SENSTIVITY OF SYSTEM RELIABILITY INDICES TO FORECASTED LOAD UNCERTAINTY.

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ABSTRACT :

The paper presents a comprehensive analysis for the effect of forecasted peak load uncertainty on power system reliability. The uncertainty is described by a normal distribution and the forecasted load is represented by its mean and standard deviation . Equations for the loss of load expectation and variance in terms of peak load uncertainty are given. These equations are distinguished by simplicity and low computer time required for computations .

Senstivity analysis for the reliability indices due to finite changes in system loading is introduced .

1. INTRODUCTION :

The power system operational planning has been strongly affected by the uncertainties in the daily forecasted peak loads (FPL) as well as the forced outage rates (FOR) of generating units . Since the uncertainty in FPL results in uncertainty of computed reliability indices of a power system, it is required to quantitatively asses a reserve margin that should be allocated in order to satisfy system security constraints .

Recent publications studied the problem of load forecasting in order to reduce the load forecast uncertainty (1) . Patton(2) analyzed the effect of uncertainty in FOR on the system reliability indices (while the FPL is assumed deterministic). The effect of uncertainty in FPL is studied in (3,4). An approximate method was described by Billinton (3) for calculating the expected loss of load probability when uncertainties in FPL, but the FOR is fixed. This method (3) does not provide informations on the variance of LOLP
a method for calculating the mean an
uncertainties exist in the FPL only.
method for analyzing the general ca
considered random variables is give

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2. Sources of load uncertainty :

Although it is possible now to reduce the maximum error of forecasted load to 4.0 % with 1.5% standard deviation, there are an amount of uncertainty which could not be avoided. This uncertainty is a result of the following sources :-

1) RandOm component in the load behaviour.
2) Random component in the weather sensitive load .
3) Recording and measurement errors of load and/or weather data.
4) Extreme weather effects .
5) Assumptions in load froecasting models .
6) Faults in the power system .

3. Effect of uncertainties on system reliability :

3.1. LOLP as a measure of reliability level :

The loss of load probabilty (LOLP) is generally defined as the long run average number of days in a period of time that load exeeds the available installed capacity and thereby capacity deficiency occurs . During a state of capacity deficiency the load demand is not met by an adequate generation due to either unexpected loss of generation as a result of technical problems or unexpected increase in the daily peak load caused by the uncertainty in load forecast .

The LOLP, for a single area system and for N-day period is :

$$\text{LOLP} = \sum_{i=1}^{N} p_c\,(C_i - L_i) \qquad \text{.........} (1)$$

Where :

C_i = total generating capacity scheduled for service on day i.

L_i = forecasted peak load on day i.

$P_c\,(C_i - L_i)$ = probability that $(C_i - L_i)$ is less or equal to zero.

$i = 1, 2, 3, ..., N.$

If there are uncertainties in FOR, the values of C_i and Li must be represented by a set of random variables. Then the LOLP is a random variable defined on both C_i and L_i with non-Zero variance.

3.2. Mathematical formulations for the mean & variance of LOLP:

It is interesting for power system operational planners ...ve exact formulations for the loss of load expectation (LOLE) ...s of load variance (LOLV) for the general case when both ... are subjected to uncertainties. The FOR and FPL are ass...ly independent of each other (4) . This assumption is ...cause forced outages of generating units are usually ...when making load forecast .

The peak loads are reasonably assumed normally distributed around its mean. It may be scattered on a base of small fraction of standard deviation (k = 0, 1 , 2, ..., ∞). The probability of having the peak load in certaing level k can be supplied from the normal probability table. The loss of load expectation is :

$$\mathrm{LOLE} = \sum_{i=1}^{N} \int_0^{\infty} \int_0^{\infty} P_c\,(C_i - L_i)\, f(R,K)\, dR\, dk \quad \ldots\ldots(2)$$

Where :

f (R, k) = joint density function of FOR and FPL.

= f(R) f(K) by the assumption of mutual independence.

Therefore,

$$\mathrm{LOLE} = \int_0^{\infty} f(R)\, dR . \sum_{i=1}^{N} \int_0^{\infty} f(K)\, P_c(C_i - L_i)\, dk \ldots\ldots (3)$$

$P_c(C_i - L_i)$ is a momotonously increasing " staircase" function of k with discontinuities at regular intervals equal to S(the step size of the cumulative capacity outage probability table). Because of this property, equation (3) can be rewritten as:

$$\mathrm{LoLE} = \sum_{i=1}^{N} \sum_{k=0}^{\infty} P_{L_i}(k)\, p_c\,(c_i - L_i) \quad \ldots\ldots\ldots\ldots (4)$$

Where P_{L_i} (k) is the probability of having the peak load in certain level k.

