PROJECT SCHEDULING WITH UNCERTAIN DATA USING FUZZY LOGIC

جدولة المشروعات ذات المعلومات غير المؤكدة باستخدام المنطق الضبابي

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ملخص عربي

الهدف من جدولة المشروعات هو عمل جدول زمني كامل لأنشطة المشروع و معرفة الأنشطة الحرجة عن طريق تحديد المسار الحرج للمشروع. وفي حالة عدم وجود معلومات محددة عن أزمنة الأنشطة يتم استخدام المنطق الضبابي حيث تمثل الأزمنة الازمة لتنفيذ الأنشطة في صورة أرقام ضبابية. هذا البحث يستخدم طريقة جدولة الشبكات الضبابية لحساب الأوقات المتأخرة للأنشطة و المرونة الوقتية الكلية و ايجاد المسار الحرج بدون استخدام حسابات المسار الخلفي مع الأخذ في الاعتبار التداخلات بين الأنشطة و أيضا يمكن من خلالها حساب أحتمالية انتهاء المشروع عند وقت معين و أيضا عند درجة ضبابية معينة و الي اي مدي يكون النشاط حرجا. و تم توضيح خطوات الحسابات و مقارنة النتائج مع الطرق الأخري من خلال مثال محلول. وهي طريقة سهلة و بسيطة اذا ما قورنت بالطرق الأخري.

ABSTRACT

The aim of project scheduling is to provide a complete timing analysis of the activities involved and identify the critical activities according to the critical path. To deal with vague and imprecise data, the concept of fuzzy logic is employed, where the activity times can be represented by fuzzy numbers. In this paper, a Fuzzy Scheduling Network Method that provides a complete fuzzy network analysis is developed with a different way which enables us calculating late starting times and slack times, and to get the critical path without using backward path calculation, also it deals with overlaps and lags between activities. Project completion time at given degree of fuzziness and criticality index for activities are calculated. An example application is solved to show the step by step calculation and to compare the new method with other methods. The developed model has been automated and applied on commercial software that is customary to practitioners. Based on the results obtained the proposed method is simple, fast and effective to provide a complete fuzzy network analysis compared with other methods.

Keywords: Project Scheduling, Fuzzy Sets, Late Start, Critical Path.

1. INTRODUCTION

Project schedulers are facing a serious problem when trying to find the most appropriate tool for scheduling their projects. In real projects, accurate data are not always available, so classical methods of scheduling would be useless. Different methods have been used to deal with uncertainty and risks including the analytical methods like Program Evaluation and Review Technique [PERT], and simulation models like Monte Carlo simulation.

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Over the years there have been successful applications and implementations of fuzzy set theory in project management.

Prade (1979) applied fuzzy set theory when data are not precisely known, fuzzy set theory is sh own to be relevant to the exact nature of the problem rather than probabilistic PERT or CPM [1]. Lootsma (1989) identifies that human judgment plays a dominant role in PERT due to the estimation of activity durations and the requirement that the resulting plan be tight [2]. Buckley (1989) provides detailed definitions of the possibility distributions

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and solution algorithm required for using fuzzy PERT [3].

Nasution (1993) considered the interactive fuzzy subtraction and observed that only the positive part of the fuzzy numbers can have physical meaning. He showed that fuzzy number can be exploited in the network [4]. Lorterapong and Moselhi (1996) presented a new method called fuzzy network scheduling (FNET) based on fuzzy set theory. It incorporates a number of new techniques that facilitate the representation of imprecise activity duration, the calculation of scheduling parameters, and the interpretation of the fuzzy results generated [5].

Hapke *et al.* (1997, 1998) proposed an approach to the multi objective fuzzy resource constraint project scheduling (RCPS) problem which organizes an interactive search for the compromise scheduling [6, 7].

Kuchta (2001) proposed a new approach for measuring the degree of criticality of project activities, the degree of criticality of the whole project, and slack times using fuzzy numbers, in case that there is one path or many paths in the project depending on the number of activities in every path, taking into account the possible delay that would be accepted by the client [8].

Chanas and Zielinski (2001) presented a new approach to generalize the criticality notion for the case of network with fuzzy activities duration times. This generalization is made directly without using generalized arithmetic operation on fuzzy numbers [9]. Dubois *et al.* (2002) introduced and analyzed the notion of necessary criticality for both path and activity with imprecise activity duration times (by means of intervals or fuzzy interval numbers). They developed the concept of criticality in project networks with imprecise durations [10]. Liberatore (2002) offered the fuzzy logic as an alternative approach for modelling uncertainty in project scheduling analysis. He compared and contrasted the fuzzy logic approach with the probability theory [11].

