TURBULENT FLOW BETWEEN CONCENTRIC ANNULUS WITH ROTATING INNER CYLINDER

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السريان المضطرب من خلال الحلقات متحدة المركز مع دوران الاسطوانة الداخلية

الخلاصة:

يقدم هذا البحث دراسة معملية للانسياب المضطرب بين اسطوانتين متحدتى المركز الاسطوانة الداخليــة منهما تدور. تم انشاء الجهاز البحثى اللازم لذلك بحيث يمكن ادارة الاسطوانة الداخلية بسرعات دوران مختلفة. استخدمت اسطوانتين داخليتين مع اسطوانة خارجية ثابتة لتكوين حلقتان النسبة بين قطر الاســطوانة الداخليــة والخارجية هما(٣١٥, & ٤٢٥,). استخدم الماء كمائع للاختبار أثناء اجراء التجارب المعملية.

فى هذا البحث تم دراسة تأثير تغيير كل من: سرعة السريان المحورية و ســرعة الـدوران المماسـية للاسطوانة الداخلية ونسبة الاقطار على كل من شكل توزيع السرعة وقيمة معامل الاحتكاك.

قد بينت الدراسة أن معامل الاحتكاك فى حالة دوران الاسطوانة الداخلية أعلى منه فى حالة الاسطوانتين الساكنتين كما يقل معامل الاحتكاك بزيادة كل من: النسبة بين الاقطار وسرعة السريان المحورى. بينت الدراسة أيضا أن قيمة السرعة المحورية القصوى تزيد زيادة طفيفة مع زيادة سرعة دوران الاسطوانة الداخليــة كمــا تميل الى الاتجاه ناحية محور الدوران كلما زادت نسبة الاقطار أو زادت سرعة دوران الاســطوانة الداخليــة. لايوجد اى تأثير لزيادة سرعة السريان على شكل توزيع السرعة.

ABSTRACT:

This study presents an experimental investigation of the turbulent axial flow through concentric annulus of rotating inner cylinder. The experimental apparatus was buildup so that the inner cylinder can be rotated at different speeds. The experimental apparatus consists of two annulus the ratio between the inner cylinder diameter and the outer diameter (Radii ratio) are 0.315 and 0.425. Water was used as a working fluid.

The effect of radii ratio (*M*), the axial Reynolds number (*Re*) and the rotational Reynolds number (R_θ) on both axial mean velocity distribution and hydraulic friction coefficient were studied experimentally.

The study indicated that, the friction coefficient, in the case of rotating inner cylinder, is higher that for stationary cylinders. It also indicated that, the friction coefficient decreases with increasing both radii ratio and the axial mean velocity. The maximum axial mean velocity increases slightly with increasing the rotating speed of the inner cylinder and it tends to shift towards the axis of rotation with increasing both the radii ratio and the speed of rotation of the inner cylinder. There is no effect of increasing axial Reynolds number on the axial mean velocity profile.

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1. INTRODUCTION:

Considerable interest has been shown in recent years in the problem of viscous flow between concentric cylinders with one or both of cylinders rotating. Such systems are of interest in the design of cooling systems for rotating electrical machinery; chemical mixing or drying machinery and they are suggested possible application to compact rotary heat exchangers.

Since the pioneer work of Taylor [1], there is a very large number of experimental and theoretical studies have considered different aspects of instability and transitions of the flow of fluid confined through the annulus between concentric, rotating cylinders [2-4]. Various modifications of this problem have also received considerable attention and these include the influence of axial flow [5-6], unsteadiness of the rotation rate [7] and the effect of radial temperature or density variations with and without an axial gravitational field [8-11].

Yamada and Imao [12] studied flow of a fluid contained between concentric cylinders both rotating in same or opposite directions by flow visualization and velocity measurements. They used slit illumination for flow observations and Pitot and hot wire probes for velocity measurements. The results indicated that, when the outer cylinder rotates faster than the inner one in same direction, the flow is stabilized and laminar flow region is fairly wide but there is a fluctuation which is too small to observe. The boundary beyond which a spiral turbulence occurs has marked hysteresis and changes with the fluid used. When the two cylinders rotate in opposite directions, Taylor vortices and spiral turbulence arise almost simultaneously and both exist together in the gap at same Reynolds number.

