



[1]

A. Define

- I. Controllable system
- II. Observable system
- III. Regulator Poles
- IV. Eigenvalues

B. Consider that a third-order system has the coefficient matrices

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & -4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Check system **controllability**?

C. For the system that have

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad C = [1 \quad 0]$$

Check system **Observability**?

[2]

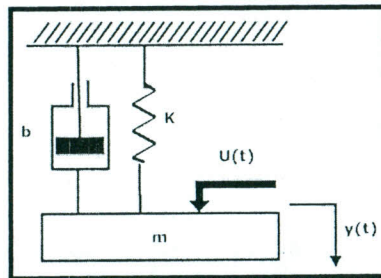
A. Find out if the matrix  $\begin{bmatrix} -e^{-t} & 0 \\ 0 & 1 - e^{-t} \end{bmatrix}$  can be **state-transition** matrix

B. For the spring-mass-damper system shown below. The state equations are

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{C}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Given:  $m = 1 \text{ kg}$ ,  $C = 3 \text{ Ns/m}$ ,  $K = 2 \text{ N/m}$ ,  $u(t) = 0$ . Evaluate,

- The characteristic equation, its roots
- The transition matrices  $\varphi(s)$  and  $\varphi(t)$
- The transient response of the state variables from the set of initial conditions  $y(0) = 1.0$ ,  $\dot{y}(0) = 0$



[3]

A. Define

- I. State observer
- II. Pole Placement

B. Consider a system defined by

$$\dot{X} = AX + BU$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = [1 \quad 0]$$

It is desired to have eigenvalues at -3 and -5 by using state-feedback control  $u = -Kx$ . Determine the state-feedback gain matrix

[4]

Use MATLAB to compute and plot the impulse and step responses of the state-space model

$$A = \begin{bmatrix} -2 & -2.5 & -0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 1.5 \quad 1], \quad D = [0]$$

Then simulate and plot the state response  $x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T$  when the input is  $u(t) = \begin{cases} 2 & 0 \leq t \leq 2 \\ 0.5 & t \geq 2 \end{cases}$  and the initial condition is  $x(0) = [1 \quad 0 \quad 2]^T$

[5]

Consider the system defined by

$$\dot{X} = AX + BU$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By using the state feedback control  $u = -Kx$ , It is desired to have the closed loop poles at  $s = -2 \pm j4$  and  $s = -10$ . Determine the state feedback gain matrix using **MATLAB**

[6]

Consider a system defined by

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX \end{aligned}$$

Where

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 1 \quad 0]$$

Transform the system into

- Controllable canonical form
- Draw state diagram of Controllable canonical form
- Observable canonical form
- Find eigenvalues

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Dr. Mostafa A. El-hosseini  
[www.melhosseini.net](http://www.melhosseini.net)