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MIXING OF TWO INCOMPRESSIBLE PLANE JETS

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Abstract

Mixing of two dimensional ventilated twin jets is analysed in this paper. An integral method approach is employed to solve this problem. The aim is to provide information on the interaction process of low speed jets and the dependence of this process upon the spacing between the two jet nozzles, the width of each nozzle as well as the angle between the two axes of nozzles. The effect of secondary flow generated between the two jets is also provided. The effect of the above mentioned parameters is shown to play an important role in the interaction process of the jets.

Nomenclature

A	: area ratio	l	: nozzle length
m	: mass flow rate	p	: static pressure
S	: nozzle spacing	t_p	: nozzle width
u	: x component of velocity	U_s	: secondary flow velocity
λ	: velocity ratio	ϕ	: gross thrust augmentation
ρ	: density of the fluid	θ	: angle defined in Fig.1

J_p : momentum flux at nozzle exit plane

J_x : momentum flux for downstream stations

R : initial radius of curvature of jet trajectory

U_p : the primary velocity of the fluid at the nozzle exit

x_p : fictitious merging point

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x : stream wise coordinate along centerline of the system

y : coordinate normal to centerline of the system

1, 2: upstream and downstream stations

α : coefficient related to merging point

β : angle between nozzle axes and system centerline

σ, σ_1 , σ_c : jet spreading parameters

ξ_1, ξ_r: ratios of spreading parameters

Introduction

The mixing of jet flows is one of the important problems of shear flow and is used in a wide variety of engineering applications, such as thrust augmenting ejectors for V/STOL aircraft or the development of burners and fluidics. Thus, the interaction of twin incompressible plane jet has been the subject of experimental research for over thirty years. Early investigations reported distributions of mean properties such as velocity and pressure. Later, the availability of hot-wire anemometry led to the time resolved measurements performed by many researchers. Miller and Comings (1960) carried out experiments for a dual jet flow. The mean velocity and velocity fluctuation were measured by Tanaka (1970, 1974). Krothapalli et al (1980) used X-wire probes to measure the mean velocity and turbulent intensities for the central nozzle with the five nozzles blowing. Research was done on the interaction of two axisymmetric parallel subsonic and supersonic free jets (Gamal, 1993). Because of the entrainment of surrounding air, the two jets (either subsonic or supersonic) attracted each other, merged, and eventually combined to resemble a single jet. Thus, the flow field could be defined by three distinct regions: the converging, merging and combined regions.

In fact, there are two types of the twin jet: twin jet issuing from two nozzles or slots placed on a common end wall (unventilated jets) and the ventilated jets; jets issuing from free standing nozzles or slots. In the first case, reattaching jets result in recirculating flow in a low pressure region between the nozzle exit plane. In the second one, secondary flow entrains between the two jets which affects the mixing process. The unventilated jets were studied by Lin and sheu (1991) and Gamal (1993), while ventilated jets were investigated by Marsters (1977) and Elbanna and Gahin (1983).

The characteristics of the twin jet flow field depend upon the aspect ratio of the nozzle, inlet geometry of the nozzle, the magnitude of the turbulence intensity at the exit plane of the nozzle, the Reynolds number at the nozzle exit, the angle between the two nozzle axes, and conditions of the ambient medium into which the jet is issuing. Sawyer (1963) and Marsters (1977) made their investigations at one value of the spacing between the nozzles and at zero divergence angle. The aim of the present study is to find out the effect of the spacing between the nozzles and the angle between the two nozzle axes on the interaction process. Therefore, in this study, analysis of ventilated incompressible twin plane jets based on integral methods was made. Since the flow field associated with the interaction of the two jets is very complex, the intgral method is easy of handle this problem. Comparison was made between the present results and those of ventilated jets given by earlier investigators, Elbanna and Gahin (1983).