Flow of Newtonian and non-Newtonian fluids in a concentric annulus with rotation of the inner cylinder were measured by Nouri and Whitelaw [13]. Mean velocity and the corresponding Reynolds shear stresses of Newtonian and non-Newtonian fluids have been measured in a fully developed concentric flow with a diameter ratio of 0.5 and at an inner cylinder rotational speed of 300 rpm. With the Newtonian fluid in laminar flow, the effect of the inner shaft rotation were a uniform increase in the drag coefficient by about 28 percent, a flatter and less skewed axial mean velocity and a swirl profile with a narrow boundary close to the inner wall with a thickness of about 22 percent of the gap between the pipes. These effects reduced gradually with bulk flow Reynolds number so that, in the turbulent flow region with a Rossby number of 10, the drag coefficient and profiles of axial mean velocity with and without rotation were similar. The intensity of the turbulence quantities was enhanced by rotation particularly close to the inner wall at a Reynolds number of 9000 and similar to that of the non rotating flow at the higher Reynolds number. Comparison between the results of the Newtonian and non-Newtonian fluids with rotation at a Reynolds number of 9000 showed similar features to those of non rotating flows with an extension of non-turbulent flow, a drag reduction of up to 67 percent, and suppression of all fluctuation velocities compared with Newtonian values particularly the cross-flow components. The results also showed that the swirl velocity profiles of both fluids were the same at a similar Rossby number.

Nouri et al [14] studied experimentally the velocity information for the flow of a Newtonian fluid in concentric and eccentric annuli, at Reynolds numbers up to those of fully turbulent flow. A basis for comparison with corresponding results obtained in concentric and eccentric annuli with a non-Newtonian fluid was also developed. The distribution in the Newtonian fluid of static pressure measurements on the outer wall was found to be linear. The friction factor for concentric-annulus flows was 8 percent higher than in a smooth round pipe, whereas for the eccentric flows of eccentricities of 0.5 and 1.0 it was 8 percent and 22.5 percent lower, respectively, than that of the concentric-annulus flow. The average wall shear stress coefficient for the non-Newtonian fluid was similar to that of the Newtonian fluid in the laminar region of the concentric-annulus flow, but it was higher for the two eccentric-annulus flows.

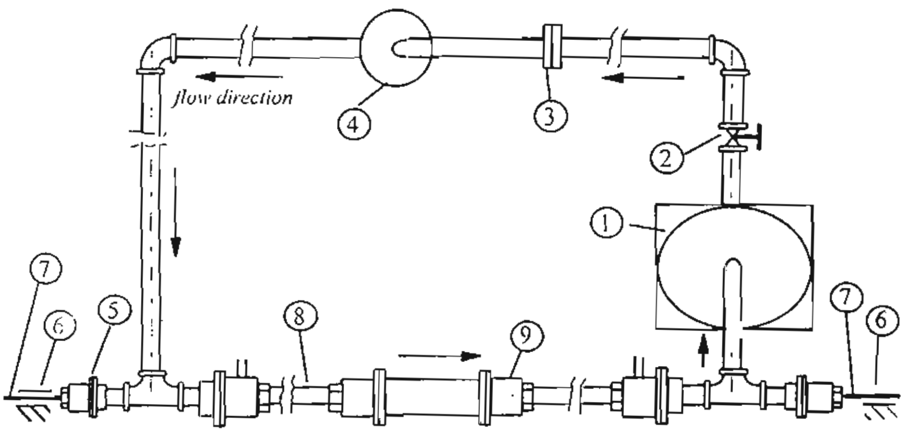
Polkowski [15] studied theoretically the turbulent flow between coaxial cylinders with the inner cylinder rotating. He used Dorfman's solution of the momentum equation (developed for a purely circumferential motion) in solving systems with axial through flow. He also applied Prandtl mixing length model of turbulence to solve the momentum and energy equations. It is shown that the axial and circumferential flows are independent of each other after moment of momentum of the axial flow acquires its final value along certain initial distance of the duct.

Mohamed, M. S. [16] studied numerically the turbulent flow between concentric rotating cylinders both rotating in the same or opposite directions in the ranges of flow at axial Reynolds number $6.6\times10^5 < Re < 5.16\times10^6$ and rotational Reynolds number $2.43\times10^3 < R_0 < 7.48\times10^4$. A finite difference scheme is developed to solve the boundary layer equations coverning the flow. A two - equation model of turbulence was employed in his study. Modifications have been made to the model to take into account the effect of rotation of the cylinders and the existing of the wall. The effect of changing the radius ratio, axial Reynolds number and angular velocity ratio on the hydraulic friction head loss of air is studied. Distributions for axial mean velocity, tangential mean velocity, turbulence intensity and shear stress in both the entrance region and fully developed region are predicted.