Dubois *et al* (2003) proposed a rigorous formalization of fuzzy PERT. It highlights the relationships between different definitions of the notion of eriticality. They presented the basis for a correet calculation of latest starting dates, slack times and criticality degrees of tasks in task networks with fuzzy processing times [12]. Zhang *et al.* (2005) incorporate fuzzy set theory and fuzzy ranking measure with discrete-event simulation in order to model uncertain activity when insufficient or no sample data are available. They used fuzzy numbers to describe uncertain activity durations, reflecting vagueness, imprecision and subjectivity in their estimation [13].

Zielinski (2005) dealt with the problems of computing the intervals of possible values of the latest starting times and floats of activities in networks with imprecise durations, represented by means of interval or fuzzy numbers. Four polynomial algorithms were proposed [14]. Soltani and Haji (2007) developed a new method based on fuzzy theory to solve the project scheduling problem under fuzzy environment. They assumed that the durations of activities were trapezoidal fuzzy numbers (TFN); they computed the project characteristics such as earliest times, latest times, and, slack times in term of TFN. A new approach called modified backward path (MBP) was introduced [15].

Yousefli *et al.* (2008) presented a novel model to resource constrained project scheduling (RCPS) problem in fuzzy environment. In that model durations, resource availability and resource demand of each activity were uncertain so ranking fuzzy numbers were used to generate the priority list. Three dimensional Gantt chart was introduced to graphically depict project scheduling results [16]. Liberatore (2008) presented a new methodology for fuzzy critical path analysis consists with the extension of fuzzy logic. It contributes to theory in that it is the direct generalization of critical path analysis to the fuzzy domain, and resolves some of the problems expressed in the fuzzy critical path literature, especially in computing the fuzzy backward pass of the project network and fuzzy activity criticality [17].

Zammori *et al.* (2009) identified the critical path in complex projects networks. Integrating Fuzzy Logic (FL) and Multi Criteria Decision Making (MCDM) techniques, a framework was presented to determine the critical path taking into account not only the expected duration of the tasks, but also additional critical parameters such as: duration variability, costs, shared resources, risk of major design revisions and external risks[18].

Yakhchali and Ghodsypour (2009, 2010) proposed a series studies on the topic of the project scheduling problems in the networks with generalized precedence relations and imprecise durations. They have completely solved the problem of determining the latest starting times in a cyclic networks and discussed problems of the criticality of paths in such networks [19, 20].

Shanker and Sireesha (2010) proposed fuzzy critical path method to compute total float time in a fuzzy project network without computing forward and backward pass calculations, but they do not calculate the late start and do not deal with overlaps and lags between activities [21].

In this paper, a Fuzzy Scheduling Network Method (FSNM) that provides a complete fuzzy network analysis is developed for calculating late starting time and slack time, and to get the critical path without using backward path calculation, also it deals with overlaps and lags between activities. An example application is solved to show the steps of the calculations, and to compare the new method with other methods

2. FUZZY SETS AND ARITHMETIC OPERATIONS

In a fuzzy set, elements are described in a way to permit a gradual transition from being a member of a set to non-member. Each element has a degree of membership ranging from zero to one, where zero signifies no membership and one indicates full membership.

A fuzzy set, A, is defined as a set of pairs, $[x, Y_A(x)]$ where x is an element in the universe of discourse X, and $Y_A(x)$ is the degree of membership associated with element x [21].

A fuzzy set A in this universe is denoted by:

$$A = \left\{ \frac{y_A(x_1)}{x_1} + \frac{y_A(x_2)}{x_2} + \dots + \frac{y_A(x_n)}{x_n} \right. \quad (1)$$

Trapezoidal Fuzzy number shown in Fig. 1 is a convex fuzzy set which is defined as $A^{\sim} = (x, \mu_A(x))$ where

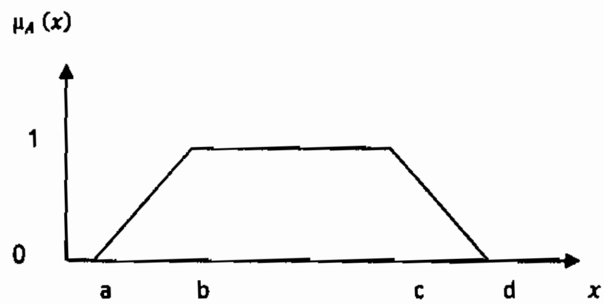


Fig. 1: Convex fuzzy set

$$\mu_A(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x \le b \\ 1 & b < x \le c \\ \frac{x-d}{c-d} & c < x \le d \\ 0 & x > d \end{cases} \quad (2)$$