Analysis

Fig. 1 shows the nozzle geometry and flow field configuration of thetwin plane jets. Incompressible air flow is assumed to issue from the two nozzles with no common end wall between them. Thus, the secondary flow velocity, denoted U_s, is assumed to be uniform across the plane between the nozzles and the velocity profiles of the primary nozzles (U_p) are assumed to be flat. S is the distance between the two nozzles. The jets are assumed to merge at some finite downstream location X_m beyond which the static pressure is uniform; the combined jet behaves as a single jet. The angle between the two nozzles is assumed to be ≤ 30°; (i.e. to make sure the two jet get mergeing). The transverse momentum of each jet is neglected.

The continuity and X-momentum equations are written for the flow between sections 1 and 2 (Fig. 1). Assuming that the entrainment on the outer surface of the jet is $m_e(x)$, thus :

$$\rho \int_{-\infty}^{\infty} u \, dy = 2 \rho U_p t_p + \rho U_s (S - t_p \cos \beta) + m_e(x) \qquad (1)$$

$$p_1(S + t_p \cos \beta) - p_2(S + t_p \cos \beta) = \rho \int_{-\infty}^{\infty} u^2 dy - \rho U_s^2 (S - t_p \cos \beta) - 2 \rho U_p^2 t_p \cos \beta \qquad (2)$$

The static pressure at station 1, outside the nozzle boundaries, is assumed to be ambient (p_a). Therefore, at station 1, the static pressure between the nozzles and over the nozzle exit is

$$p_1 = p_a - 1/2\ \rho\ U_s^2 + \Delta\ p \qquad (3)$$

where Δ p represents the pressure losses of the secondary flow upstream of station 1.
Note that station 2 is taken to be sufficiently far downstream to insure that the static pressure is uniform everywhere and equal to p_a. Combining equations (2) and (3) with Δ p = 0 yields

$$2 \cos \beta + \frac{A - 2 \cos \beta}{2 \lambda^2} = \frac{1}{U_p^2\ t_p} \int_{-\infty}^{\infty} u^2\ dy$$

where A is the area ratio ($A = \dfrac{S - t_p \cos \beta}{t_p}$)

and λ is the velocity ratio ($\lambda = U_p / U_s$)

The integral is to be evaluated at some sufficiently large value of x where the merged jets behave like a single free jet, and so according to Bourque and Newman (1960) the velocity distribution may express as :

$$u = \sqrt{3\ J_x\ (\sigma / 4 \rho x)}\ \ \mathrm{sech}^2\ (\sigma y / x) \qquad (4)$$

where J_x is the momentum flux of the merged jet evaluated at $x > x_m$ and is constant. Also, σ is the spreading rate and $U_p^2 t_p = J_p / \rho$; where J_p is the momentum flux of one of the primary jets. Thus, the integral becomes

$$\frac{3}{4}\ \frac{J_x}{J_p} \int_{-\infty}^{\infty} \mathrm{sech}^4\ \eta\ d\eta$$

Where $\eta = \dfrac{\sigma\ y}{x}$. One then obtains the momentum equation in the following form :

$$2 \cos \beta + (A - 2 \cos \beta) / 2\lambda^2 = \frac{J_x}{J_p} \qquad (5)$$

Proceeding with the continuity equation, we examine the total flow into the jet over the external (outside and inside) jet surfaces. Up to the point x_m on the outer surfaces, entrainment takes place at a rate determined by the curvature; beyond this point, the entrainment parameter is assumed to be σ . Thus, for the outer surfaces,

$$m_e(x) = \int_0^{x_m} \left(\frac{dm_e}{d\,x}\right) dx + \int_{x_m}^{x} \left(\frac{dm_e}{d\,x}\right) dx$$

For stations downstream of x_m,

$$\frac{dm_e}{d\,x} = \frac{d}{d\,x}\left(\rho \int_{-\infty}^{\infty} \sqrt{\frac{3\,J_x\,x}{4\,\rho\,\sigma}}\ \mathrm{sech}^2\,\frac{\sigma\ y}{x}\ d\left(\frac{\sigma\ y}{x}\right)\right)$$

Thus,

$$\int_{x_m}^{x} \left(\frac{dm_e}{d\,x}\right) dx = 2\sqrt{\frac{3\,\rho\ J_x}{4\,\sigma}}\left(\sqrt{x} - \sqrt{x_m}\right)$$