The LOLV, can be determined and is given by :

$$\mathrm{LOLV} = \sum_i \sum_k P_{L_i}(k) \left[P_c^2\,(C_i - L_i) + \mathrm{VAR}\left\{ P_c(C_i - L_i) \right\} \right]$$

$$+ \sum_i \sum_j \sum_k \sum_l P_L(k)\, P_{L_j}(l)\, \mathrm{CoV}\left[p_c(C_i - L_i), P_c(C_j - L_j) \right]$$

$$- \sum_i \mathrm{LOLE}_i^{\,2} \quad \ldots\ldots (5)$$

Where :

VAR $P_c(C_i - L_i)$ = Variance of load loss probability on day i,

COV $p_c(C_i - L_i)$, $p_c(C_j - L_j)$ = Covariance of probabilities $p_c(C_i - L_i)$ on day i and $P_c(C_j - L_j)$ on day c_j,

LOLEi = Loss of load expectation on day i.

4. Sensitivity of reliability indices to load uncertainty

As it is useful for power system operation to study the effect of incremental load changes on power system reliability indices, an efficient method for estimating the changes in reliability indicas due to incremental changes in the power system loading is given. Equation (4) for LOLE can be rewritten as :

$$LOLE = \sum_{i} \int_{0}^{\infty} f(L)\, P_c\,(C_i - L_i)\, dL \quad \ldots\ldots (6)$$

Where f(L) is the density function of the forecasted load L which is assumed normally distributed .

$$f(L) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-1/2(L-\bar{L})^2/\sigma^2} \quad \ldots\ldots (7)$$

Where is the standard deviation . For the i^{th} day, the $LOLE_i$ is:

$$LOLE_i = P_c\,(C_i - \bar{L}_i \pm S) \int_{0}^{S} f(L)\, dL$$

$$+ P_c\,(C_i - L_i \pm 2s) \int_{S}^{2S} f(L)\, dL$$

$$+ \ldots + P_c(C_i - \bar{L}_i \pm nS) \int_{(N-1)S}^{nS} f(L)\, dL \ \ldots ; (8)$$

So,

$$LOLE_i = \sum_{n=1}^{N} P_c\,(C_i - \bar{L}_i \pm nS) \int_{(n-1)S}^{nS} f(L)\, dL \quad \ldots\ldots (9)$$

Where n = 1, 2, ... , N and N is sufficient integer number at which $f(L_i + (N-1)S)$ is equal to zero. The sensitivity of LOLE to a small variation in the mean load of the i^{th} day is :

$$\frac{\partial LOLE}{\partial \bar{L}_i} = \sum_{n=1}^{N} p_c\,(C_i - \bar{L}_i \pm nS) \int_{(n-1)S}^{nS} \frac{\partial f(L)}{\partial L_i}\, dL \ \ldots . (10)$$

Taking the partial derivative of f(L) w.r.t. $\bar{L}_i$ in equation (7)

$$\frac{\partial f(L)}{\partial L_i} = \frac{L - \bar{L}}{\sigma^2} \quad f(L) \qquad ....(11)$$

Substituting equation (11) in equation (12),

$$\frac{\partial LOLE}{\partial L_i} = \sum_{n=1}^{N} p_c (C_i - \bar{L}_i \pm nS) \left[f(L) \right]_{(n-1)s}^{ns} \qquad ....(12)$$

Therefore,

$$\frac{\partial LOLE}{\partial \bar{L}_i} = \sum_{n=1}^{N} (P_c (C_i - \bar{L}_i - nS) - P_c (C_i - \bar{L}_i + nS)$$

$$. (f(\bar{L}_i + (n-1)S) - f(\bar{L}_i + nS)) \qquad .....(13)$$

The LOLV for the i^{th} day is :

$$LOLV_i = \int_0^{\infty} f(L) \, P_c^2 (C_i - L_i) \, dL - LOLE_i^2 \qquad .... (14)$$

For the sensitivity evaluation of the LOLV to variation in the mean load of the i^{th} day, a similar procedure results to :

$$\frac{\partial LOLV}{\partial L_i} = \sum_{1}^{N} (P_c^2 (C_i - \bar{L}_i - nS) - P_c^2 (C_i - \bar{L}_i + nS)$$

$$. \ (f(\bar{L}_i + (n-1)S) - f(\bar{L}_i + nS)$$

$$- 2 \, LOLE_i \frac{\partial LOLE}{\partial \bar{L}_i} \qquad ..... (15)$$

5. Economical reliability level

The most economical reliability level is affected by the uncertaities in forecasted load and forced outage of generating units. An accurate power reserve calculation ensures good service continuity and saves the service interruption cost. However, this power reserve depends on the load forecast uncertainty and hence the power reserve determination is uncertain.

5.1. Cost of power reserve

Estimation of reserve cost of power system depends on loading procedure, availability of units and the forecasted l load demand .

Figure (1) shows the relationship between the cost of power reserve and system availability at different load forecast standard deviation .