Let $A_1^{\sim}$ and $A_2^{\sim}$ be two trapezoidal fuzzy numbers parameterized by the quadruple (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2) , respectively. The simplified fuzzy number arithmetic operations between the

trapezoidal fuzzy numbers $A_1^{\sim}$ and $A_2^{\sim}$ are as follows [21]:
Fuzzy numbers addition ⊕:
$A_1^{\sim} \oplus A_2^{\sim} = (a_1 + a_2\ ,\ b_1 + b_2\ ,\ c_1 + c_2\ ,\ d_1 + d_2)$. (3)
Fuzzy numbers subtraction ⊖:
$A_1^{\sim} \ominus A_2^{\sim} = (a_1 - d_2\ ,\ b_1 - c_2\ ,\ c_1 - b_2\ ,\ d_1 - a_2)$. (4)
While solving decision problems with fuzzy data, there often appear situations in which two or more fuzzy numbers are to be compared. The comparison of fuzzy numbers takes place in the two situations: while solving precedence problems of fuzzy events and while comparing fuzzy results of optimization.
Let $A_1^{\sim} = (a_1, b_1, c_1, d_1)$ and $A_2^{\sim} = (a_2, b_2, c_2, d_2)$ be two fuzzy numbers. If $a_1 < a_2$, $b_1 < b_2$, $c_1 < c_2$ and $d_1 < d_2$, the ranking of $A_1^{\sim}$ and $A_2^{\sim}$ is straightforward, $A_2^{\sim}$ is said to be strongly greater than $A_1^{\sim}$. This relation si denoted by $A_2^{\sim} >>= A_1^{\sim}$. The rule determining $A_2^{\sim} >>= A_1^{\sim}$ will be called a strong comparison rule (SCR). If one or two of these four inequalities are not true, the strong comparison rule must be abandoned to the advantage of the so-called weak comparison rule (WCR). This rule answers not only which one of two fuzzy numbers is greater but also what is the degree to which one fuzzy number is greater than another. The degree to which $A_1^{\sim}$ is greater than $A_2^{\sim}$ is denoted by $R\ (A_1^{\sim} \geq A_2^{\sim})$ and calculated as follows:

$$R\ (A_1^{\sim} \geq A_2^{\sim}) = \frac{1}{4}\ \{(a_1 + b_1 + c_1 + d_1) - (a_2 + b_2 + c_2 + d_2)\} \quad (5)$$

Using this degree, we can define three relations between A˜ and B˜:

$$A_1^{\sim} \geq A_2^{\sim}\ if\ \ R\ (A_1^{\sim} \geq A_2^{\sim}) \geq 0 \quad (6)$$

$$A_1^{\sim} > A_2^{\sim}\ if\ \ R\ (A_1^{\sim} > A_2^{\sim}) \geq 0 \quad (7)$$

$$A_1^{\sim} \sim A_2^{\sim}\ if\ \ R\ (A_1^{\sim} \geq A_2^{\sim}) = (A_2^{\sim} \geq A_1^{\sim}) = 0 \quad (8)$$

If the first relation holds, we will say that $A_1^{\sim}$ is weakly greater than or equal to $A_2^{\sim}$. The proposed WCR is related to the comparison of "mean values of fuzzy numbers" [21].

A natural way of defuzzifying such an interval is then to calculate its gravity centre.

$$F(A_n^{\sim}) = \frac{1}{2}(a_n + b_n) \quad (9)$$

When the fuzzy intervals to compare are trapezoidal intervals, then the Eq. 10 applies [22]

$$F(A_n^{\sim}) = \frac{1}{4}(a_n + b_n + c_n + d_n) \quad (10)$$

3. FUZZY SCHEDULING NETWORK METHOD (FSNM)

In this paper, a FSNM model is developed to schedule projects with uncertain duration. In this development, early and late times are calculated without using backward path calculation, which is not sound if the durations are described by means of fuzzy numbers or fuzzy intervals. Overlaps and lags between activities are also considered.
In the proposed model, relationships among activities and activities fuzzy durations should be known to construct the fuzzy project network. To get the critical path, paths lengths are calculated considering overlaps and lags between activities, the path which has the highest length is considered as the critical path. To get the total floats for the project activities, the total float of each path is calculated from the difference between the critical path length and the length of each path excluding those activities that has been assigned a total float from another path. Finally early times and late times are calculated. Model details are described in the following sub-sections.