For stations upstream of x_m, a value of $\sigma_c = 13.2$ is assumed (Sawyer, 1963). Then the entrainment rate is given by

$$\frac{dm_e}{d\,x} = \frac{d}{d\,x}\left(\rho\sqrt{\frac{3\,J_p\,x}{4\,\rho\,\sigma_c}}\int_{-\infty}^{\infty} \mathrm{sech}^2\,\frac{\sigma_c\ y}{x}\ d\left(\frac{\sigma_c\ y}{x}\right)\right)$$

Therefore,

$$\int_0^{x_m} \left(\frac{dm_e}{d\,x}\right) dx = \sqrt{\frac{3\,J_p\rho\,x_m}{\sigma_c}}$$

Equation (1) then becomes

$$\rho\int_{-\infty}^{\infty}\sqrt{\frac{3\,J_x\sigma}{4\,\rho\,x}}\ \mathrm{sech}^2\left(\frac{\sigma\ y}{x}\right) dy = 2\,\rho U_p t_p + \rho\ U_s(S - t_p\cos\beta) +$$

$$\sqrt{\frac{3\,J_x\,\rho}{\sigma}}\left(\sqrt{x} - \sqrt{x_m}\right) + \sqrt{\frac{3\,J_p x_m\,\rho}{\sigma_c}}$$

On evaluating the integral and dividing by J_p, the result is

$$\sqrt{\frac{3}{4}\left(\frac{x_m}{t_p}\right)}\left(\sqrt{\frac{J_x}{J_p\sigma}} - \sqrt{\frac{1}{\sigma_c}}\right) = 1 + \frac{A}{2\lambda} \qquad (6)$$

On the inner surfaces of the jets, it is clear that all the secondary flow must be entrained in a distance of, at most, $\alpha\ x_m$, where $0 < \alpha < 1$. On these surfaces, the spreading parameter is denoted σ_i ; the value of σ_i is taken to be 18.5 (Sawyer, 1960). Assuming a uniform entrainment rate on the inner surface of both primary jets,

$$\int_0^{\alpha x_m} \frac{d}{d\ x}(m_e)\ dx = \int_0^{\alpha x_m} \frac{d}{d\ x}\left(\int_{-\infty}^{\infty}\sqrt{\frac{3\ J_p\ x\ \rho}{4\ \sigma_i}}\ *\right.$$

$$\left.\operatorname{sech}^2 \frac{\sigma_i\ y}{x}\ d\left(\frac{\sigma_i\ y}{x}\right)\right) dx = \sqrt{\frac{3\ J_p \rho\ x_m\ \alpha}{\sigma_i}}$$

This must be just equal to the secondary flow $\rho\ U_s\ t_p A$, hence we find

$$\frac{A}{\lambda} = \sqrt{\frac{3\ x_m\ \alpha}{\sigma_i\ t_p}} \qquad (7)$$

Eliminating x_m from equation (6) and combining with equation (5) yields

$$\frac{A}{2\lambda} = \left[\sqrt{\left(2\cos\beta + \frac{A - 2\cos\beta}{2\ \lambda^2}\right)\frac{\xi_i}{\alpha}} - \sqrt{\xi_r} - 1\right]^{-1} \qquad (8)$$

where $\zeta_i = \sigma_i\ /\ \sigma$ and $\zeta_r = \sigma_i\ /\ \sigma_c$. Equation (8) may be solved for λ, provided that the value of α is fixed. Unfortunately, the distance x_m and αx_m are not determined easily, and little guidance is available for establishing α. To assist in estimating α, we make use of the pressure difference across the jets and denote as x_p the location of circular arcs drawn tangent to the centerlines of each jet at the nozzle exit plane. The initial radius of curvature R is estimated on the basis of the jet momentum and the pressure difference across the jets. With reference to Fig. 1, we can write

$$\frac{dp}{dr} = \frac{\rho U_p^2}{R} \qquad (9)$$

$$S = 2 R (1 - \cos \theta) \qquad (10)$$

$$x_p = R \sin \theta \qquad (11)$$

Approximating dp/dr by $\Delta p/t_p$ and recalling that $\Delta p = 1/2 \rho U_s^2$, one obtains

$$\frac{x_p}{t_p} = 2 \lambda^2 \sin \cos^{-1} \left(1 - \frac{A + \cos \beta}{4 \lambda^2} \right) \qquad (12)$$

It seems clear that x_p is a lower limit for the distance for merging of the jets, and that the merging point lies some where between x_p and x_m.