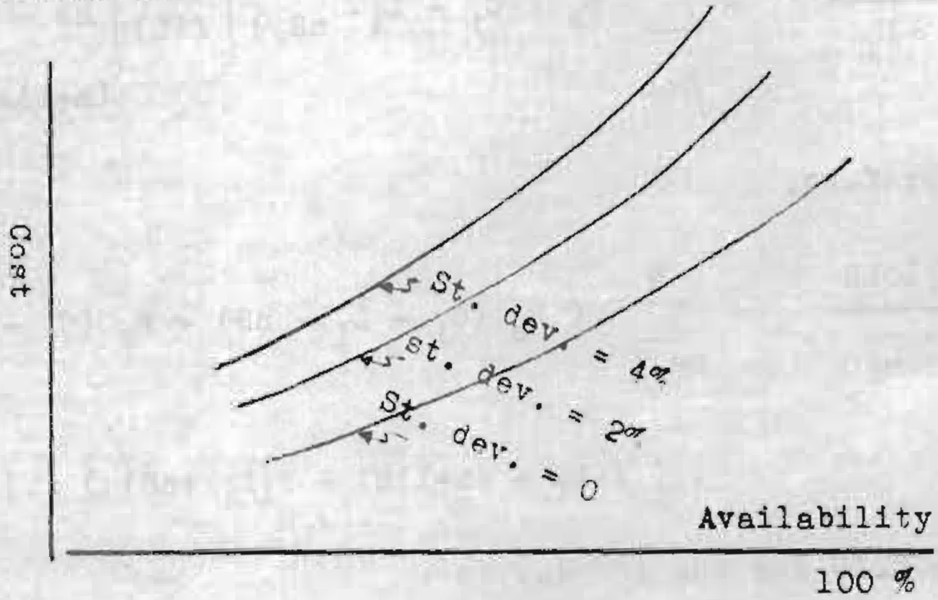

Fig.1.Cost of power reserve

5.2. Cost of service interruption

Power service interruptions result in considerable cost to both the electric utility and power customers (6), It may result in the following costs to the utility :-

1 - Loss of revenue from load not served.
2 - Loss of future potential sales due to adverse consumer reaction.

An expression for the cost of service interruption(C_i) as a function of service availability (6) is :

$$C_i = k. (1 - a) \qquad ...... (16)$$

Where a is the service availability and k is the proportionalty constant .

It should be emphasized that the straight line expression for the cost of service interruption is belived to be valied only for relatively small deviations from the level of service availability . But large reduction in the service availability would probably be represented by the hyperbolic expression(7) as :

$$C_i = \frac{k}{a-a'} - \frac{k}{1-a'}$$

Where k and a' are constants. Both the straight line and hyperbolic cost functions are shown in Figure (2).

5.3 Sensitivity of economical reliability level to load

The cost of power reserve (C_r) as a function of the system availability a may be represented by a quadratic equation as :

$$C_r = b_o + b_1 a + b_2 a^2 \quad \ldots\ldots\ldots (18)$$

Where b_o, b_1 and b_2 are constants depending on the system characteristics and the uncertainty of load demand .

For economical reliability level, the total cost (C_T) should be minimum i.e.,

$$\frac{dC_T}{da} = 0 \quad \ldots\ldots\ldots (19)$$

Where :

$$C_T = C_r + C_i \quad \ldots\ldots (20)$$

Thus,

$$\frac{dC_r}{da} = -\frac{dC_i}{da} \quad \ldots\ldots (21)$$

Substituting in equations (16 , 18), we have :

$$k = b_1 + b_2 a$$

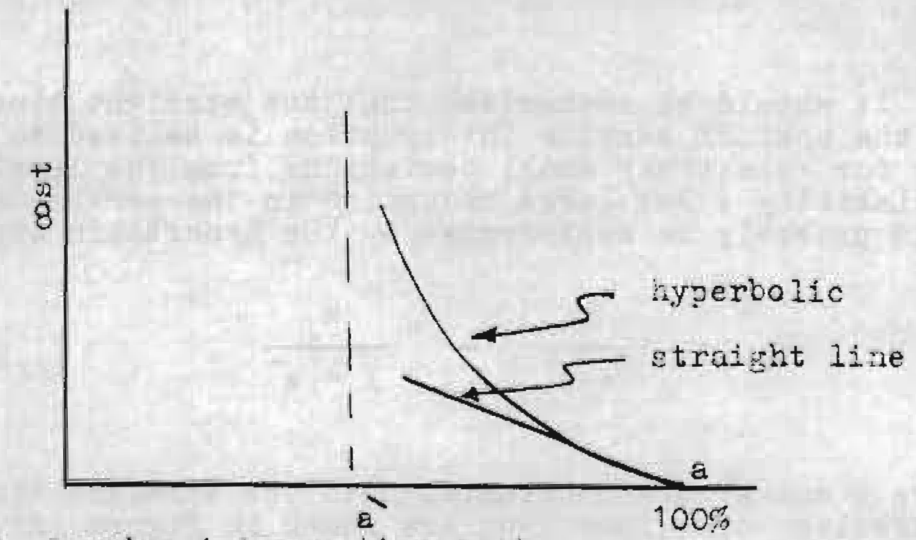

Fig. 2. Service interruption cost.