Although, There has been a very large number of experimental and theoretical studies of axial flow between concentric rotating cylinders (circular Couette flow), most of the work has been concerned with the primary instability using air as working fluid but there has been relatively little work examining the flow parameters that occur at higher cylinder-rotation rates for different radii ratio. Therefore, the object of the present investigation is to study the adiabatic flow of water in an annulus having a rotating inner cylinder, at high speeds of rotation, with stationary outer cylinder. The experimental program was conducted to determine the effect of axial Reynolds number, rotational Reynolds number and the radius ratio on the axial mean velocity profile and the hydraulic friction coefficient.

2. EXPERIMENTAL APPARATUS AND PROCEDURE :

A schematic outline of the experimental apparatus is illustrated in figure (1). The test-rig employed in this experiment is of the closed circuit type. The apparatus consist of twelve parts, shown in figure (1).



1- Cylindrical tank 2- Gate valve 3- Orfifice meter
4- Centrifugal pump 5- Stuffing box 6- Ball bearing
7- Inner cylinder 8- Outer cylinder 9- Test section

Figure (1) A schematic outline of the experimental apparatus

The dimensions of the cylinders employed are given in table (1). Two inner cylinders and one outer cylinder were used to give the two radii ratio ($M=r_1/r_2$) equals to 0.315 and 0.425. Cylinders are made of stainless steel.

Radius ratio	r_1 (cm)	r_2 (cm)	$b = r_2 - r_1$ (cm)
$M_1 = 0.425$	1.35 ± 0.01	3.175 ± 0.01	1.825 ± 0.02
$M_2 = 0.315$	1.0 ± 0.01	3.175 ± 0.01	2.175 ± 0.02

Table (1) Dimensions of the cylinders and radius ratio.

The aspect ratios, at the test section, were (L/b = 60 and 71) to ensure that the flow is fully developed. The inner cylinder was supported on ball bearing and rotated by an electric motor and a V-belt by means of pulleys with different diameters to change the speed of rotation. A stuffing box was fixed at the ends of the inner cylinder to prevent any leakage from the system during rotation. The speed of rotation was counted using a digital photo tachometer. Errors in counting were estimated to be less than ±2 percent.

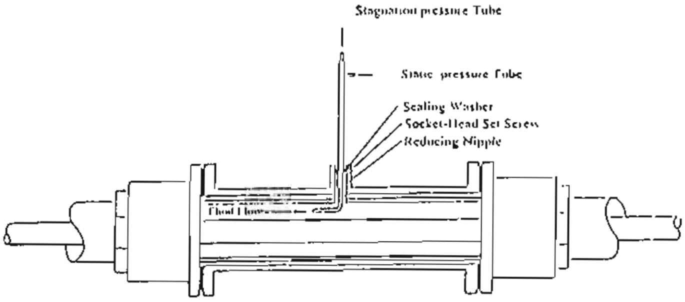


Figure (2) Test section.

The axial mean velocity profile was measured by a standard Pitot-static tube which was accommodated in a vertical plane inside the annuli gap, as shown in figure (2). The velocity was calculated using the equation:

$$V_z = C \sqrt{2 \frac{\Delta P_l}{\rho_w}} \qquad \text{...... (1)}$$

where C is the Pitot tube coefficient and $(\Delta P_1 / \rho_w \, g)$ is the difference in pressure head across the Pitot tube. The reading accuracy of the manometer associated with the Pitot tube was ±1.0 mm that produces errors in determining V_z less than ±2 percent.

Prandtl type Pitot tube was employed in this investigation, so the effect of the nose and stem can be canceled. Calibration was made for the Pitot tube in the hydraulic circut and the coefficient C was found to be unity

(C=1). The Pitot tube has an outer diameter of 3 mm and a hole of 0.85 mm in diameter with four static pressure tappings drilled at a cross section (12 mm) away from the nose.

