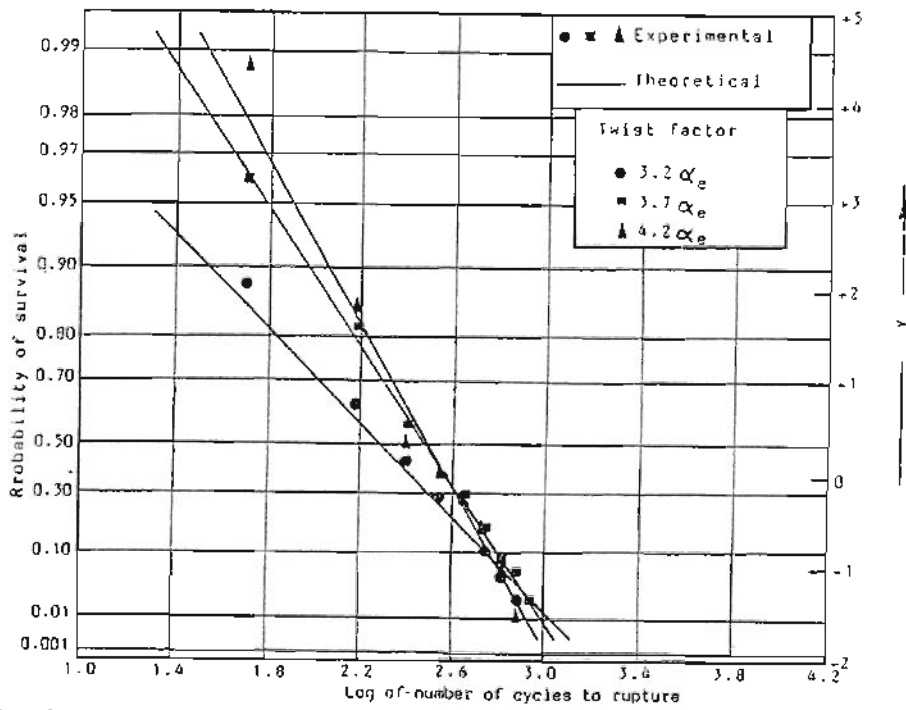


Fig. 12: Probability of survival plotted on Gumbel probability paper for blended yarns at various twist factors fatigued in cyclic abrasion.

Table 6: Estimated parameters for resistance to repeated extension, repeated bending and abrasion of blended yarns .

Estimated parameters	Resistance to repeated extension, cycles						Resistance to repeated bending, cycles						Resistance to abrasion, cycles					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
	50%P	67%P	3.2 $\alpha_e$	3.7 $\alpha_e$	4.2 $\alpha_e$	33%P	50%P	67%P	3.2 $\alpha_e$	3.7 $\alpha_e$	4.2 $\alpha_e$	33%P	50%P	67%P	3.2 $\alpha_e$	3.7 $\alpha_e$	4.2 $\alpha_e$	33%P
Measured average number of cycles to rupture, $\bar{x}$	748	2560	11520	386	2560	6680	3400	4780	5380	1392	4780	5260	169	334	1160	258	334	332
Standard deviation, $\sigma$	441.02	2310	5957	202.74	2310	6227	1836	2960	2252.9	582.7	2960	2235.9	75.26	208.2	650.2	234.3	208.2	177.4
Sample skewness, $M_3/\sigma^3$	0.3997	1.348	0.015	1.290	1.348	1.099	0.748	0.628	0.5923	0.518	0.628	0.671	-0.077	0.825	0.426	0.496	0.825	0.574
Shape parameter, $K$	1.7	1.1	2.0	1.99	1.1	1.1	1.75	1.7	2.7	2.6	1.7	2.5	2.46	1.635	1.848	1.1	1.635	1.9
Characteristic extreme life, $V$	838.57	2661.1	13002	435.7	2661.1	6943.9	3820	5358.7	6055	1565.8	5358.7	5928.4	190.53	373.8	1306.3	268.2	373.81	373.87
Minimum life, $X_0$	19.08	34.42	142.8	-1.951	34.42	-137.0	321.5	-136	-292.9	-24.5	-135.5	31.6	-5.92	3.59	0.488	1.85	1.57	7.45
Reduced average for characteristic extreme, $(V-\bar{x})/\sigma$	0.205	0.044	0.249	0.245	0.044	0.042	0.229	0.196	0.30	0.30	0.196	0.30	0.29	0.19	0.225	0.044	0.19	0.236
Reduced average for minimum life, $(V-X_0)/\sigma$	1.858	1.137	2.159	2.159	1.137	1.137	1.91	1.86	2.82	2.73	1.86	2.64	2.61	1.78	2.0	1.137	1.78	2.066
Calculated average number of cycles to rupture, cycles, $\bar{x}$	750.4	2569.8	11539	385.9	2569.8	6697.7	3435.2	4767.5	5352	1388.5	4767.5	5263.6	168.1	334.8	1160.2	258.9	334.8	332.7

\* P - means polyester fibres.



in Figures 7 - 12. It could be seen that the larger polyester ratio and twist factor the higher number of cycles to rupture.