3.1 Path Length

To find path length in project network, for each activity in the selected path, add the fuzzy durations of the activity using fuzzy addition process taking into consideration overlaps and lags between activities by adding them to the activity's duration with their signs.

$$PD = \sum_{i=1}^{m} Di\ \oplus overlap \quad (11)$$

PD = path length

m = number of activities in each path
D_i = duration of activity i

$If\ D_1 = (a_1, b_1, c_1, d_1),\ D_2 = (a_2, b_2, c_2, d_2),\ O_{12} = (o_1, o_2, o_3, o_4)$

$Then\ D_1 \oplus D_2 \oplus O_{12} = (a_1 + a_2 + o_1, b_1 + b_2 + o_2, c_1 + c_2 + o_3, d_1 + d_2 + o_4).$

Having identified the length of each path, it is necessary, in the next step, to identify the critical path.

3.2 Critical Path

The critical path represents the longest path in a project network. To find the critical path, the (WCR) is used. The length of each path is compared with zero, all paths are then arranged according to their values in ascending order, the path which had the highest length is considered as the critical path. The following pseudo code shows the steps of identifying the longest path.

$for\ PD_1 = (a_1, b_1, c_1, d_1),\ PD_2 = (a_2, b_2, c_2, d_2),\ ...,\ PD_n = (a_n, b_n, c_n, d_n),\ n = number\ of\ paths$
$Calculate$
$R\,(PD_1) = (a_1, b_1, c_1, d_1)\,/\,4$
$R\,(PD_2) = (a_2, b_2, c_2, d_2)\,/\,4$
....
$R\,(PD_n) = (a_n, b_n, c_n, d_n)\,/\,4$
$for \quad i = 1\ to\ n-1$
$If \quad R\,(PD_{i+1}) \geq R\,(PD_i)$
$Then$
$Critical\ Path = R\,(PD_{i+1})$
$else$
$Critical\ Path = R\,(PD_1)$
$End\ If$
$Write \quad Critical\ Path$

With the critical path have been identified, the next step is to calculate the total float for the project activities.

3.3 Activities Total Float

Normally, total float of an activity is either the difference between its late and early start times or the difference between its late and early finish times. In fuzzy networks, the total float time of the activities located in a given path is calculated as the difference between the length of the critical path and the length of that path.

So to get the total float of different activities, first get the total float of each path then find the total float for activities. To get the total float for the different paths in a network, subtract each fuzzy path length from the maximum fuzzy path length, using fuzzy number subtraction process. All paths in the network are arranged in an ascending order starting from the critical and the next. The first path (the critical path is assigned a total float of zero for the activities in that path). For the second path, assign the value of total float of the path as the total float of each activity in that path; discarding the activities already assigned in any path before; continue the process until all the activities assigned the float time [19]. Thus ensures that the float is assigned to the sub-paths only in order to avoid assigning a total float to activity twice.

3.4 Activity Fuzzy Times

3.4.1 Fuzzy early start and early finish times

In accordance with Critical Path method (CPM), the forward path calculations yield the fuzzy earliest start and fuzzy earliest finish times.

$$FESi = \max_{j \in P(i)}\{FES_j \oplus Fd_j \oplus overlap_{ij}\} \qquad (12)$$

$$FEFi = FESi \oplus Fdi \qquad (13)$$

Where FES_i is the fuzzy earliest start time and FEF_i is the fuzzy earliest finish time of activity i, P(i) is the set of predecessors of activity i , and Fd_i is the fuzzy duration of activity i.

3.4.2 Fuzzy late start and late finish times

The fuzzy late start time for a given activity can be calculated as follow:

$$FLSi = FESi \oplus FTFi \qquad (14)$$

Accordingly, the fuzzy late finish time for a given activity can be calculated as follow:

$$FLFi = FEFi \oplus FTFi \quad (15\text{-}a)$$

$$or\ FLFi = FLSi \oplus Fdi \quad (15\text{-}b)$$

Where FLS_i is the fuzzy late start time, FLF_i is the fuzzy late finish time, and FTF_i is the fuzzy total float.

3.5. Calculation Steps

In the proposed method, the calculations are made as described below:

1. Identify activities in a project
2. Establish precedence relationships of all activities.
3. Estimate the fuzzy activity durations.
4. Construct the project network.
5. Find all possible paths (n) in the project network.
6. Add all the fuzzy activity times in each path using fuzzy number addition which gives fuzzy path length in fuzzy number, considering overlaps and lags between activities.
7. Find the maximum fuzzy path (π_m) using WCR among all network paths. The path with the highest π_m is considered as the critical path.
8. Arrange the path length in descending order using WCR, according to the values obtained.
9. Subtract each path fuzzy number (length of each path) using fuzzy number subtraction from the maximum fuzzy path length (fuzzy number) including itself to get total floats of paths (π_1, π_2, ... π_n) .
10. Assign the total float of each activity as follow:
    Choose the path length which had a total float π_1; assign the path value of π_1 as the total float of each activity in that path.
    Choose path length π_2; assign the path value of which had a total float π_2 as the total float of each activity in the path discarding the activities already assigned before (which means that floats are assigned to the activities in the sub-paths only). Continue the process until all the activities assigned the float time.
11. Calculate FES and FEF using the fuzzy CPM method.
12. Calculate FLS and FLF using the relations mentioned before.
13. Defuzzify the fuzzy results to obtain a crisp output for the total project duration.