The gross thrust augmentation ϕ is the ratio of momentum flux at the exit plane of the device to the momentum flux of the primary stream, assuming that the primary stream expands to ambient pressure p_a and the primary mass flux remains the same. With the primary flow stagnation pressure denoted p_t ; thus the thrust augmentation available with a pair of plane jets becomes

$$\phi = \frac{J_x}{2 \dot{m}_p V_a} \qquad (13)$$

where $\dot{m}_p = \rho U_p^* t$ and $U_p^* = \sqrt{2 (p_t - p_a)/\rho}$

(where U_p^* is the nozzle exit plane velocity, expansion to freestream static pressure).

Because $V_a = \sqrt{2 (p_t - p_1)/\rho}$, we find

$$\frac{J_p}{J_p^*} = \frac{V_a^2}{U_p^{*2}} - \frac{2 (p_t - p_a) + \rho U_s^2}{2 (p_t - p_a)} = 1 - \frac{U_s^2}{V_a^2} \frac{V_a^2}{U_p^{*2}}$$

with this result, equations (13) and (5) yield

$$\phi = \frac{0.5\ \lambda}{\sqrt{\lambda^2 - 1}} \left[2 \cos \beta + \frac{A - 2 \cos \beta}{2\ \lambda^2} \right] \qquad (14)$$

Results and Discussion

Fig.2 shows the effect of centerline spacing on the velocity ratio for different values of β. It is clear that the value of λ increases as the centerline spacing is increased. The increasing rate is almost linear for all tested values of β. The effect of divergent angle on the velocity ratio is also evident in this figure. The velocity ratio decreases with increasing the divergent angle. This means that the secondary flow velocity becomes higher for higher values of divergent angle.

The two jets merge together to form a single jet near the nozzle exit when the spacing between the nozzles is small and the merging occurs far down stream when the divergent angle becomes large. This can be seen in Fig.3. This means that the curvature radius of the jet boundaries is large for the large value of β so that the jets take a longer distance to merge together. For zero degree divergence angle, the two jets merge at a distance between $x_m/t_p = 30$ and $x_m/t_p = 40$, for small values of nozzle spacing. This is in accordance with the results obtained by Elbanna and Gahin (1983).

Fig.4 shows the variation of the jet gross thrust with centerline spacing. The effect of β is also given. It is seen that the jet gross thrust is higher for small values of centerline spacing and decreases as the distance between the two jets is increased. This is due to the decrease in the momentum flux at the exit plane of the device compared to that of the primary streams. The gross thrust is large for small divergent angle and also decreases with increasing the divergent angle. This is also because the two jets take longer distance to merge.

Fig.5 shows the effect of the divergence angle on the merging point represented by x_m/t_p, gross thrust, ϕ, and velocity ratio, λ, for two values of nozzle width $t_p = 2.5$ and 5 mm, respectively. The location of merging point comes closer to the nozzle exit plane for the larger values of nozzle width. However, the velocity ratio decreases. This means that the spreading rate of the jet increases with increasing the jet width. Therefore, the two inner boundaries of the jets merge at the closed distance from the nozzle exit plane. Also, the gross thrust increases as the nozzle width is increased. This shows that the momentum flux becomes higher as the nozzle width is increased.

Conclusion

Analysis of the flow field of incompressible two dimensional ventilated twin jets has been described based on the integral method. The analysis shows that the interaction of low speed jets depends upon different parameters such as centerline spacing, divergence angle between the nozzles and the shape of nozzle exit. The secondary flow velocity increases as the divergence angle is increased. Whereas, the gross thrust decreases with increasing the divergent angle and centerline spacing. The two jets merge closer to the nozzle exit as the width of the nozzle exit is increased.

