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THEORETICAL STUDY OF INJECTION EFFECTS ON TWO-DIMENSIONAL SEPARATED TURBULENT BOUNDARY LAYER

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ABSTRACT

This paper presents the resulting of computation of turbulent boundary layer equations including the effect of transvers curvature about two-dimensional and symmetrical bodies for compressible flow with mass injection of air into air through porous surface. A transformation which combines the Probstein-Elliot and Levy-Lees transformations is used in equations of continuity, momentum and energy. The equations are solved simultaneously with variable fluid properties by a finite difference technique imploying a large scale computer. The Reynolds shear stress term is eliminated by an eddy-viscosity concept, and the time of the product of fluctuating velocity and temperature appearing in the energy equation is eliminated by an eddy-conductivity concept. The turbulent boundary layer is regarded as a composite layer consisting of inner and outer regions, for which separate expressions for eddy-viscosity are used. The eddy-conductivity term is lumped into a turbulent Prandtl number that is currently assumed to be constant. The theoretical prediction shows that, the separation point is shifted in the

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down stream direction by injection effect. This predicted values satisfy the experimental results. However, one can control the separation region through the control of injection rate and hence the drag force produced from flow around the body.

NOMENCLATURE

a	:	Half body length.
c	:	Viscosity-density parameter, $c = \rho\mu/\rho_e \mu_e$.
f	:	Dimensionless stream function, equation (12)
g	:	Dimensionless total enthalpy ratio, $g = H/H_e$
h	:	Specific enthalpy.
H	:	Total enthalpy, $H = h + u^2/2$
K	:	Flow index.
k_1, k_2	:	Constants appearing in eddy-viscosity formulas.
ℓ	:	Mixing length.
L	:	Reference body length.
M	:	Mach number.
MI	:	Mass injection ratio, $MI = \rho_w v_w/\rho_e u_e$
P	:	Pressure.
P_r	:	Prandtl number.
r	:	Radial distant from body axis.
r_o	:	Radius of body
t	:	Transverse curvature term, $t = y1/r_o \cos\alpha$.
u	:	x-component of velocity.
u^*	:	Friction velocity, $u^* = \sqrt{\tau_w/\rho}$
T	:	Local temperature.
v	:	y-component of velocity.
x_1, y_1	:	Distant along length (L) and normaly to x_1 respectively.
x	:	Distance along surface measured from leading edge.
y	:	Distance normal to x
α	:	Angle between normal to the surface y and the radius (r)

B : Dimensionless velocity gradient term, $B = 2\ \xi\ du_e/u_e\ d\xi$

b : Half body width.

γ : Intermittency factor, equation (9).

δ : Boundary layer thickness.

δ_1 : Displacement thickness

ε : Eddy-viscosity.

ε^+ : Ratio of eddy-viscosity to kinematic viscosity, $\varepsilon^+ = \varepsilon/\nu$.

η : Transformed y_1-co-ordinate.

λ : Thermal conductivity.

μ : Dynamic viscosity.

ν : Kinematic viscosity.

ρ : Density.

τ : Shear stress.

SUBSCRIBTS

c_1 : Switching point between the inner and outer eddy-viscosity formulas.

e : Outer edge of boundary layer.

t : Turbulent flow

w : Wall conditions.

∞ : Free-stream conditions.

n : Designation point in ξ -direction.

i : Designation point in r -direction.

(-): Nondimensionalized quantities.

INTRODUCTION

This paper presents the resulting computation of turbulent boundary layer equations for two dimensional flows with mass transfer and accounts for transverse-curvature effects. The main purpose of this paper is to study the effect of injection on the separated turbulent boundary layer. We first start the solution of the boundary layer equations without injection to determine the separation point, then the mass injection begins from the separation point until the last section. In principle, the present approach used for delying the separation of the turbulent boundary layer in an adverse pressure gradient is similar to the ones used by Newman and Irwin [5], Ljuboja and Rodi (6), and Tsahalis [7]. The main difference between the four methods lies in the eddy-viscosity models, as well as the method of injection and the used fluid properties equations. In addition, the transformations used to stretch the co-ordinate normal to the flow direction, as well as the numerical method used to solve the boundary layer equations, are considerably different. The results show that, the separation point shifted in the down stream direction due to the injection effect.

Basic Equation

The Boundary Layer Equations

The compressible turbulent boundary layer equations for two dimensional and axisymmetric flows can be written as in [2].

Continuity equation :

$$\frac{\partial}{\partial x} r^k (\rho u) + \frac{\partial}{\partial y} r^k (\rho v + \overline{\rho' v'}) = 0 \qquad (1)$$

Momentum equation :

$$\rho u \frac{\partial u}{\partial x} + (\rho v + \overline{\rho' v'}) \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{1}{r^k} \frac{\partial}{\partial y} r^k (\mu \frac{\partial u}{\partial y} - \rho \overline{u' v'}) \qquad \ldots . (2)$$

and ,

Energy equation :

$$\rho u \frac{\partial H}{\partial x} + (\rho v + \overline{\rho' v'}) \frac{\partial H}{\partial} = \frac{1}{r^k} \frac{\partial}{\partial y} r^k . [\frac{\mu}{P_r} \frac{\partial H}{\partial y} - \rho \overline{v' H'} + $$

$$+ \mu (1 - \frac{1}{P_r}) u \frac{\partial u}{\partial y}] \qquad \ldots . (3)$$

where k = 0.0 for two dimensional flow and k=1 for axisymmetric flow. The basic notation and scheme for co-ordinates are shown in Fig.(1) for two dimensional flow. In order to solve the equations given by (1),(2) and (3), it is necessary to relate the Reynolds shear stress, $(-\varepsilon\,\overline{u'v'})$, and the term $(-\varepsilon\,\overline{v'H'})$, to mean velocity and enthaly distributions, respectively. Here we use an eddy-viscosity (ε) concept and an eddy conductivity (λ_t) concept to relate these time-mean fluctuating quantities to mean velocity and enthalpy distributions. Hence, we define

$$\varepsilon = -\frac{\overline{u'v'}}{\partial u/\partial y}, \quad \lambda_t = -\frac{\overline{v'H'}}{\partial H/\partial y} = \frac{\varepsilon}{P_{r_t}} \qquad \ldots.(4)$$

where P_{r_t} is a turbulent prandtl number.

We next use a two-layer model and represent the eddy-viscosity within the boundary layer by separate expressions. In the inner region, we use an eddy-viscosity based on Prandtl's mixing-length theory; that is,

$$\varepsilon_i = \ell^2 \cdot \frac{\partial u}{\partial y} \qquad \ldots\ldots(5)$$

where ℓ, the mixing length, is given by =0.4 y. However, to account for the viscous sublayer close to the wall, a modified expression for ℓ is used. This modification, suggested by Van Driest [1], is

$$\ell = 0.4\, y\ [1 - \exp(-\frac{y}{A})] \qquad \ldots..(6)$$

where, A is a constant for a given stream location in the boundary layer, defined as $26\,(\tau_w/\varepsilon_w)^{-1/2}$, with subscribted denoting values at the wall. The experession given by equation (6) for a flat-plate flow with no mass transfer , therefore , it cannot be used for flows with pressure gradients or for flows with mass transfer . The former is quite obvious, since, for a flow with an adverse pressure gradient, it may approach zero (flow separation). In such case the mixing-length expression will have a discontinuity, and consequently, the velocity profile, will have discontinuities. For this reaseon, Cebeci [2] has modified equation (6) by an expression to account flows with pressure gradient and mass transfere. So, the modified expression for inner eddy-viscosity ε_i can be written as

$$\varepsilon_i = k_1\, y^2\, [1 - \exp(-y^+/A^+)]^2 \quad \frac{\partial u}{\partial y} \qquad \ldots\ldots(7)$$

where k_1 is constant $= 0.16$.