4. PROJECT COMPLETION TIME at A GIVEN DEGREE OF FUZZINESS

In this paper, planners are allowed to find the project duration at a given degree of fuzziness (DF). The DF reflects the confidence of meeting a given date (e.g., DF=0 means high confidence of completing the project at its minimum duration) [23]. This DF reflects the assurance level of completing the project. As the DF decreases, thus means high assurance level, and vice versa.

In fuzzy project scheduling, the project duration is calculated as a trapezoidal fuzzy number (a, b, c, d) (Fig. 2). Project completion time at a given DF can be calculated using Eq. (16)

$$P_\alpha = a + (d - a) * \alpha \quad (16)$$

Where:

α is the degree of fuzziness, the range of the degree of fuzziness is from 0 to 1, P_α is the project completion time at DF α, a is the earliest finish time and d is the latest finish time.

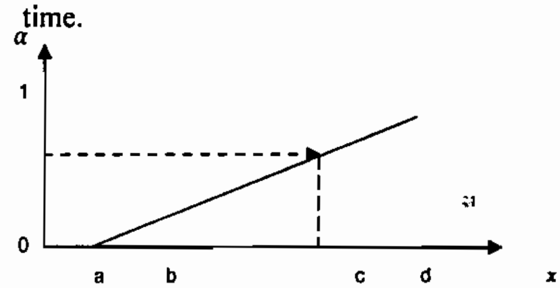


Fig. 2: project completion time at degree of fuzziness α

5. CRITICALITY INDEX FOR PROJECT ACTIVITIES

When the durations of project activities are estimated with uncertainty, the determination of their respective degree of criticality is not as obvious as in case where the durations are estimated with certainty. The criticality index reflects the possibility of an activity to be critical if the duration of the activity is changed in its fuzzy duration range. After identifying activities total float, the criticality index (y) could be calculated as follow (Fig. 3):

$$y = 1 - \frac{x}{t} \tag{17}$$

Where: t is the highest value of total float,
x is the total float of a certain activity

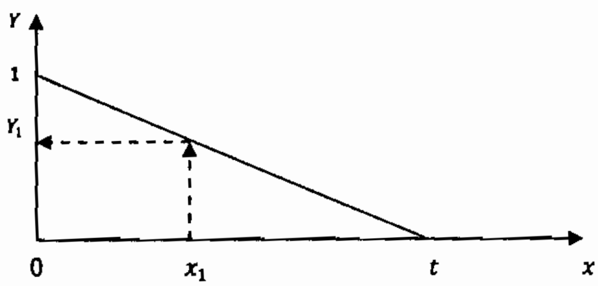


Fig. 3: Criticality index for activities

6. FSNM AUTOMATION

In order to facilitate the use of the developed mode, especially for large real-life projects, an automated system of the FSNM is developed. The developed system is coded using VB.Net and applied on a commercial software package "Microsoft Project", which is customary to many construction practitioners. On Microsoft Project, users are simply required to provide the data related to project activities, relations, fuzzy durations and overlaps (by double click on predecessors room) in the designated fields as shown in Fig. 4.

After entering the data to Microsoft Project, users may use a standalone application designed using VB.Net as shown in Fig. (5) to run the open the Microsoft Project file and run the program to perform all the calculations required to identify the different paths in the project network, the fuzzy start and finish times, the floats, the criticality index for the activities.

The implementation steps of the FSNM system are listed as follow:

1. Identify the project activities and then establish relationships among activities.
2. Write the values of activities fuzzy durations in a Microsoft Project file as (a, b, c, d) = (Duration1, Duration2, Duration3, Duration4), (Fig. 4).
3. From the application file, shown in Fig 5, choose the button "select project" and enter the Microsoft Project file name and then run the application.
4. The results of paths ,their floats, and critical paths will be presented to the user as shown in Fig. 5.

7. EXAMPLE APPLICATION

In order to validate the new developed model and to experiment with its developed system, an example application is used. This example was solved by Lorterapong and Moselhi [3] using the fuzzy network scheduling method (FNET), Monte Carlo simulation method. A comparison of the results using the proposed method will be presented.

Data of the example application are listed in Table (1). The calculation starts by identifying the different paths that of the project network, calculating each path fuzzy total duration and then application the WCR to rank the path from the longest to the shortest as presented in Table 2 (steps 1-9 presented in section 3.5).