The flow rate through the test section was measured by an orifice meter which is perpendicular to the axis of the pipe. The hole of the orifice meter is concentric with the pipe. Two pressure tappings are located on the sides of the orifice plate. The upstream tapping is one diameter distance and the downstream is half diameter distance. The ratio between the orifice meter diameter to the pipe diameter is equal to 0.5. The orifice meter was designed and constructed according to ASME recommendations [17]. The discharge, Q, was calculated using the equation:

$$Q = C_d \frac{a_1 \; a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2 \frac{\Delta P_2}{\rho_w}} \qquad \text{....... (2)}$$

where a_1 is the area of the pipe, a_2 is the area of the orifice meter and $(\Delta P_2/\rho_w g)$ is difference in pressure head across the orifice meter which is measured using U-tube Hg manometer. It was estimated that the flow rates in the test section might be in error by as mush as ±3 percent.

Two pressure tappings, one upstream and the other is at downstream of the rotating cylinder, were drilled on the outer cylinder of the annulus where the distance between taps is 2.9 ± 0.001 meters which is sufficient enough to make a pressure drop through the flow during the rotation of the inner cylinder. The pressure tappings were connected to the manometer board by means of a transparent plastic hose in the location specified for measuring the friction head loss. Air bubbles in the transparent plastic hose are removed. Both stems of the manometer are filled with water above the mercury surface level. The difference in mercury level, referred to as y, was measured. The friction coefficient f, for stationary cylinders, can be calculated from the Darcy-Weisbach equation:

$$h_L = f \frac{L}{D_h} \frac{U^2}{2g} = y \left(\frac{\gamma_{Hg}}{\gamma_{Water}} - 1 \right) = 12.6\, y \qquad \text{....... (3)}$$

where h_L is the difference in pressure head between the pressure tappings, L is the distance between the pressure tappings, $D_h = 2\,(r_2 - r_1)$ is the hydraulic diameter and U is the mean axial velocity. Errors in reading the mercury level was estimated to be ±1 percent, that produces errors in f less than ±1.0 percent.

The friction coefficient for axial flow with rotating inner cylinder is denoted as λ which can be obtained from equation (3) by using the pressure head loss for the case of rotating inner cylinder.

3. EXPERIMENTAL CONDITIONS:

The aim of this investigation is to study the effect of the radius ratio M, axial Reynolds number ($Re = UD_h / \nu$) and rotational Reynolds number ($R_\theta = V \ D_h / \nu$) on the mean velocity profile and the friction coefficient. Thus, Two different annulus of radii ratio ($M_1 = 0.425$ and $M_2 = 0.315$) were employed to study the effect of radii ratio. Also, nine sets of the inner cylinder speeds of rotation (N = 166, 190, 230, 275, 312, 360, 410, 456 and 675 rpm) were employed. It corresponds to the rotational Reynolds numbers (R_θ= 8.78x10^3, 1.0x10^4, 1.21x10^4, 1.45x10^4, 1.65x10^4, 1.9x10^4, 2.16x10^4, 2.4x10^4 and 3.56x10^4) for the radius ratio $M_1 = 0.425$ and corresponds to the rotational Reynolds numbers (R_θ = 7.46x10^3, 8.62x10^3, 1.07x10^4, 1.26x10^4, 1.44x10^4, 1.65x10^4, 1.9x10^4, 2.1x10^4 and 3.18x10^4) for the radii ratio $M_2 = 0.315$.

Eight sets of water flow rate (Q = 6.017, 5.859, 5.642, 5.474, 5.181, 4.255, 3.352 and 1.117 lit/s) were used in this study. The corresponding mean axial velocities ($U = Q/A_1$, where A_1 is the annulus cross-sectional area for first annuli), for the radius ratio ($M_1 = 0.425$) were 2.320, 2.260, 2.176, 2.111, 1.998, 1.641 and 1.293 m/s; that corresponds to the axial Reynolds numbers (Re = 8.67 x 10^4, 8.44 x 10^4, 8.13 x 10^4, 7.89 x 10^4, 7.46 x 10^4, 6.13 x 10^4, and 4.83 x 10^4) respectively.

In the second radii ratio ($M_2 = 0.315$), for the same rates of discharge, the mean axial velocity values ($U = Q/A_2$, where A_2 is the annulus cross-sectional area for second annuli) were 2.110, 2.055, 1.979, 1.920, 1.817, 1.492 and 1.176 m/s; and the corresponding axial Reynolds number were (Re = 9.40 x 10^4, 9.15 x 10^4, 8.81 x 10^4, 8.55 x 10^4, 8.09 x 10^4, 6.64 x 10^4 and 5.23 x 10^4) respectively.