The damping constant A^+, is given by

$$A^+ = 26\{-\frac{P^+}{v_w^+}[\exp(11.8\, v_w^+) - 1] + \exp(11.8\, v_w^+)\}^{-\frac{1}{2}} \ldots(8)$$

Here,

$$P^+ = -\frac{dp}{dx}\,\frac{\nu}{\rho_w (u^*)^{3/2}}, \quad y^+ = \frac{yu^*}{\nu}, \quad u^* = \sqrt{\frac{\tau_w}{\rho}} \quad \text{and}$$

$$v_w^+ = \frac{v_w}{u^*}$$

In the outer region, a constant eddy-viscosity, ε_o is used

where; $$\varepsilon_o = k_2 \cdot u_e\, \delta_1 \cdot \gamma \qquad \ldots(9)$$

k_2 is constant $= 0.0168$, and

γ, the intermittency factor.

The modified intermittency factor approximated is given by [3] as :

$$\gamma = [1 + 5.5\,(y/\delta)^6\,]^{-1} \qquad \ldots\ldots(10)$$

Transformation of boundary-layer equations by the applications of the Probstein-Elliot/Levy-Lees transformations are given for ξ and η by the relations

$$d\xi = \rho_e\, \mu_e\, u_e\, (\frac{r_o}{L})^{2k} dx, \quad d\eta = \frac{\rho u_e}{\sqrt{2\xi}} (\frac{r}{L})^k \; dy \ldots(11)$$

using the relations in equation (11), momentum and energy equations after simplification we get the transformed equations in the following forms.

Momentum Equation :

$$[(1+t)^{2k}.c.(1+\varepsilon^{+})f'']' + ff'' + B[\frac{\rho e}{\rho} - (f')^2]$$
$$= 2\xi \;.[f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi}] \qquad \ldots(12)$$

Energy Eauation :

$$\{(1+t)^{2k}.c.[(1+\varepsilon^{+}\frac{P_r}{P_{r_t}})\frac{g'}{P_r} + \frac{u_e}{H_e}(1-\frac{1}{P_r})f'f'']\} + f\,g'$$
$$= 2\xi \;.[f' \frac{\partial g}{\partial \xi} - g' \frac{\partial f}{\partial \xi}] \qquad \ldots.(13)$$

The boundary conditions of equations (12) and (13) in the $\xi-\eta$ plane are as follows :

For momentum equation :

$$f(\xi,o) = f_w = -\frac{1}{\sqrt{2\xi}} \int_o^{\xi} (\frac{L}{r_o})^k \frac{\rho_w\, v_w}{\rho_e\, \mu_e\, u_e}\, d\,\xi$$
$$f'\;(\xi,o) = 0.0,\; \lim_{\eta\to\infty} f(\xi,\eta) = 1 \qquad \ldots(14)$$

and

For Energy Equation

$$g(\xi,o) = \frac{H_w}{H_e} = g_w,\quad \lim_{\eta\to\infty} g(\xi,\eta) = 1 \qquad \ldots(15)$$

METHOD OF SOLUTION

The calculation begins at the leading edge, where $\xi= 0.0$,and proceeds downstream.Consider, the case when initial turbulent velocity and constant temperature profiles are known at $\xi=\xi_{n-1}$, and we seek a solution to momentum and energy equations with the boundary conditions at $\xi=\xi_n$. We first calculate the fluid properties from the temperature profiles at $\xi=\xi_{n-1}$ and then start the computation. For this, it is necessary to establish the inner and outer regions for the eddy-viscosity formulas. For the first iteration, δ_1 and f" are obtained from the solution at ξ_{n-1}, and inner and outer regions are established by the continuity and the eddy-viscosity equations. With this information, a solution of the momentum is obtained and consequently a solution of the energy equation. Then fluid properties are obtained for that particular solution, inner and outer regions for the eddy-viscosity formulas are established and the momentum and energy equations are solved in succession. An iteration procedure based on the convergence of δ_1 and f_w is used. A finite difference pattern in the shape of a horizontal involving three points in the ξ-direction and five points in the η-direction is used as shown in the Fig.2. The resulting algebraic equations are solved by the Cheleski matrix method.

DISCUSSION OF THE RESULTS

In this paper, the effect of mass injection through porous surface for two-dimensional, compressible flows on the separated turbulent boundary layer is studied theoretically. The mass injected is described by (MI). The computer program designed for the solution of the turbulent boundary layer on a curved surface was used to predict the separation point. So, we shall be discus the results of computation for velocity and temperature without injection, and then the effect of injection on these results. The problem is studied here for mass injection ratio MI, $(0.0 \leq MI \leq 0.01)$. The physical properties of used fluid in this study, are taken as follows :

$$\mu_\infty = 1.59 \times 10^{-5} \text{ N.S/m} , \; t_\infty = 50\,°C, \; M_\infty = 0.35$$

and,

$R_e = 8 \times 10^5$. This Reynolds number corresponds a free stream velocity (u_∞) of about 125 m/s. The problem was solved on an adiabatic surface and steady flow conditions. The fluid used in injection was also air.

EFFECTS OF MASS INJECTION RATIO (MI) ON VELOCITY PROFILES

Figures 3,4,5,6 and 7, show the effect of (MI) variation on the velocity profiles distributions developed on the body shown in Fig. (1). The predicted separation profile is obtained, as shown in Fig.(3),at $\bar{x}$ = 0.7, where the velocity gradient on the wall becomes zero.The mass injected starts from the predicted separation point until the last section of the body normally to the wall through a porous surface as shown in Figure(1). Fig. (4) shows the velocity profiles after injection with a ratio, MI = 0.7×10^{-3} at $\bar{x}$ = 0.65 , 0.7 and 1.0 . Figures 5,6 and 7 show also that velocity profiles of different positions of $\bar{x}$ in the neighbourhood of the predicted separation point for different values of MI (0.1×10^{-2}, 0.5×10^{-2} and 0.01). The effect of pressure gradient obtained due to the curvature of the surface on the velocity profile distribution without injection as shown in Fig. 3. One can see that, the local velocity decreases in the downstream direction due to friction and pressure gradient at the same distance from the wall,so at a certain section the flow starts to separate from the wall. Downstream of that predicted separation point, the assumption of equations of motion are not more valied, so,the solution stops at the separation point. To move the separation point downwards, a mass of air is injected. In figures 4,5,6 and 7 one can see easily that, the value of injection ratio influences very strong the velocity distribution in the wall region. One can divide the velocity profile after injection into three distinguish layers,namely, the inner layer,which starts from zero velocity at the wall to the first maximum velocity,then the intermediate layer which starts from the first maximum velocity to the minimum velocity in the profile and lastly, the outer layer or wake layer, starts from the minimum velocity to the outer edge velocity, where ($u_e = 0.99\ u_\infty$). The figures indicate clearly that , an increase of mass injection leads to an increase of maximum and minimum velocities,since the mass flow rate increases. So, the separation point shifted in downstream direction by injection.