The next step is to calculate the activities FES and FEF times, activities floats and then FLS and FLF times as shown in Fig. 5.

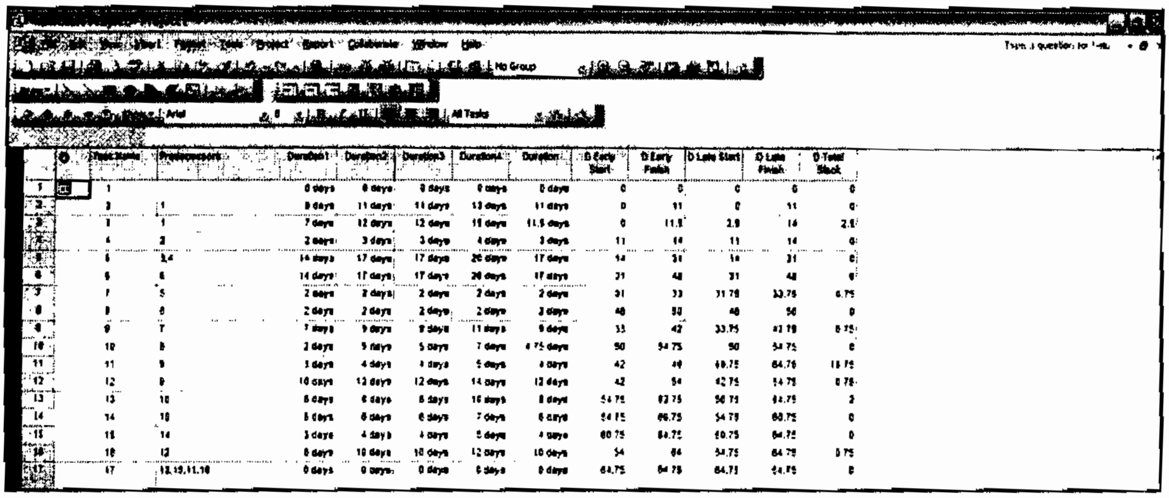
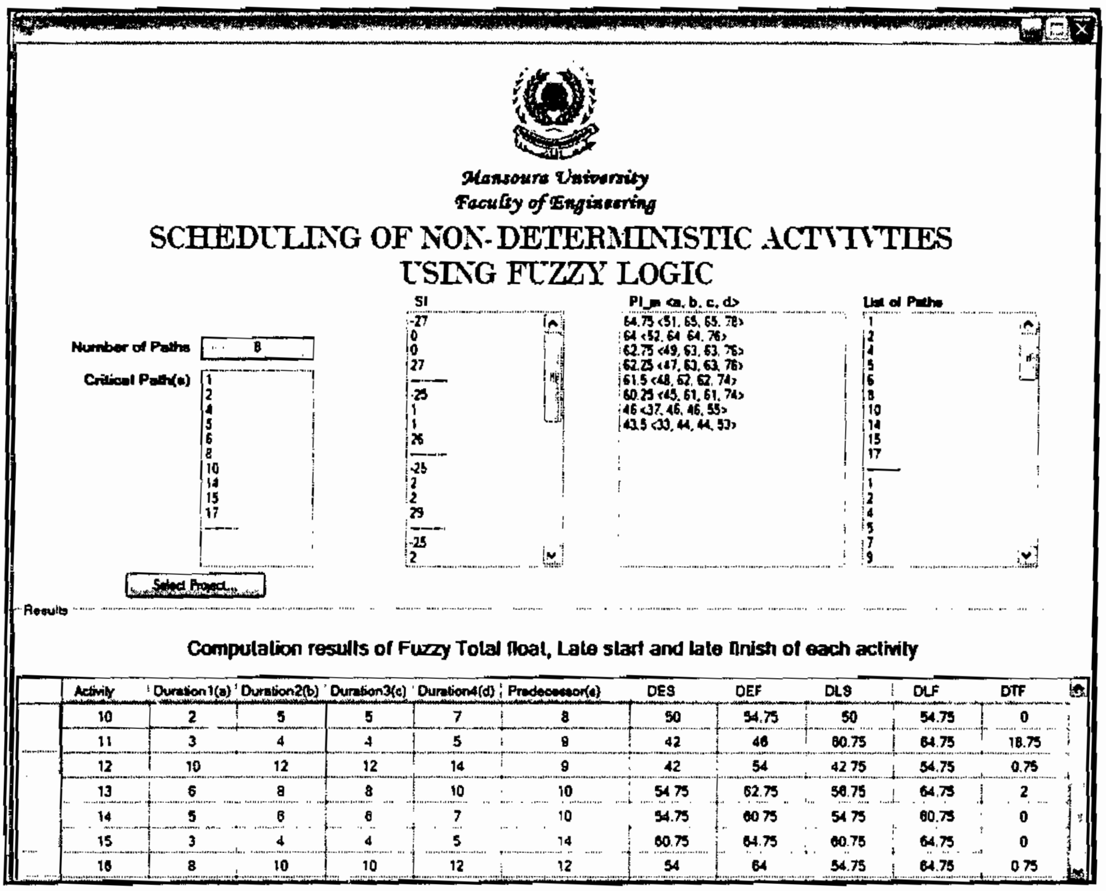