4. RESULTS AND DISCUSSION:

Comparison between the present results and the work of Yamada and Imao [12] and Polkowski [15] indicated that the results are consistent with each other.

Measurements were made to determine the effect of changing the rotational Reynolds number R_0, the axial Reynolds number Re and radius ratio M on the hydraulic friction head loss of water and on the distribution

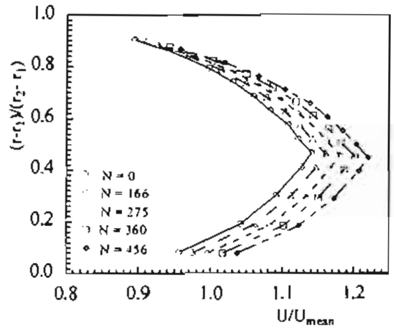


Figure (3) Dimensionless velocity profile for annulus with radius ratio $M = 0.425$ for stationary cylinders (N = 0) and rotating inner cylinder.

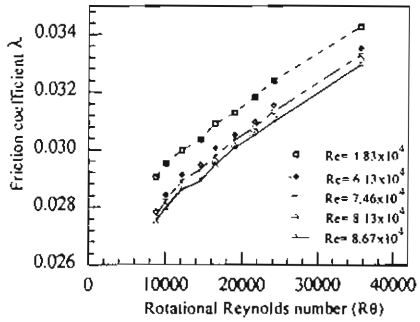


Figure (4) Variation of the rotational friction coefficient factor and the rotational Reynolds number R , for different axial Reynolds number Re.

of axial mean velocity. The effect of each parameter can be discussed as follows:

4.1. Effect of Rotational Reynolds number

When water enters an annulus with inner cylinder rotating about its axis, tangential forces acting between the rotating cylinder wall and the water cause the water to rotate with the cylinder, resulting in a rather different flow pattern from that observed in stationary cylinders. Figure (3) represents the dimensionless velocity profile for annulus with radius ratio $M = 0.425$ for stationary cylinders ($N = 0$) and rotating inner cylinder at different rotating speeds. The comparison indicates that, there is a small increase in the maximum mean velocity value for annulus with inner cylinder rotating. It also indicates that the velocity profile tends to shift slightly towards the axis of rotation.

Figure (4) represents the variation of the rotational friction coefficient λ and the rotational Reynolds number R_θ, for different axial Reynolds number Re. The figure indicates that the rotational friction coefficient factor λ increases with increasing the rotational Reynolds number R_θ or decreasing the axial Reynolds number Re. The more rotation the more swirling velocity given to the flow by rotating the inner cylinder that produces higher tangential velocity gradient which in turn increases the friction coefficient λ.

Figure (5) represents the variation of the friction coefficient ratio (λ/f) with the rotational Reynolds number R_θ for radius ratio M =0.425 and Re

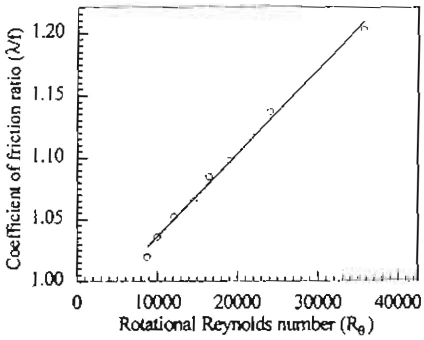


Figure (5) variation of the friction coefficient ratio (/*f*) with the rotational Reynolds number *R* for radius ratio *M* =0.425 and *Re* = 8.67x10^4.

= 8.67x10^4. The figure shows linear increase of the friction coefficient ratio (λ/f) with increasing R_θ.

4.2. Effect of Axial Reynolds Number

Figure (6) shows the dimensionless velocity profile for the first annulus, $M_1 = 0.425$, at different axial Reynolds number Re at N=360 rpm. The figure indicates that there is no change in the profile with increasing the axial Reynolds number Re, since the profiles coincide with each other.