EFFECTS OF MASS INJECTION RATIO (MI) ON TEMPERATURE PROFILES

Figure 8 shows the predicted temperature distribution for the computed flow over the wall till the predicted separation point for MI = 0.0. Figures 9, 10, 11 and 12 show the predicted distribution, for MI=0.7×10^{-3} 0.1×10^{-2}, 0.005 and 0.01 at positions of ($0.65 \leq \bar{x} \leq 1.0$). Here also, one see that, the local temperature increases in the downstream direction at a constant distant from the wall inside the boundary layer due to friction and turbulence effects. Figures 9, 10, 11 and 12 show that, the influence of the injection on the temperature profile is small compard with its effect on the velocity distribution where the Mach number is still small in that flow ($M_\infty = 0.35$). So, the main variation of temperature profiles due to friction only.

CONCLUSIONS

The mathematical model for calculation of the fluid flow with mass injection of air into air through porous surface, in case of turbulent flow for two dimensional compressible flow enabled to study the effect of mass injection ratio (MI) on the predicted separation point of boundary layer. The following conclusions are obtained from the present work.

1] The mass injection ratio (MI) has very strong influence on the velocity profiles in the wall region.

2] An increase of mass injection ratio leads to an increase of maximum and minimum velocities, So, the separation point shifted in the down-stream direction by injection.

3] The first maximum velocity decays along the injection region, in the same manner, distance from the wall where the maximum velocity exists is contineously departing the wall. The minimum velocity increases also along the injection region and its location is departing the wall.

4] The influence of the mass injection on the temperature profiles is small compared with its effect on the velocity distribution, because of the assumtion of wall adiabatic and small Mach number. So, the main variation of temperature profiles is due to friction only.

This paper presents the initial results for Dr. thesis about the influence of injection on the separated turbulent boundary layers.

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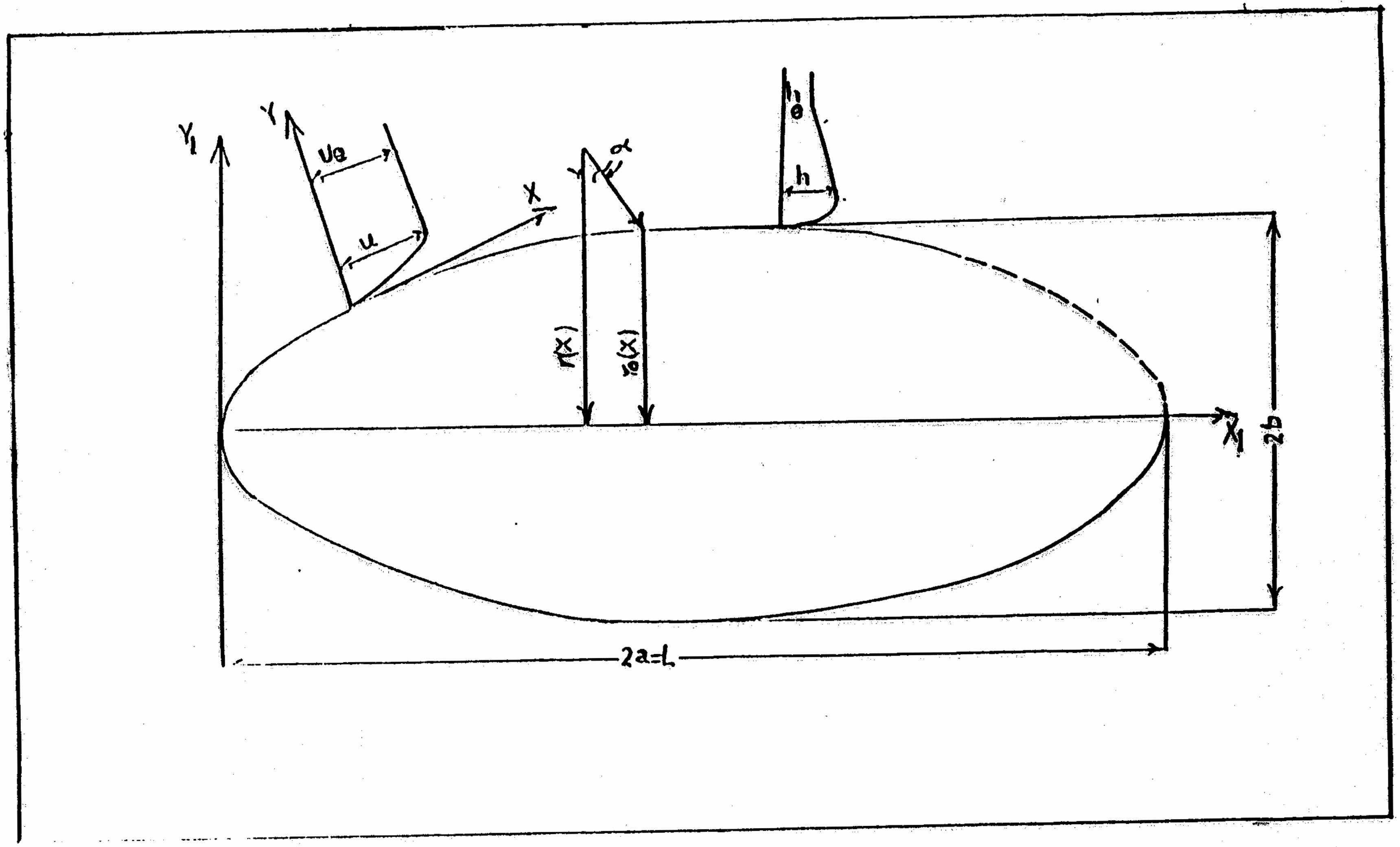

FIG.(1): CO-ORDINATE SYSTEM FOR THE BOUNDARY LAYER ON TWO DIMENSIONAL BODY.