Fig. 4: Project data



Activity	Duration1(a)	Duration2(b)	Duration3(c)	Duration4(d)	Predecessor(s)	DES	DEF	DLS	DLF	DTF
10	2	5	5	7	8	50	54.75	50	54.75	0
11	3	4	4	5	9	42	46	60.75	64.75	18.75
12	10	12	12	14	9	42	54	42.75	54.75	0.75
13	6	8	8	10	10	54.75	62.75	56.75	64.75	2
14	5	6	6	7	10	54.75	60.75	54.75	60.75	0
15	3	4	4	5	14	60.75	64.75	60.75	64.75	0
16	8	10	10	12	12	54	64	54.75	64.75	0.75

Fig. 5: FSNM main menu

7.1 Comparison with Monte Carlo Simulation

Monte Carlo simulation is performed on this example. It is assumed that activity duration described using a triangular marginal distribution. The results of this example application are based on 5000 simulations expressed in terms of mean and standard deviation for each activity. The average project duration is found to be 65.28 days with a standard deviation of 1.98 days.

Fuzzy Scheduling Network Method (FSNM) is performed on this example. It is assumed that activity durations are independent and can be described using a trapezoidal fuzzy number. The project completion time is determined to be 64.75 days.

The degree of fuzziness refers to how much the managers are sure of the problems that may face the project in the future. The degree of fuzziness depends on Project budget, .nanpower and technology difficulty. The project completion time is 64.5 days at degree of fuzziness 0.5.

7.2 Comparison with Fuzzy Network Scheduling Method (FNET)

Fuzzy network scheduling method (FNET) is performed on this example. The criticality index for activities by (FNET) and (FSNM) is listed in Table 4

Table1: Activities and their fuzzy durations

Activity	Fuzzy Duration	Predecessors
1	(0,0,0,0)	-
2	(9,11,11,13)	1
3	(7,12,12,15)	1
4	(2,3,3,4)	2
5	(14,17,17,20)	3,4
6	(14,17,17,20)	5
7	(2,2,2,2)	5
8	(2,2,2,2)	6
9	(7,9,9,11)	7
10	(2,5,5,7)	8
11	(3,4,4,5)	9
12	(10,12,12,14)	9
13	(6,8,8,10)	10
14	(5,6,6,7)	10
15	(3,4,4,5)	14
16	(8,10,10,12)	12
17	(0,0,0,0)	13,15,11,16

Table2: Path ranking of the example application

Path	Total fuzzy time for path (a, b, c, d)	Rank	Path rank	$\pi_m \Theta$ (a, b, c, d)
1-2-4-5-6-8-10-14-15-17	(51,65,65,78) π_m	64.75	1	(-27,0,0,27)
1-2-4-5-6-8-10-13-17	(49,63,63,76)	62.75	3	(-25,2,2,26)
1-2-4-5-7-9-12-16-17	(52,64,64,76)	64.00	2	(-25,1,1,29)
1-2-4-5-7-9-11-17	(37,46,46,55)	46.00	7	(-4,19,19,41)
1-3-5-6-8-10-14-15-17	(47,63,63,76)	62.25	4	(-25,2,2,31)
1-3-5-6-8-10-13-17	(45,61,61,74)	60.25	6	(-23,4,4,33)
1-3-5-7-9-12-16-17	(48,62,62,74)	61.50	5	(-3,3,3,30)
1-3-5-7-9-11-17	(33,44,44,53)	43.50	8	(-2,21,21,45)

Table 3: Fuzzy Total float, late start and late finish of the example application activates