Figure (7) represents the variation of friction coefficient λ with the axial Reynolds number Re for stationary cylinders (N = 0) and rotating inner cylinder at different speeds of rotation. The figure indicates that the

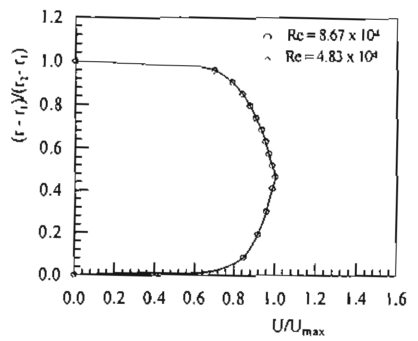


Figure (6) Dimensionless velocity profile for annulus, M_1 = 0.425, at different axial Reynolds number *Re* at *N* = 360 rpm.

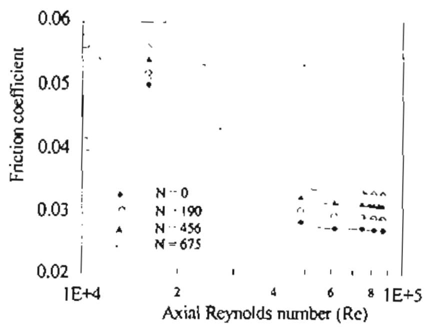


Figure (7) Variation of friction coefficient with the axial Reynolds number *Re* at different rotational speeds.

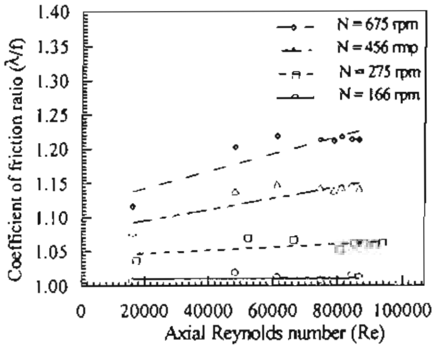


Figure (8) Variation of the friction coefficient ratio with the axial Reynolds number *Re* for different rotation speeds.

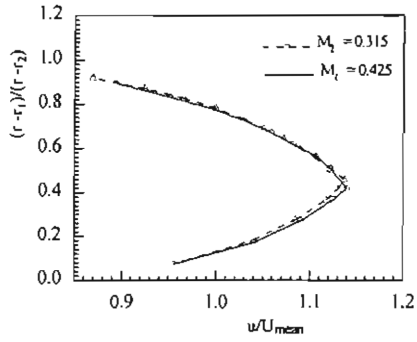


Figure (9) Dimensionless velocity profile for annuls M_1 = 0.425 and M_2 = 0.315 rotating at N = 275 *rpm*.

friction coefficient λ decreases with increasing the axial Reynolds number *Re* and it is generally higher than that for stationary cylinders (λ > *f*). As the axial Reynolds number *Re* increases, the flow becomes more turbulent which reduces the friction coefficient λ until it approaches the complete turbulent flow in which λ has almost constant value with increasing the axial Reynolds number *Re*.

Figure (8) shows the variation of the friction coefficient ratio (λ/*f*) with the axial Reynolds number *Re* for different rotational speeds. The figure indicates that the friction coefficient ratio (λ/*f*) increases with increasing both the axial Reynolds number *Re* and the rotation speed *N*. The slope increases as the rotational speed increases. The friction coefficient ratio is generally greater than unity (λ/*f* > 1).

4.3. Effect of Radius Ratio:

Two radii ratio (M_1 = 0.425 and M_2 = 0.315) were employed in this investigation and similar results were obtained but they are not included in this paper only the comparison between them is included to indicate the effect of changing the radius ratio. Figure (9) represents comparison between the dimensionless velocity profile for annuls M_1 = 0.425 and M_2 = 0.315 rotating at N = 275 *rpm*. The velocity distribution indicates that the maximum mean axial velocity $(V_z)_{max}$ tends to shift slightly towards the axis of rotation as the radius ratio increases. Increasing the radius ratio *M*, for annulus of constant outer cylinder, produces higher radius for the inner cylinder which in turn increases the tangential velocity that makes the maximum velocity tends to shift towards the axis of rotation.