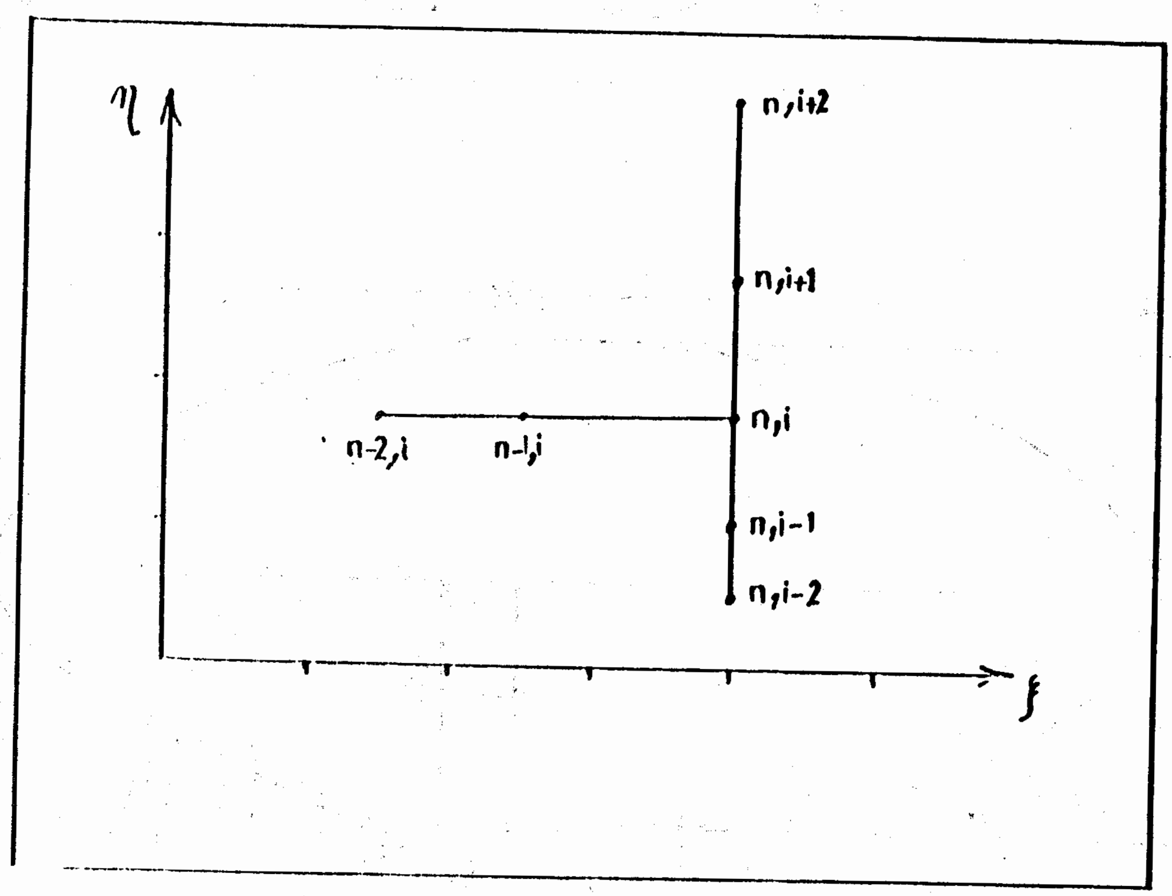

FIG.(2): A FINITE DIFFERENCE MODEL FOR SOLVING THE MOMENTUM AND ENERGY EQUATIONS.

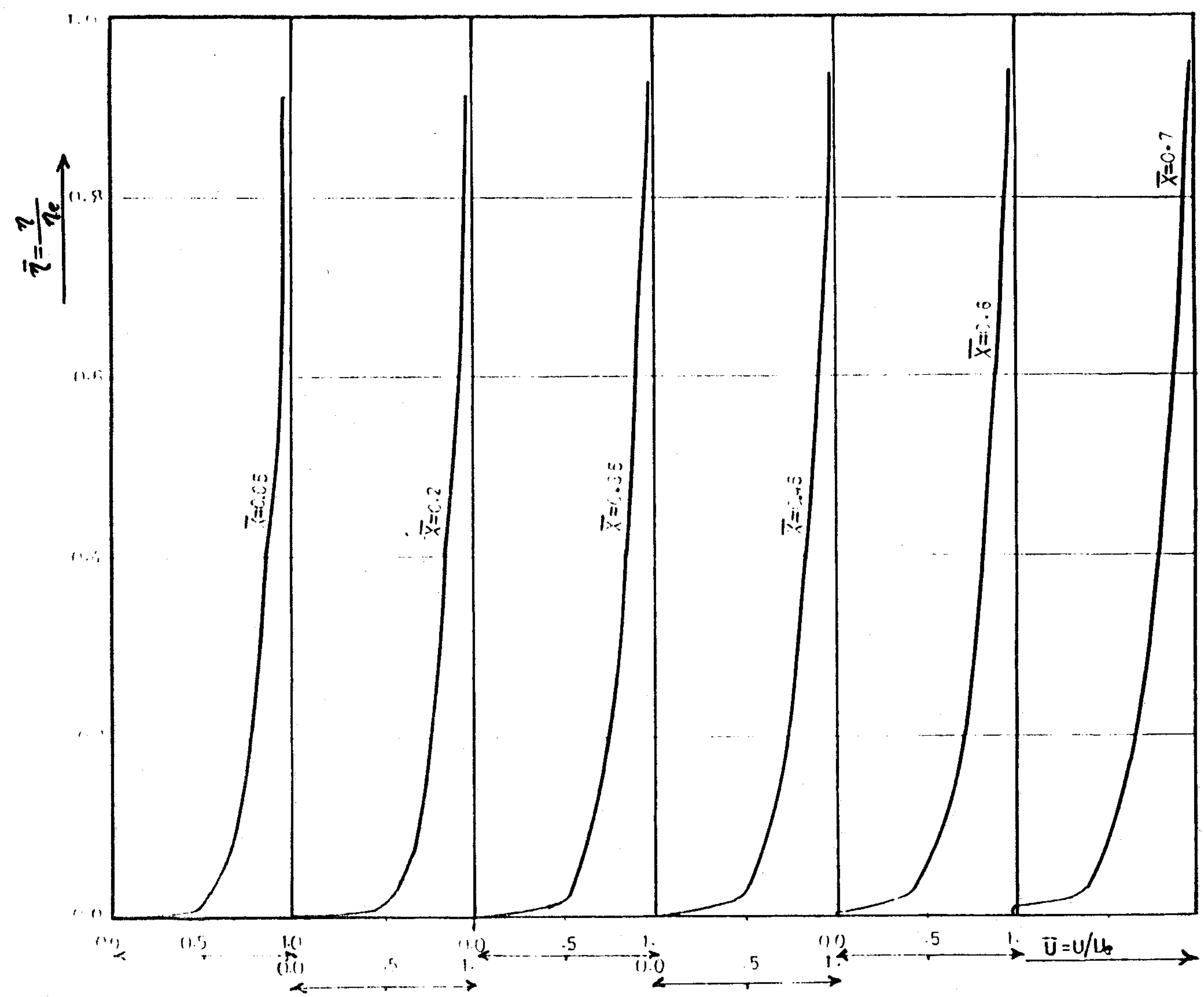

FIG.(3): DISTRIBUTION OF THE VELOCITY PROFILES ALONG THE SURFACE UNTIL THE SEPARATION POINT.
(RE $=8.0 \times 10^5$, $t_\infty = 50°C$, $M_\infty = 0.35$, MI = 0.0.)