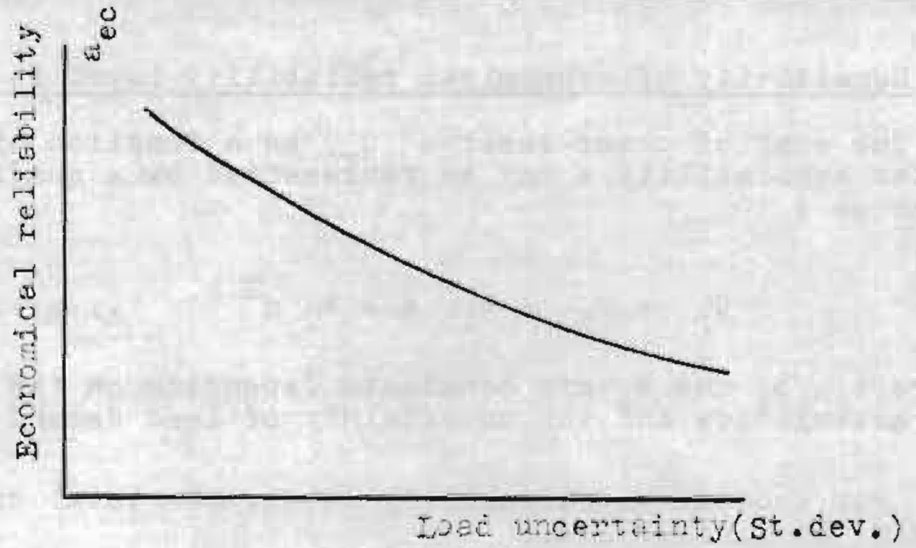

Fig.3. Effect of load uncertainty on economical rel. level.

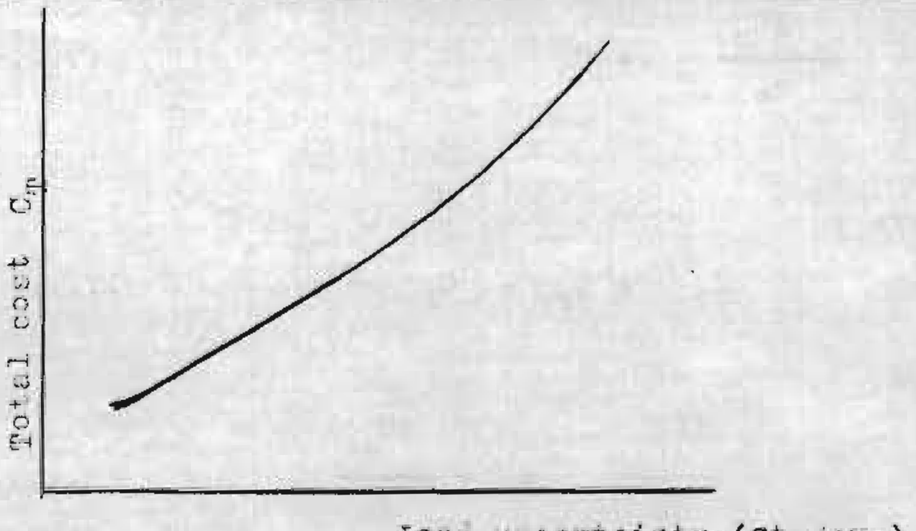

Fig.4. Effect of load uncertainty on total cost.

Therefore, the economical reliability level ($a_{ec.}$) is ;

$$a_{ec.} = (k - b_1) / b_2 \qquad .... (23)$$

It is shown from equation (23) that if there is uncertainty in load forecast (where b_1 and b_2 are increased), the economical reliability level will be decreased. Thus, the uncertainty in load forecast results in an increase in the power system cost and also decreases the system reliability level.

Figure (3) shows the effect of load forecast uncertainty on the economical reliability level. Figure(4) shows the effect load uncertainty on the total cost of both power reserve & service interruption .

6. CONCLUSIONS :

As the power system reliability indicas depend on uncertainties in the estimated FOR of generating units and the uncertainty of FPL, equations for LOLE and LOLV are troduced. These equations have the advantage of simplicity and low computer time required for computations .

The presented senstivity analysis of power system riliability indices (LOLE & LOLV) to forecasted load uncertainty show that an increase in the system loading level results in a decrease in system reliability level.

7. REFERENCES :

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