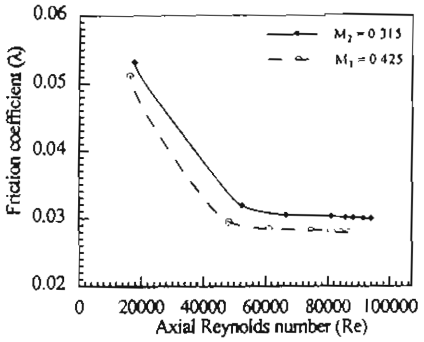


Figure (10) Comparison between the friction coefficient ratio with the axial Reynolds number *Re* for rotating speed (N = 166 rpm).

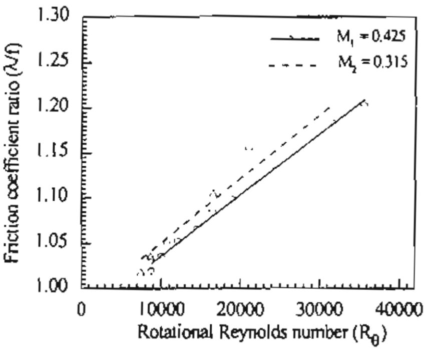


Figure (11) Variation of friction coefficient ratio ($/f$) with the rotational Reynolds number R at (Re_1 = $8.44\text{x}10^4$ and $Re_2 = 8.55\text{x}10^4$)

Figure (10) shows the comparison between the variation of the rotational friction coefficient λ and axial Reynolds number Re for annuls M_1 = 0.425 and M_2 = 0.315 with inner cylinder rotating at N = 166 rpm. The comparison reveled that increasing the radius ratio M, reduces the friction coefficient λ. Increasing the radius ratio produces narrow annulus which makes the flow more turbulent which decreases the friction coefficient.

Figure (11) represents the comparison between the variation of friction coefficient ratio (λ/f) with the rotational Reynolds number R_θ for annulus M_1 = 0.425 and M_2 = 0.315 at (Re_1 = $8.44\text{x}10^4$ and $Re_2 = 8.55\text{x}10^4$ respectively). The comparison indicates that the friction coefficient ratio (λ/f) decreases with increasing the radius ratio. Increasing the radius ration produces narrow annulus that makes the flow more turbulent which, in turn, decreases the friction coefficient.

5. CONCLUSIONS:

This study is concerned with the experimental study for the turbulent flow between concentric cylinders with rotating inner cylinder. Two annulus with radii ratio 0.425 and 0.315 were employed. Water flows at range of rotational Reynolds number ($7.5\text{x}10^3 < R_0 < 3.2\text{x}10^4$) and axial Reynolds number ($4.83\text{x}10^4 < R_\theta < 9.4\text{x}10^4$) were used in this investigation.

The results can be concluded in the following points:

(1) The axial maximum mean velocity increases slightly with increasing rotating inner cylinder.
(2) The axial maximum velocity tends to shift towards the axis of rotation as the radii ratio increase or speed of rotation of inner cylinder increases.
(3) There is no effect of increasing the axial Reynolds number, in the range tested, on the dimensionless axial mean velocity profile.
(4) The friction coefficient in an annulus with rotating inner cylinder is higher than that for stationary cylinders.
(5) The friction coefficient ratio increase with increasing the axial Reynolds number and increase with increasing the inner cylinder rotating speed.
(6) The friction coefficient decreases with increasing radii ratio.

6. NOMENCLATURE:

A	Cross section area of annuli = $\frac{\pi}{4}(D_2^2 - D_1^2)$
a_1	Area of the pipe.
a_2	Area of the orifce meter.
A_1	Cross section area of first annuli $M_1 = 0.425$
A_2	Cross section area of second annuli $M_2 = 0.315$
b	Gap width = $r_2 - r_1$.
C_d	Coefficient of discharge for the orifice meter.
D	Pipe diameter.
d	Orifice diameter.
D_h	Hydraulic diameter.
f	Hydraulic friction coefficient for stationary cylinder.
h_L	friction head loss.
L	Pipe length.
$M = r_1/r_2$	Radius ratio.
N	Speed of rotation.
Q	Discharge.
r	Arbitrary radius in the gap.
r_1, r_2	Radius of inner and outer cylinder respectively.
r_h	Hydraulic radius = $2(r_2 - r_1) = 2b$.
R_e	Axial Reynolds number = $2Ub/\nu$.
R_θ	Rotational Reynolds number = $2V_\theta b/\nu$.
U	Mean axial velocity.
V_z	Axial mean velocity.
V_θ	Tangential mean velocity of the inner cylinder = ωr_1.
ω	Angular velocity of the inner cylinder.