Act.	Early start			Early finish			Late start			Late finish			Total float		
	MC mean	FNET mode	FSNM	MC mean	FNET mode	FSNM	MC mean	FNET mode	FSNM	MC mean	FNET mode	FSN M	MC mean	FNET mode	FSNM
1	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.04	0.00	0.00	0.04	0.00	0.00
2	0.00	0.00	0.00	11.03	11.00	11.00	0.04	0.00	0.00	11.07	11.00	11.00	0.04	0.00	0.00
3	0.00	0.00	0.00	11.18	12.00	11.50	2.86	2.00	2.50	14.04	14.00	14.00	2.86	2.00	2.50
4	11.03	11.00	11.00	14.00	14.00	14.00	11.07	11.00	11.00	14.04	14.00	14.00	0.04	0.00	0.00
5	14.04	14.00	14.00	31.08	31.00	31.00	14.04	14.00	14.00	31.08	31.00	31.00	0.00	0.00	0.00
6	31.08	31.00	31.00	48.05	48.00	48.00	31.64	31.00	31.00	48.63	48.00	48.00	0.57	0.00	0.00
7	31.08	31.00	31.00	33.07	33.00	33.00	32.33	31.00	31.75	34.33	33.00	33.75	1.26	0.67	0.75
8	48.05	48.00	48.00	50.05	50.00	50.00	48.63	48.00	48.00	50.63	50.00	50.00	0.57	0.00	0.00
9	33.08	33.00	33.00	42.09	42.00	42.00	34.33	33.00	33.75	43.34	42.00	42.75	1.26	0.67	0.75
10	50.05	50.00	50.00	54.68	55.00	54.75	50.62	50.00	50.00	55.25	55.00	54.75	0.57	0.00	0.00
11	42.09	42.00	42.00	46.06	46.00	46.00	61.30	62.00	60.75	65.28	66.00	64.75	19.21	21.00	18.75
12	42.09	42.00	42.00	54.04	56.00	54.00	43.34	42.00	42.75	55.29	56.00	54.75	1.26	0.67	0.75
13	54.68	55.00	54.75	62.67	63.00	62.75	57.28	57.00	56.75	65.28	65.00	64.75	2.60	2.00	2.00
14	54.68	55.00	54.75	60.70	61.00	60.75	55.26	55.00	54.75	61.28	61.00	60.75	0.58	0.00	0.00
15	60.70	61.00	60.75	60.70	65.00	64.75	61.28	61.00	60.75	65.28	65.00	64.75	0.58	0.00	0.00
16	54.04	56.00	54.00	64.02	66.00	64.00	55.29	56.00	54.75	65.28	66.00	64.75	1.26	0.67	0.75
17	64.70	66.00	64.75	64.70	66.00	64.75	65.28	66.00	64.75	65.28	66.00	64.75	0.58	0.00	0.00

8. ADVANTAGES AND LIMITATIONS OF the FSNM

As demonstrated in the example application, the calculations in FSNM are direct, simple, and transparent. By contrast, Monte Carlo simulation is relatively difficult to follow, partly because most practitioners have a vague understanding of probability distributions.

FSNM, like most fuzzy methods, does not require historical data, because durations may be estimated without needing historical data. In practice, such historical data is difficult to compile and maintain. The trapezoidal fuzzy numbers, expressed in the quadruple form are simple, effective, and practical in representing activity durations.

Table 4: Activities criticality index

Activity	Criticality measurement (FNET)		Criticality index (FSNM)
	Possibility measure	Agreement index	
1	0.96	0.93	1.00
2	0.96	0.93	1.00
3	0.89	0.74	0.87
4	0.96	0.93	1.00
5	0.96	0.93	1.00
6	0.96	0.93	1.00
7	0.92	0.92	0.96
8	0.96	0.93	1.00
9	0.92	0.92	0.96
10	0.96	0.93	1.00
11	0.14	0.02	0.00
12	0.92	0.92	0.96
13	0.89	0.79	0.89
14	0.96	0.93	1.00
15	0.96	0.93	1.00
16	0.92	0.92	0.96
17	0.96	0.93	1.00

On the other hand, probabilistic scheduling methods such as PERT and Monte Carlo simulation ideally require historical data. If historical data are available, various steps would have to be carried out to compile the data before they can be used on new projects. Since such data are not readily available in practice, subjective estimates of activity durations are often used as input to probabilistic methods.

Like most other methods, FSNM has some limitations such as estimating the activities fuzzy numbers may require some experience from the planner to judge the range of such numbers.

9. CONCLUSION

The paper presents a Fuzzy Scheduling Network Method (FSNM) that provides a complete fuzzy network analysis in a different way which enables planners to calculate late starting time and slack time, and to get the critical path without using backward path calculation. Also, the developed model enables dealing with overlaps and lags between activities. An automated system of the FSNM is developed to facilitate its use and its application on large project network calculations. The developed computerized system incorporated a number of algorithms designed to support FSNM calculations. Project completion time at given degree of fuzziness and criticality index for activities are, also, calculated. Compared with other fuzzy critical method and Monte Carlo simulation, the proposed method is proved to be simple, fast and effective to provide a complete fuzzy network analysis.

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