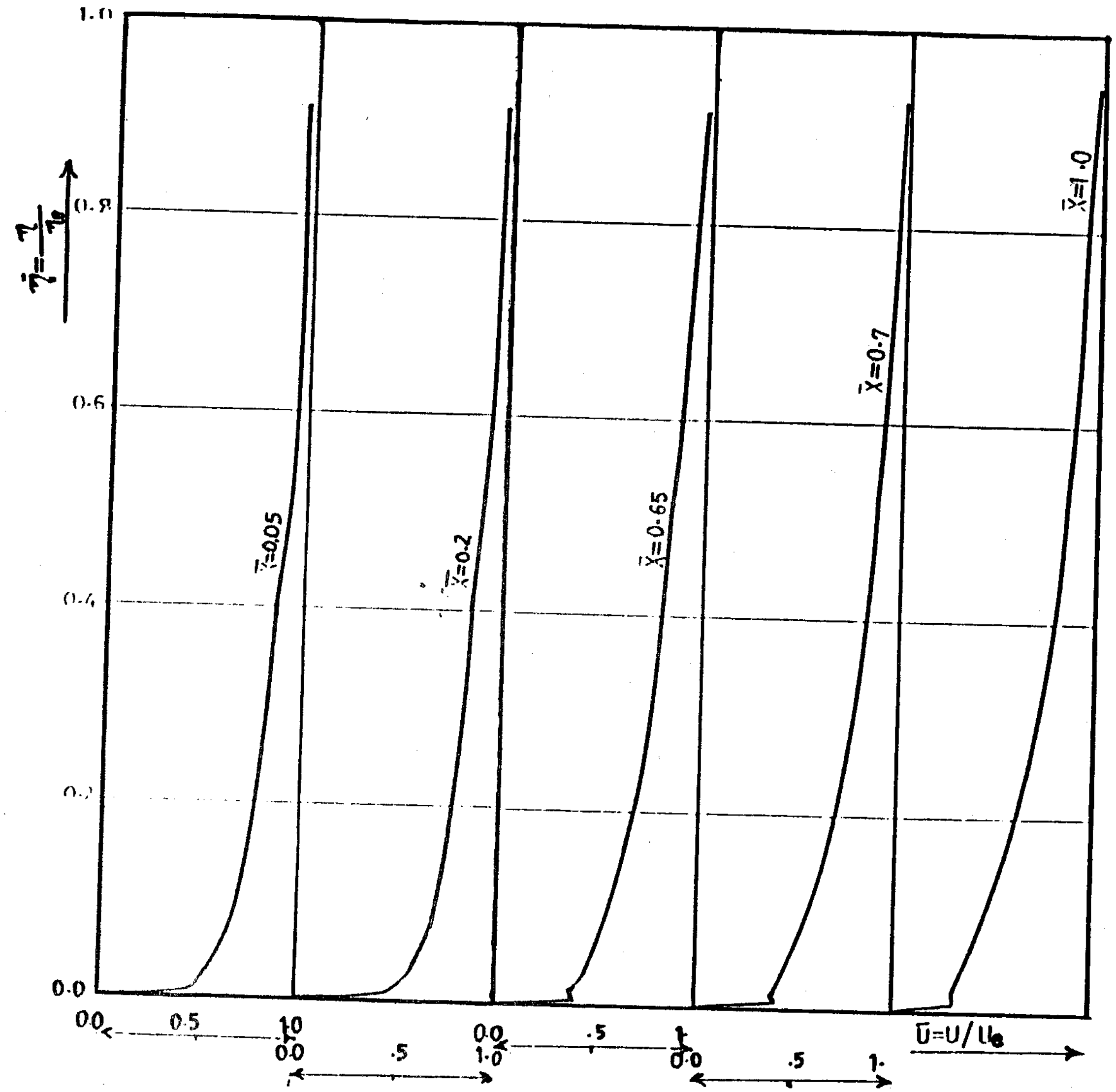

FIG.(4): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION .

(RE = 8.0 x 10^5 , t_∞ = 50°C , M_∞ = 0.35,MI=.7 x 10^{-3})

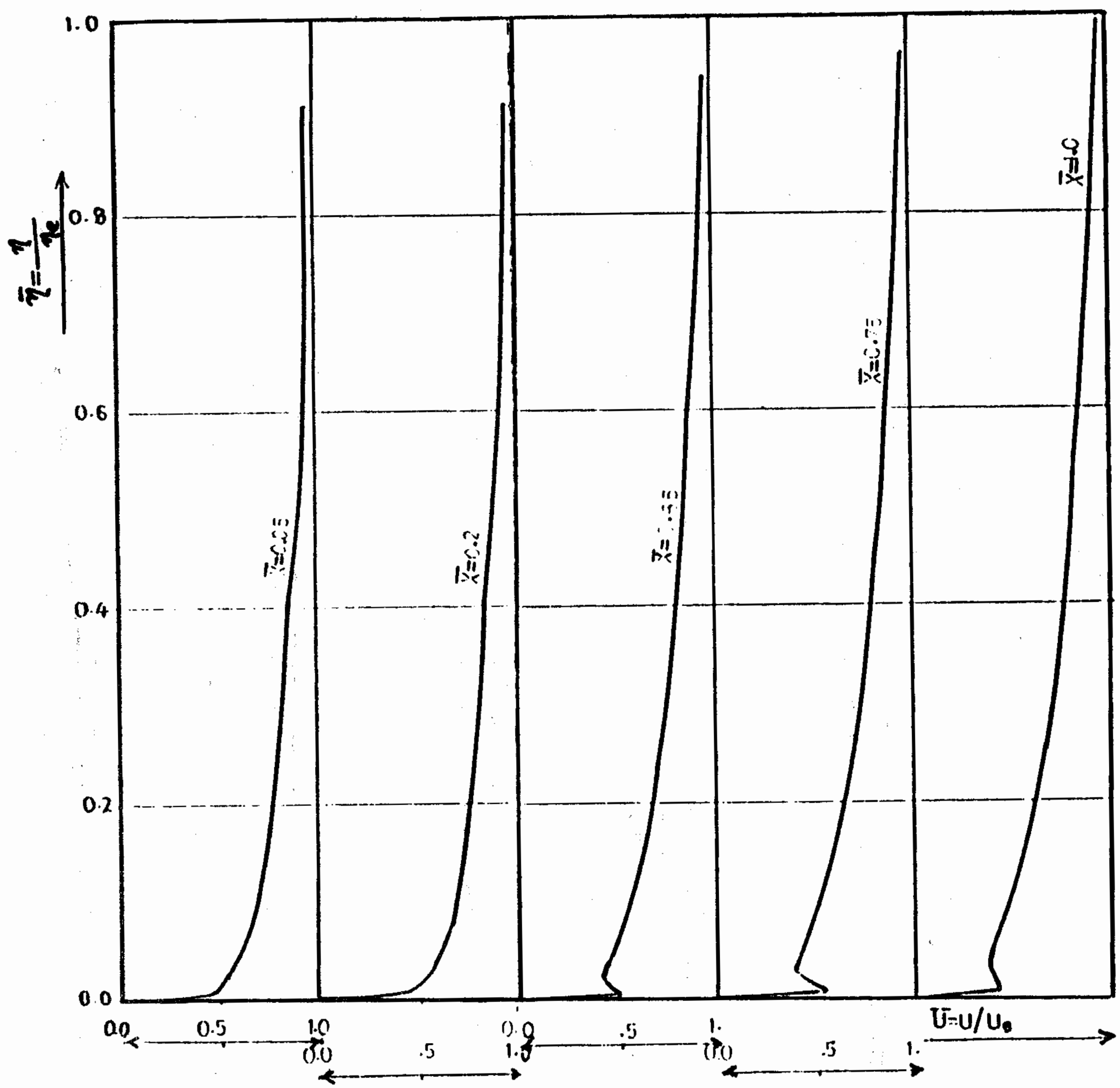

FIG.(5): DISTRIBUTION OF THE VELOCITY PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE = 8.0×10^5 , t_{oo} = 50°C , M_{oo} = 0.35 , MI = $.5 \times 10^{-2}$).