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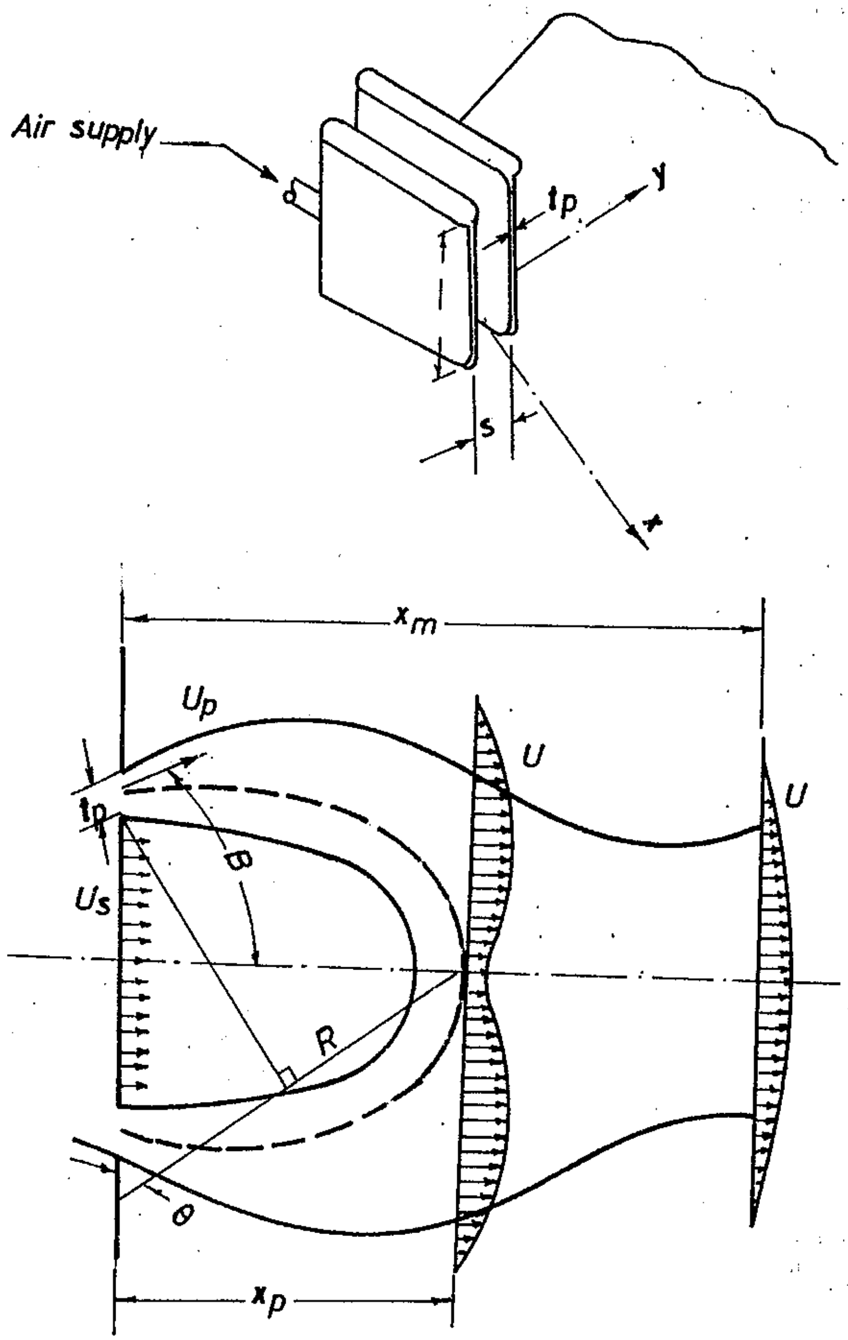

Fig.1. Nozzle geometry of the twin plane jet.