ρ	Fluid density.
ν	Fluid kinematic viscosity.
λ	hydraulic friction coefficient for rotating inner cylinder with axial flow.

7. REFERENCES:

1- Taylor, G. I, "Fluid Friction Between Rotating Cylinders II- Distribution of Velocity Between Concentric Cylinders When Outer One is Rotating and Inner One is at Rest," *Proceedings of the Royal society of London*, series A, Vol. 157, 31 August, 1936.

2- Boubnov, B. M., Gledzer, E. B., and Hopfinger, E. J., "Stratified Circular Couette Flow Instability and Flow Regimes," *Journal of Fluid Mechanics*, Vol. 292, pp 333-358, Jun. 1995.

3- Andereck, D. A., Liu, S. S., and Swinney, H. , "Flow Regimes in a Circular Couette System with Independently Rotating Cylinders," *Journal of Fluid Mechanics*, Vol. 164, pp 155-183, 1986.

4- Donnelly, R. J., "Experiments on the Stability of Viscous Flow Between Rotating Cylinders I- Torque Measurements," *Proceedings of the Royal society of London*, Series A, Vol. 246, pp 312-325, March 1958.

5- Pfitzer, H. and Beer, H., "Heat transfer in an Annulus Between Independently Rotating Tubes with Turbulent Axial Flow," *International Journal of Heat and Mass Transfer*, Vol. 35, No. 3, pp 623-633, 1992.

6- Smith, G. P., and Townsend, A. A. , "Turbulent Couette Flow Between Concentric Cylinders at Large Taylor Number," *Journal of Fluid Mechanics*, vol. 123, pp 187-217, 1982.

7- Cooper, E. R., Jankowski, D. F, Neitzel, G. P. and Squire, T. H., "Experiments on the Onset of Instability in Unsteady Couette Flow", *Journal of Fluid Mechanics*, Vol. 161, pp 97-114, 1985.

8- Ali, M., and Weidman, P. D., "On the Stability of Circular Coutte Flow with Radial Heating," *Journal of Fluid Mechanics*, Vol. 220, pp 53-84, 1990.

9- Becker, K. M., and Kaye, J., "The Influence of Radial Temperature Gradient on the Stability of Fluid Flow in an Annulus with Inner Rotating Cylinder," *Journal of Heat Transfer, ASME Transactions*, Vol. 84 , pp 106-110, 1962.

10- Childs, P. R. and Long, C. A., "A Review of Forced Convective Heat Transfer in Stationary and Rotating Annuli," *Proceedings of the Institution of Mechanical Engineers*, Part C, Journal of Mechanical Engineering Science, Vol. 210, No. C2, pp 123-134, 1996.

11- Kuzay, T. M., and Scott, C. J., "Turbulent Heat Transfer Studies in Annulus with Inner Cylinder Rotation," *Journal of Heat Transfer*, Trans. ASME, Vol. 99, pp 12-19, 1977.

12- Yamada, Y., and Imao, S., "Flow of a Fluid Contained Between Concentric Cylinders Both Rotating," *Bulletin of JSME*, Vol. 29, No. 252, pp 1691-1697, 1986.

13- Nouri, J. M and Whitelaw, J. H., "Flow of Newtonian and Non-Newtonian Fluids in a Concentric Annulus with Rotation of the Inner Cylinder," *Journal of Fluids Engineering*, Vol. 116, pp 821-827, Dec. 1994.

14- Nouri, J. M., Umur, H. and Whitelaw, J. H., "Flow of Newtonian and non - Newtonian fluids in Concentric and Eccentric Annuli," *Journal of Fluid Mechanics*, Vol. 253, pp 617-641, Aug. 1993.

15- Polkowshi, J. W., "Turbulent Flow Between Coaxial Cylinders with the Inner Cylinder Rotating," *Transactions of the ASME, Journal of Engineering for Gas Turbines and Power*, Vol. 106, January 1984. pp 128-135.

16- Mohamed, M. S., "Turbulent Flow Between Concentric Rotating Cylinders," *Proceedings of ASME Fluids Engineering Division Summer Meeting*, FEDSM'98-4961, pp 1-9, Washington, June 1998.

17- Holman, J. P. and Gajda, W. J., "Experimental Methods for Engineers" International Student Edition, McGraw-Hill 1958.