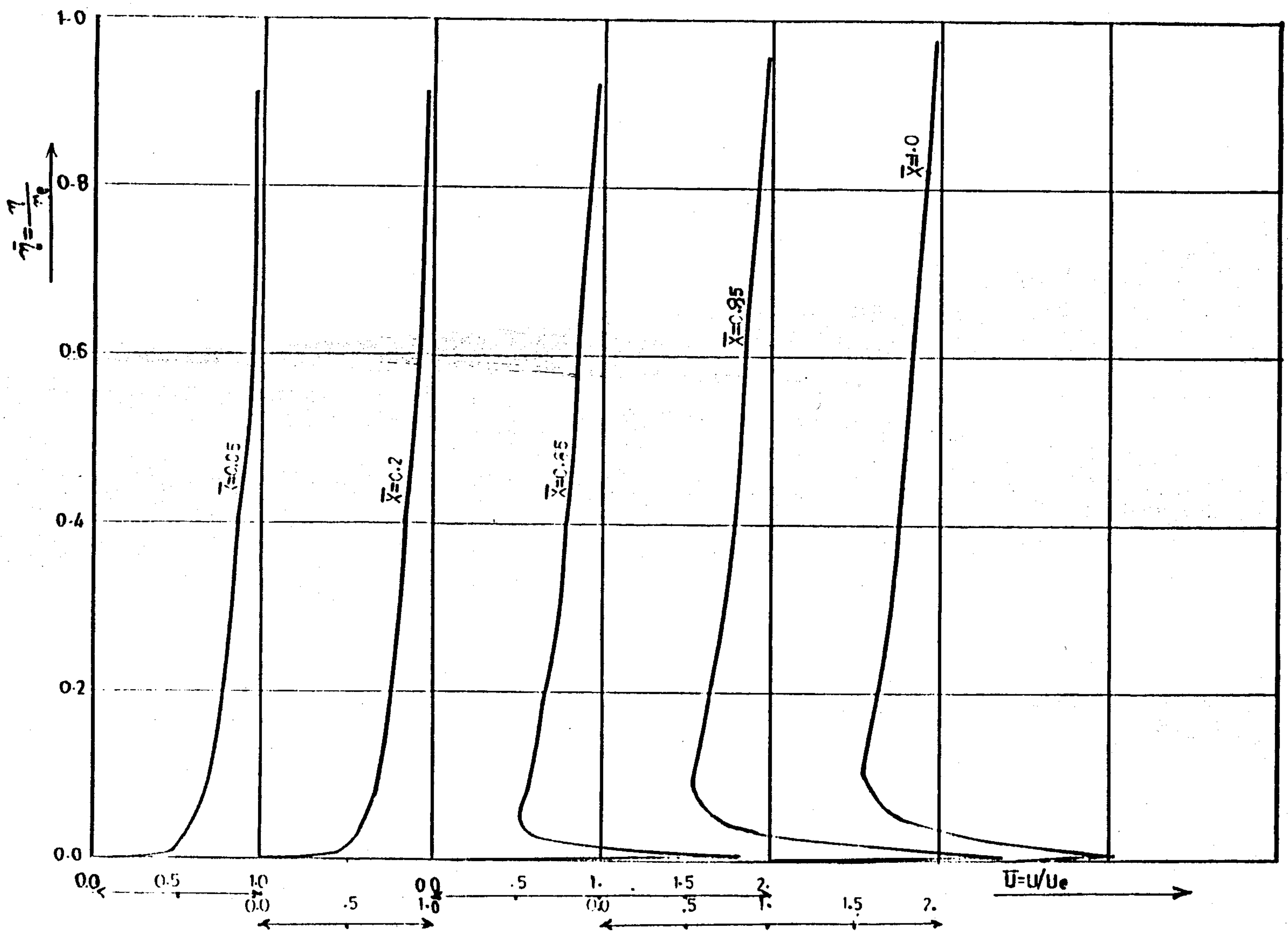

FIG.(6): DISTRIBUTION OF THE VELOCITY PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE = 8.0 x 10^5 , t_{∞} = 50°C , M_{∞} = 0.35 , MI = 0.001)

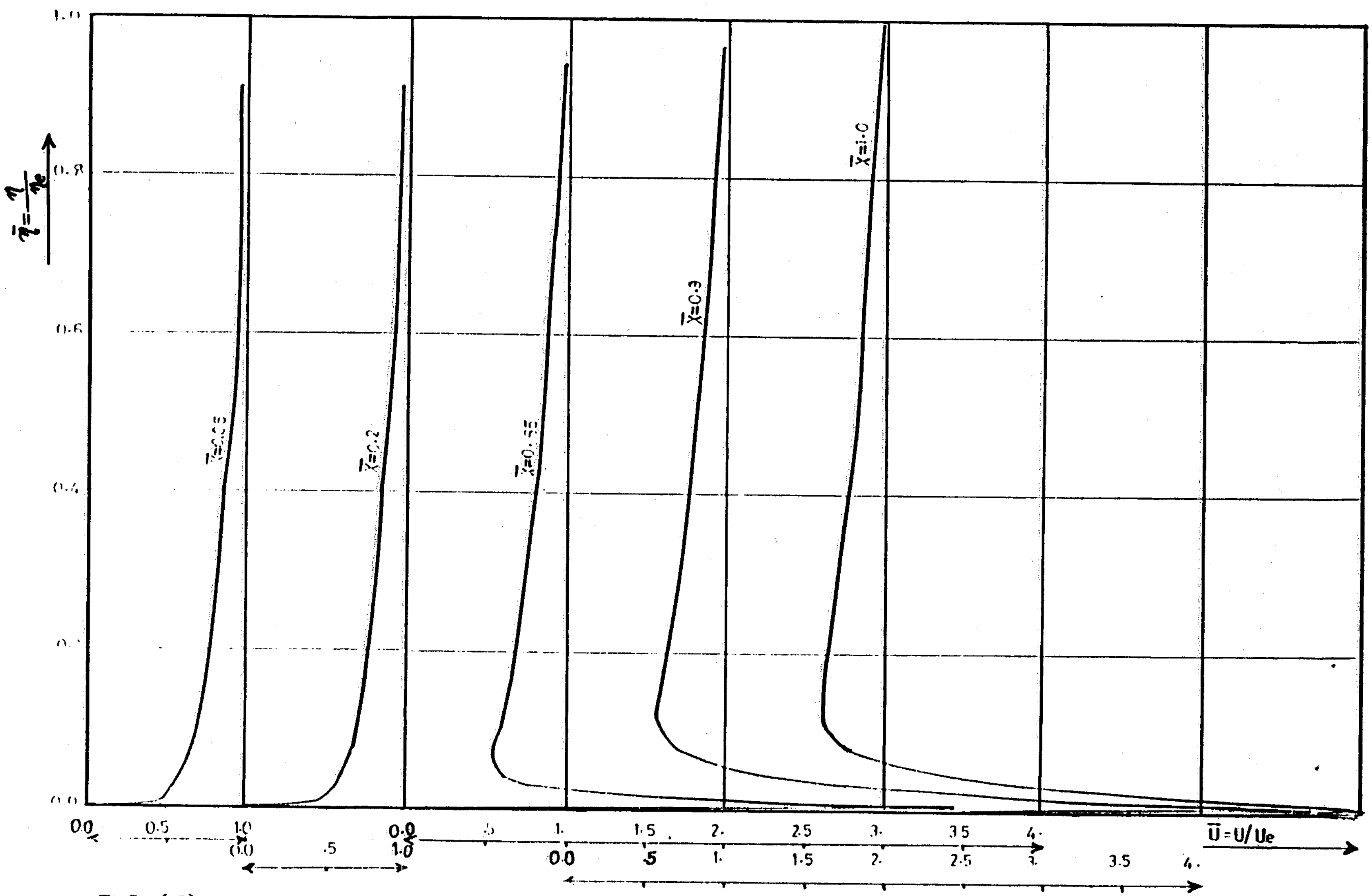

FIG.(7): DISTRIBUTION OF THE VELOCITY PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE = 8.0 x 10^5 , t_∞ = 50°C , M_∞ = 0.35 , MI = 0.01)