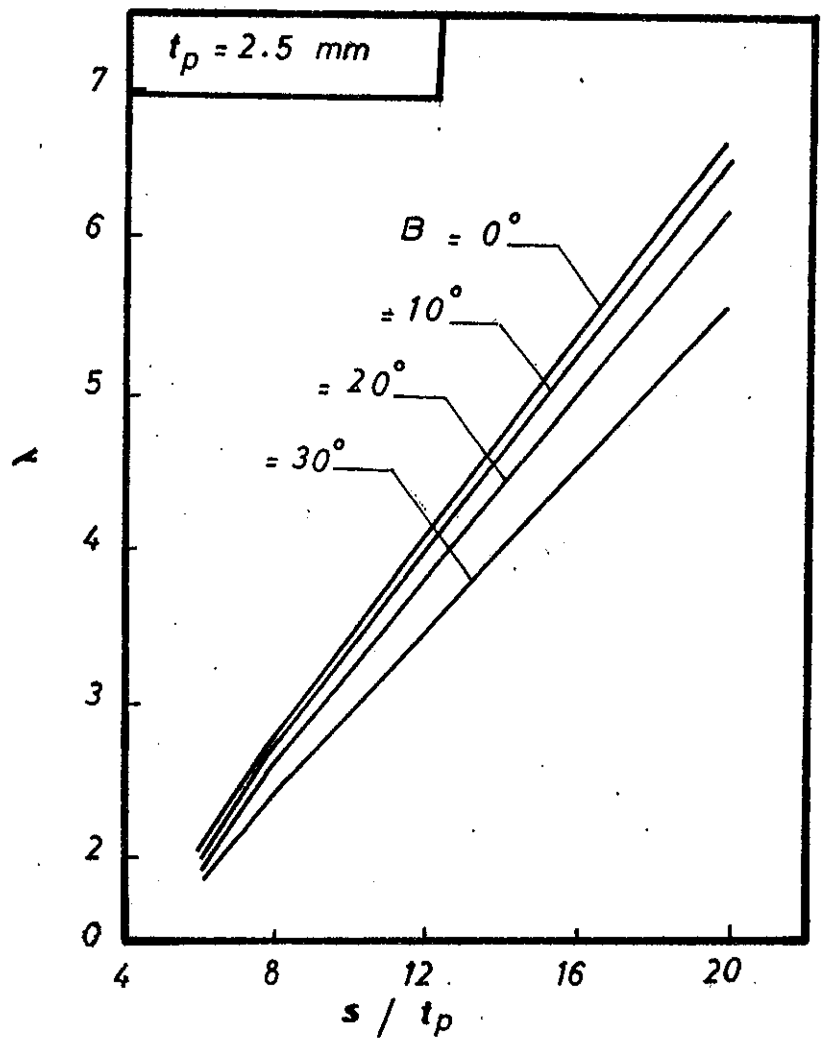

Fig. 2. Effect of center line spacing on the velocity ratio.

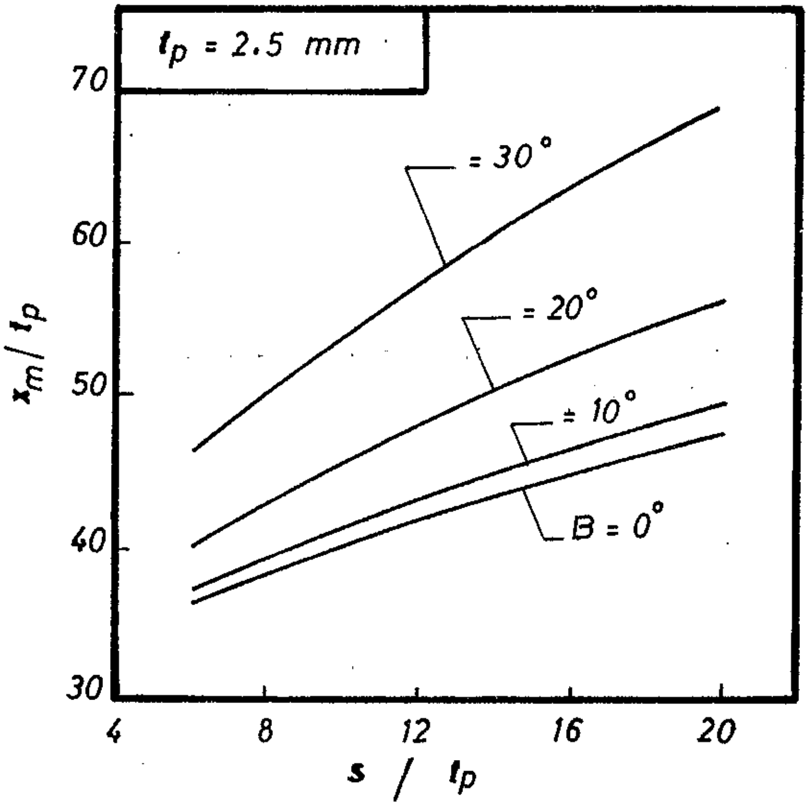

Fig. 3. Variation of merging location with center line spacing.

