



Answer the following questions

Question 1 (25 MARKS)

(A) Check if the given pair of functions are linearly dependent or not

- (i) $f(t) = e^t$ and $g(t) = e^{-t}$
- (ii) $f(t) = \sin t$ and $g(t) = \cos t$
- (iii) $f(t) = t + 1$ and $g(t) = 4t + 4$
- (iv) $f(t) = 2t$ and $g(t) = t + 1$

(10 Marks)

(B) If y_1 and y_2 are two solutions of the ordinary differential equation

$$t y'' + 2 y' + t e^t y = 0 \text{ and } w(y_1, y_2)(1) = 2, \text{ Find } w(y_1, y_2)(5)$$

(15 Marks)

Question 2 (40 MARKS)

(A) For the ODE $y'' + 4y' + 4y = 0$, if we know that $y_1 = e^{-2t}$ is a solution of this equation, Find y_2 by using Abel's theorem. (20 Marks)

(B) Consider the ODE $t^2 y'' - t(t + 2)y' + (t + 2)y = 0$, $t > 0$, if we know that $y_1 = t$ is a solution of this equation, Find the general solution. (20 Marks)

Question 3 (35 MARKS)

(A) By using undetermined coefficients, Find the general solution of the following ODEs

- (i) $t^2 y'' - 3y' - 4y = 2e^{-t}$
- (ii) $y'' - 3y' - 4y = \sin 4t + 2e^{4t} + e^{5t} - t$
- (iii) $y'' + 16y = \sin 4t + \cos t - 4 \cos 4t + 4$

(15 Marks)

(B) By using Laplace Transformation, Solve the following initial value problems

(i) $y'' + y' + y = g(t)$ where $g(t) = \begin{cases} 0 & 0 \leq t < 1 \\ 1 & 1 \leq t \end{cases}$, $y(0) = 1, y'(0) = 0$

(ii) $y'' + 4y = g(t)$ where $g(t) = \begin{cases} 0 & 0 \leq t < 5 \\ \frac{t-5}{5} & 5 \leq t < 10 \\ 1 & 10 \leq t \end{cases}$, $y(0) = 1, y'(0) = 0$ (20 Marks)

This exam measures the following ILOs

Question Number	Q1-a	Q2-a	Q3-b		Q2-b	Q3-b			Q1-b	Q3-a		
Skills			b-ii			b-i						
	Knowledge & understanding skills				Intellectual Skills				Professional Skills			

With my best wishes

Dr. Elsayed M. Zaki