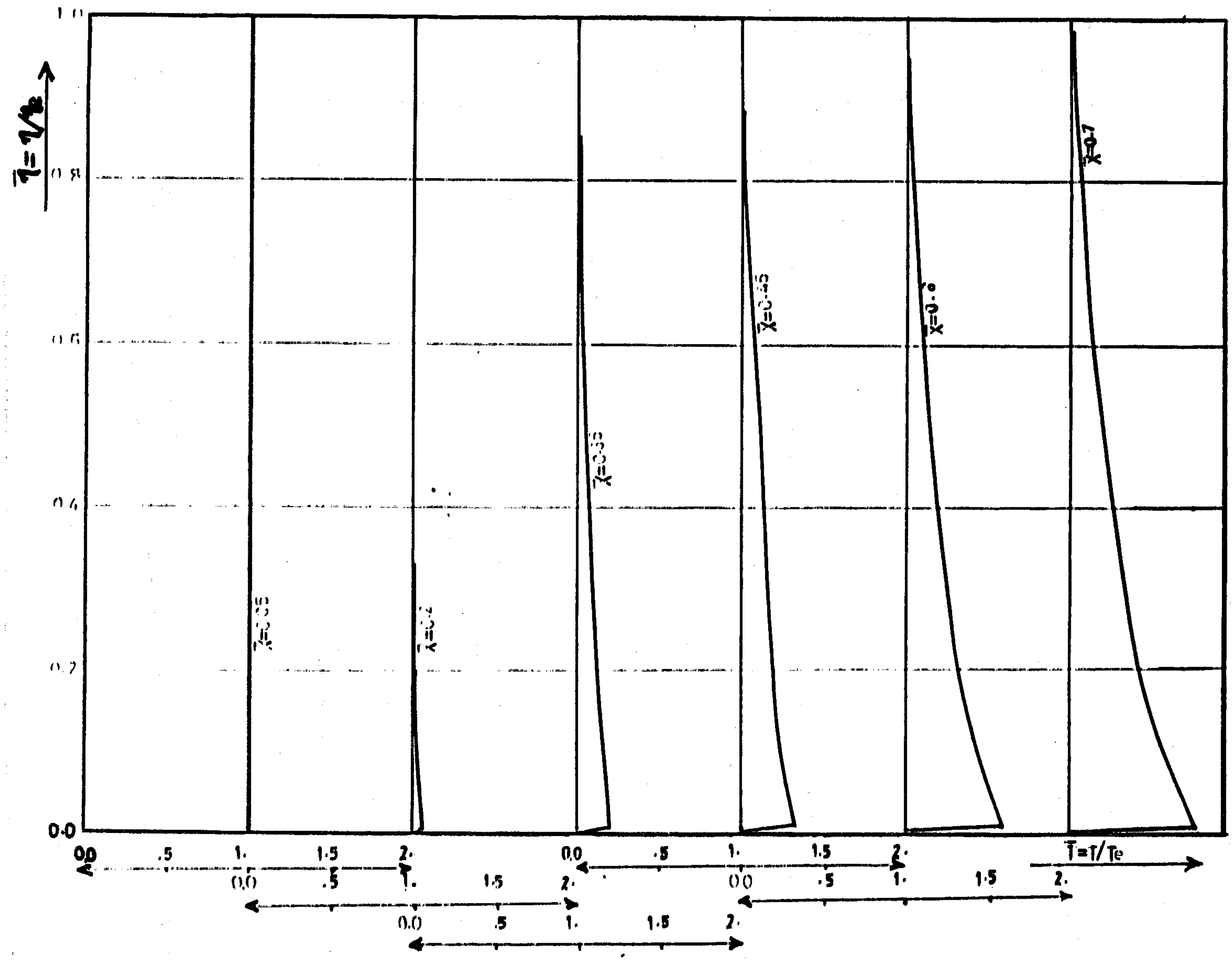

FIG.(8): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE UNTIL THE SEPARATION POINT.
(RE = 8.0 x 10^5, t_∞ = 50°C , M_∞ = 0.35, MI = 0.0).

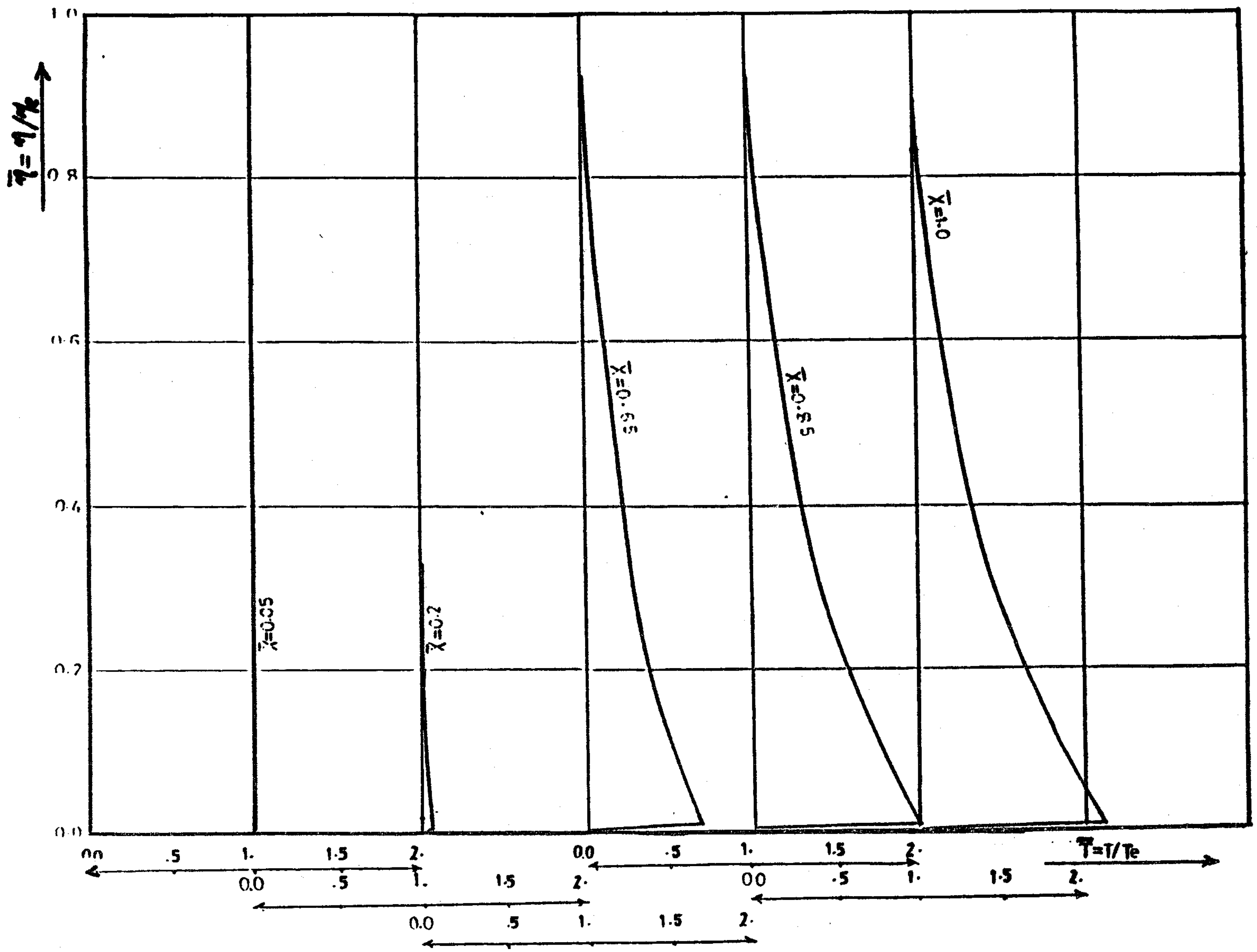

FIG.(9): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE =8.0 x 10^5 , t_∞ = 50°C , M_∞ = 0.35 , MI = .7 x 10^{-3}).