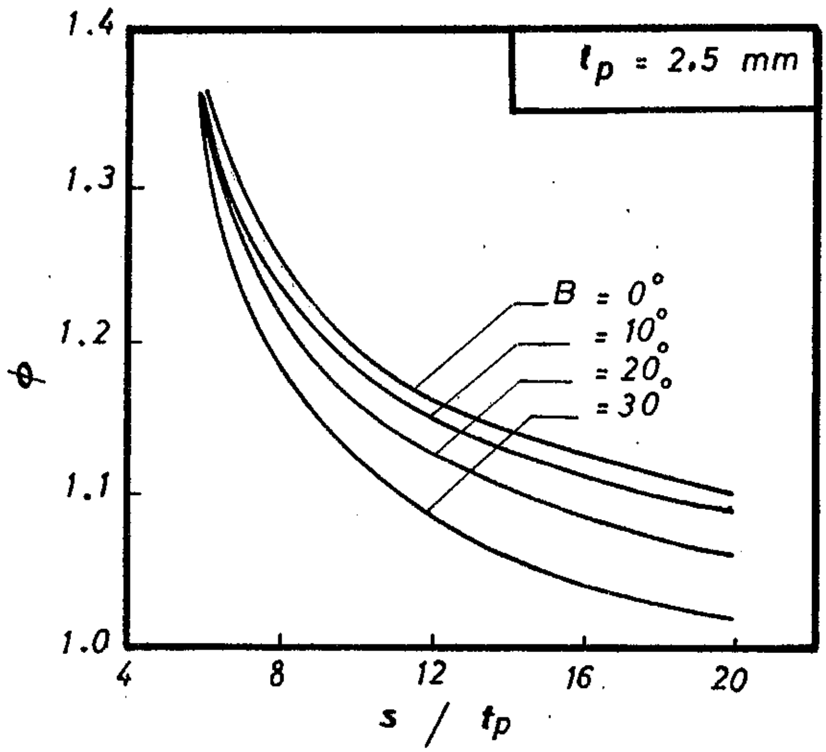

Fig. 4. Variation of gross thrust with center line spacing.