#### 5- DISCUSSION

Figures 7 - 12 show that Weibull distribution seems to agree well with the observed phenomena. A plot of this distribution on Gumbel's probability paper is linear. In relation to the parameters of Weibull's distribution it could be observed that the value of shape parameter (K) was larger than unity and ranges from 1.1 to 2.7. The characteristic extreme life (V) being larger than the average. In all cases, the value of the characteristic extreme life, V, increases as the twist factor and polyester ratio of the yarn increases.

#### 6- CONCLUSIONS

A simplified method for application of Weibull's distribution to a set of experimental results, obtained in the fatigue testing of blended yarns, has been established. Weibull distribution gives a good fit for the resistance to repeated extension, repeated bending and abrasion of blended yarns. By means of Gumbel's probability paper, it is easy to evaluate the quality of yarns. The number of early failures of yarns can be expected (at a selected probability) to be reduced substantially especially on the weaving machine (the loom) which the yarns are exposed to repeated stresses continually. Thus, it is more useful for adjusting the tension on warp yarns without more breaks in order to improve loom efficiency. The application of Weibull's distribution to industrial problems and to certain laboratory tests leads to satisfactory results.

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## APPENDIX

Table I: Calculation of the recurrence number,  $y_i$ .

$\omega_i$	$y_i$	$\omega_i$	$y_i$	$\omega_i$	$y_i$
0.0000	-	0.30	-0.18	0.93	2.62
0.0001	-2.22	0.35	-0.05	0.94	2.78
0.0005	-2.03	0.37	0.00	0.95	2.97
0.001	-1.93	0.40	0.09	0.96	3.20
0.003	-1.76	0.45	0.22	0.97	3.49
0.005	-1.67	0.50	0.36	0.975	3.67
0.010	-1.53	0.55	0.51	0.980	3.90
0.015	-1.44	0.60	0.67	0.985	4.19
0.020	-1.36	0.65	0.84	0.987	4.33
0.025	-1.30	0.70	1.03	0.990	4.60
0.030	-1.25	0.75	1.24	0.991	4.71
0.040	-1.17	0.77	1.34	0.992	4.82
0.050	-1.10	0.80	1.50	0.993	4.96
0.070	-0.98	0.83	1.68	0.994	5.11
0.100	-0.83	0.85	1.82	0.995	5.30
0.130	-0.71	0.87	1.97	0.996	5.52
0.150	-0.64	0.89	2.15	0.997	5.81
0.170	-0.57	0.90	2.25	0.998	6.21
0.200	-0.48	0.91	2.36	0.999	6.91
0.250	-0.33	0.92	2.48	0.9999	9.21

Table II: Calculation of the shape parameter,  $K$ .

$\bar{x}/\sigma$	0.063	0.19	0.32	0.45	0.58	0.68	0.79	0.90	1.0	1.48	1.93	2.34	2.70	3.5
$L_1$	120	8.86	3.32	2.00	1.50	1.27	1.13	1.05	1.0	0.90	0.89	0.89	0.89	0.9
$L_2$	1900	46.9	10.4	4.47	2.61	1.86	1.43	1.17	1.0	0.61	0.46	0.38	0.33	0.2
$K$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0	4.0

Table III: Calculation of the value,  $\Sigma W$ 

t	$\Sigma W$	t	$\Sigma W$	t	$\Sigma W$	t	$\Sigma W$
-3.1	0.001	-1.5	0.067	0.0	0.500	1.6	0.945
-3.0	0.001	-1.4	0.081	0.1	0.540	1.7	0.955
-2.9	0.002	-1.3	0.096	0.2	0.579	1.8	0.964
-2.8	0.003	-1.2	0.115	0.3	0.618	1.9	0.971
-2.7	0.003	-1.1	0.136	0.4	0.655	2.0	0.977
-2.6	0.005	-1.0	0.159	0.5	0.691	2.1	0.982
-2.5	0.006	-0.9	0.184	0.6	0.726	2.2	0.986
-2.4	0.008	-0.8	0.212	0.7	0.758	2.3	0.989
-2.3	0.011	-0.7	0.242	0.8	0.788	2.4	0.992
-2.2	0.014	-0.6	0.274	0.9	0.816	2.5	0.994
-2.1	0.018	-0.5	0.308	1.0	0.841	2.6	0.995
-2.0	0.023	-0.4	0.345	1.1	0.864	2.7	0.997
-1.9	0.029	-0.3	0.382	1.2	0.885	2.8	0.997
-1.8	0.036	-0.2	0.421	1.3	0.903	2.9	0.998
-1.7	0.045	-0.1	0.460	1.4	0.919	3.0	0.999
-1.6	0.055	0.0	0.500	1.5	0.933	3.1	0.999

Table IV: Calculation of the value,  $P(\lambda)$ 

$\lambda$	$P(\lambda)$	$\lambda$	$P(\lambda)$	$\lambda$	$P(\lambda)$	$\lambda$	$P(\lambda)$
0.3	0.999	0.9	0.393	1.4	0.040	2.0	0.0007
0.4	0.997	1.0	0.270	1.5	0.022	2.1	0.0003
0.5	0.964	1.1	0.178	1.6	0.012	2.2	0.0001
0.6	0.864	1.2	0.112	1.7	0.006	2.3	0.0001
0.7	0.711	1.3	0.068	1.8	0.003	2.4	0.0000
0.8	0.544	1.36	0.050	1.9	0.002	2.5	0.0000