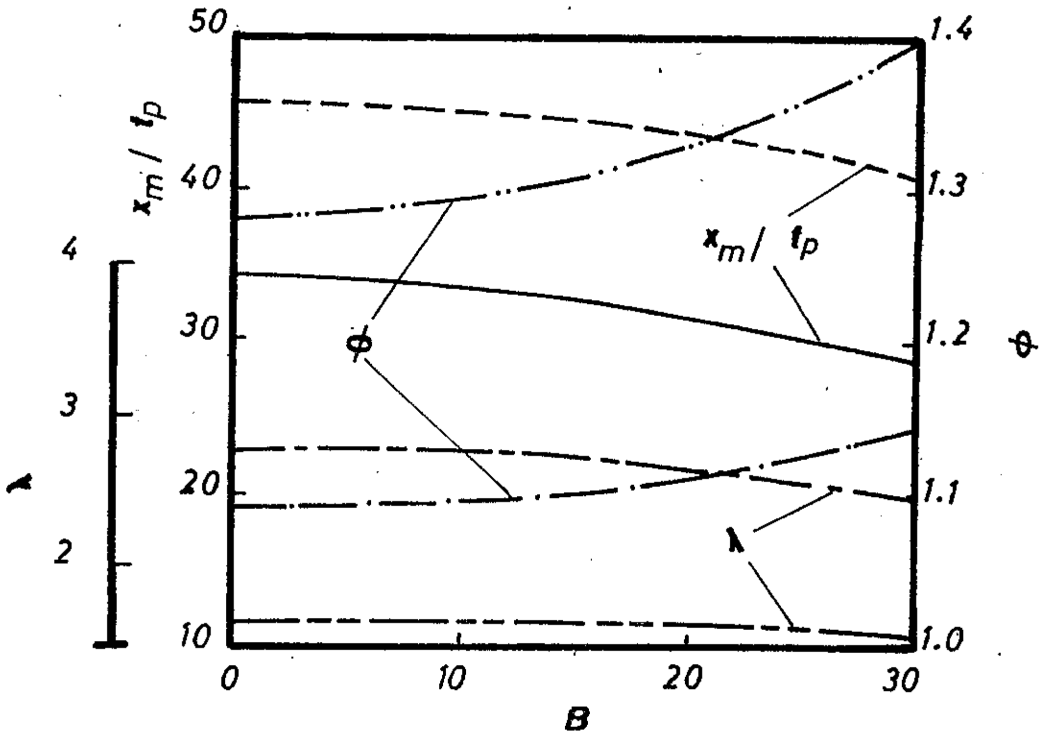

Fig. 5. Effect of jet divergence angle on the velocity ratio, merging point and gross thrust.

t_p = 2.5 mm		t_p = 5 mm
————————	x_m / t_p	— — — — — —
——— — ———	λ	———— — ————
———··———	φ	————·————

عنوان البحث

" عملية الخلط بين أثنين من النفث الغير قابل للإنضغاط ذات أبعاد مستطيلة "

تناول البحث دراسه نظريه لعمليه الخلط بين إثنين من النفث (Jet) الغير قابل للإنضغاط والمبعوثين من إثنين من الأبواق ذات مقطع مستطيل عند المخرج وليس بينهما حائط مشترك . أستخدم فى هذه الدراسه طريقه تكامليه لمعادلات الحركه والإستمراريه . فوجد أن سرعة المائع المنبعث من كل بوق متماثله ومستويه وأن سرعة المائع الثانوى المتولده بين الأبواق منتظمه .

تمت الدراسه لبيان تأثير كلا من المسافه بين الأبواق وزاوية ميل محور وضع الأبواق وعرض مخرج البوق على أداء الخلط بين النفثين . فرض أيضا أن النفثين يحدث بينهما إندماج على مسافه معينه من مخرج البوق وأن النفث الناتج بعد الدمج له نفس خواص النفث المنفرد أى المنبعث من بوق واحد فقط .

بينت الدراسة أن نسبه السرعة (النسبه بين سرعة المائع المنبعث من كل بوق إلى سرعة المائع الثانوى المتولد بين الأبواق) تتغير مع المسافة بين الأبواق ومع تغير الزاويه بين محورى الأبواق . فإتضح أن نسبة السرعة تزداد كلما زادت المسافة بين الأبواق . وأن نسبة السرعة تقل مع زيادة مقدار زاوية ميل المحورين . هذا يدل على أن المائع الثانوى المتولد بين الأبواق يكون بكميه أكبر فى حالة زاوية الميل الكبيرة لكلا المحورين . إتضح أيضا أن النفثين المنبعثين من كلا البوقين يندمجا سويا ليكونا نفثا واحدا قريبا من مخرج البوق عندما تكون المسافة بين الأبواق صغيره ويحدث الدمج بعيدا عندما تكون زاوية ميل محورى البوق كبيره . هذا يدل على أن نصف قطر إنحناء حدود النفث المنبعث من كل بوق يزداد مع زيادة زاوية ميل محورى البوقين مما يؤدى إلى ان يأخذا مسافة أطول ليندمجا سويا . ووجد من النتائج أن النفثين يندمجا سويا على مسافة تعادل ٣٠ مرة من عرض مخرج البوق عندما تكون المسافة بين البوقين صغيرة و يكون الدمج على مسافة تعادل أكثر من ٤٠ مرة من عرض مخرج البوق عندما تكون المسافة بين البوقين كبيرة . هذة النتائج تتفق مع النتائج المنشورة سابقا فى هذا المجال .