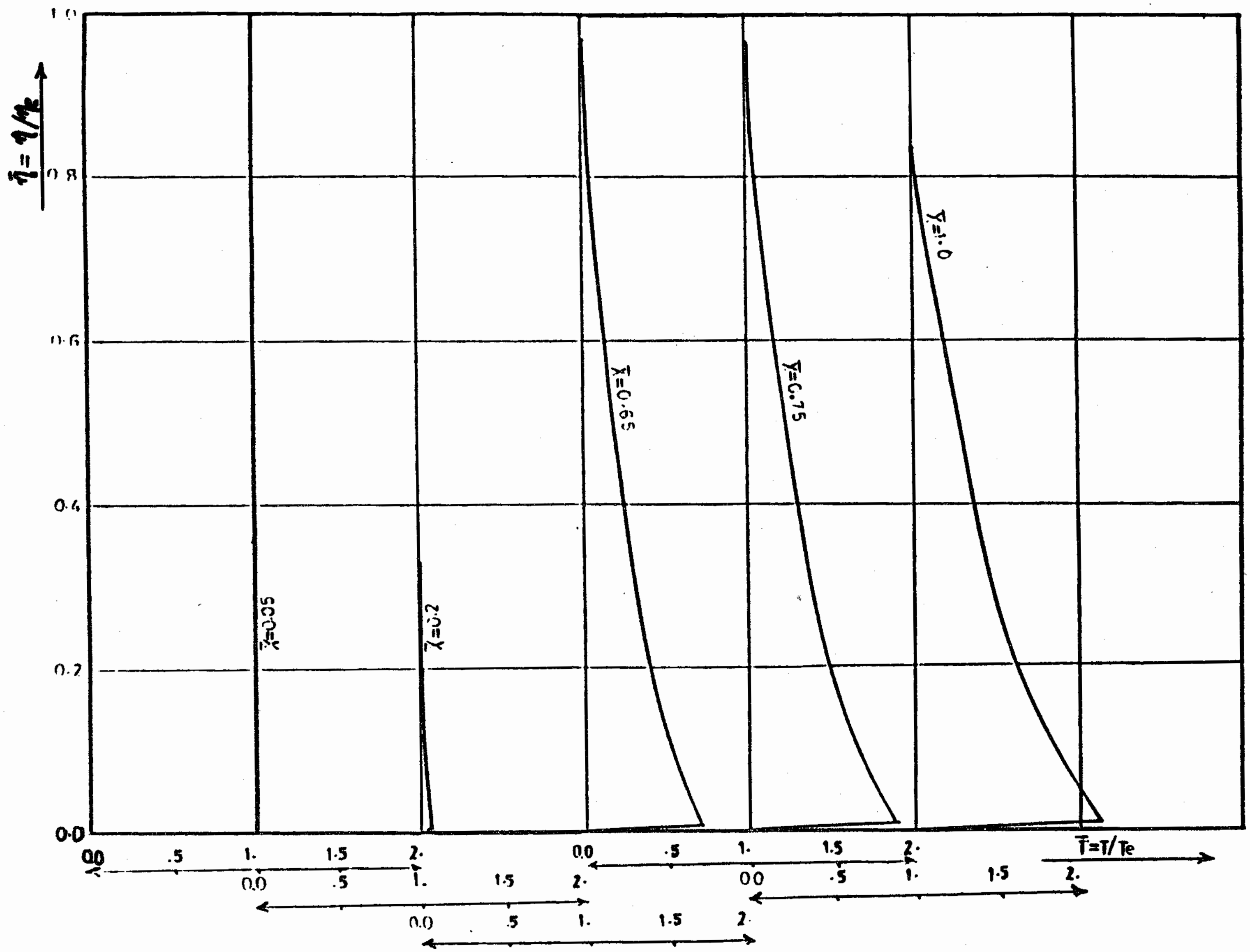

FIG.(10): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE = 8.0 x 10^5, t_∞ = 50°C , M_∞ = 0.35 , MI = .5 x 10^{-2}).

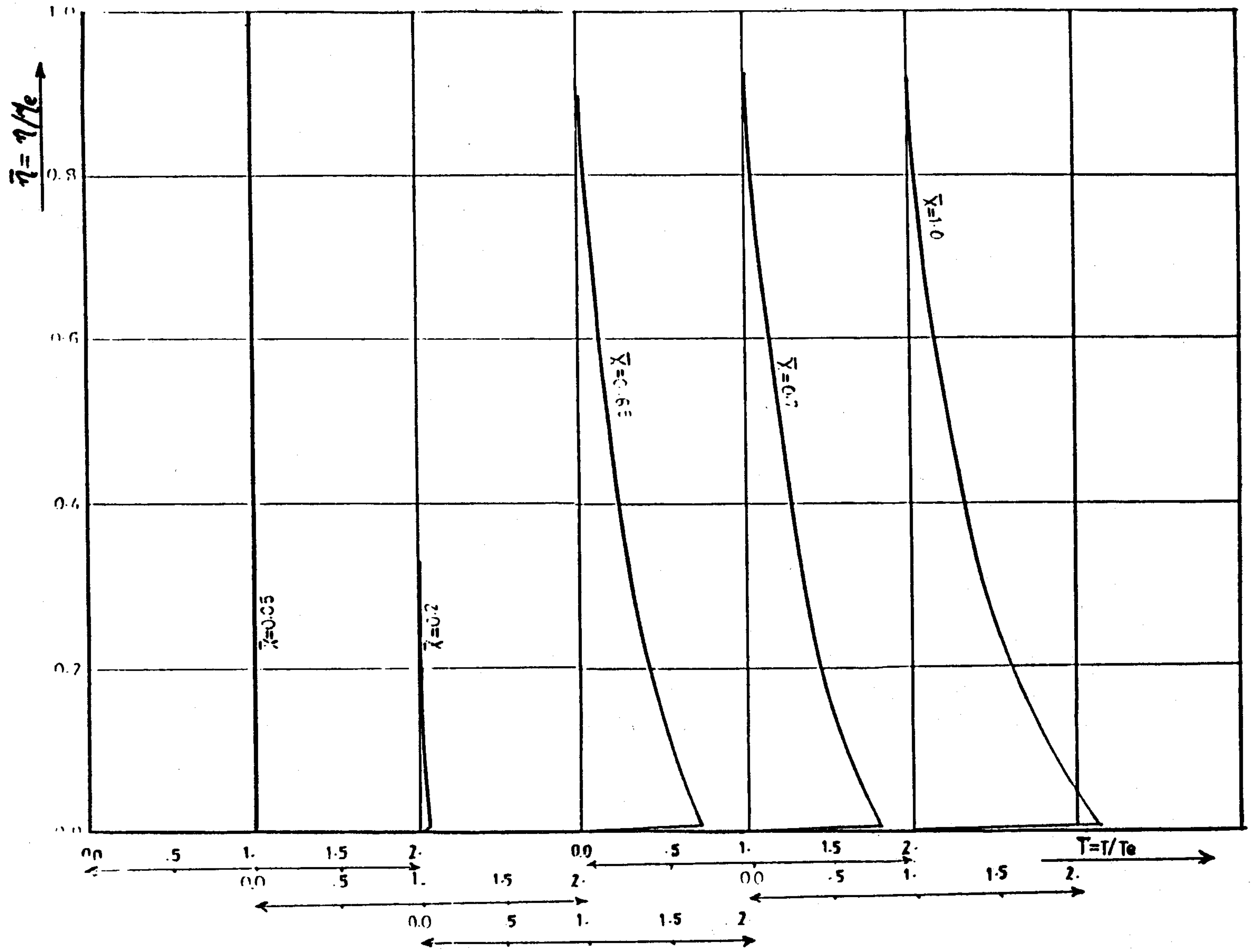

FIG.(11): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION .
(RE = 8.0 x 10^5 , t_∞ = 50°C , M_∞ = 0.35 , MI = 0.001).

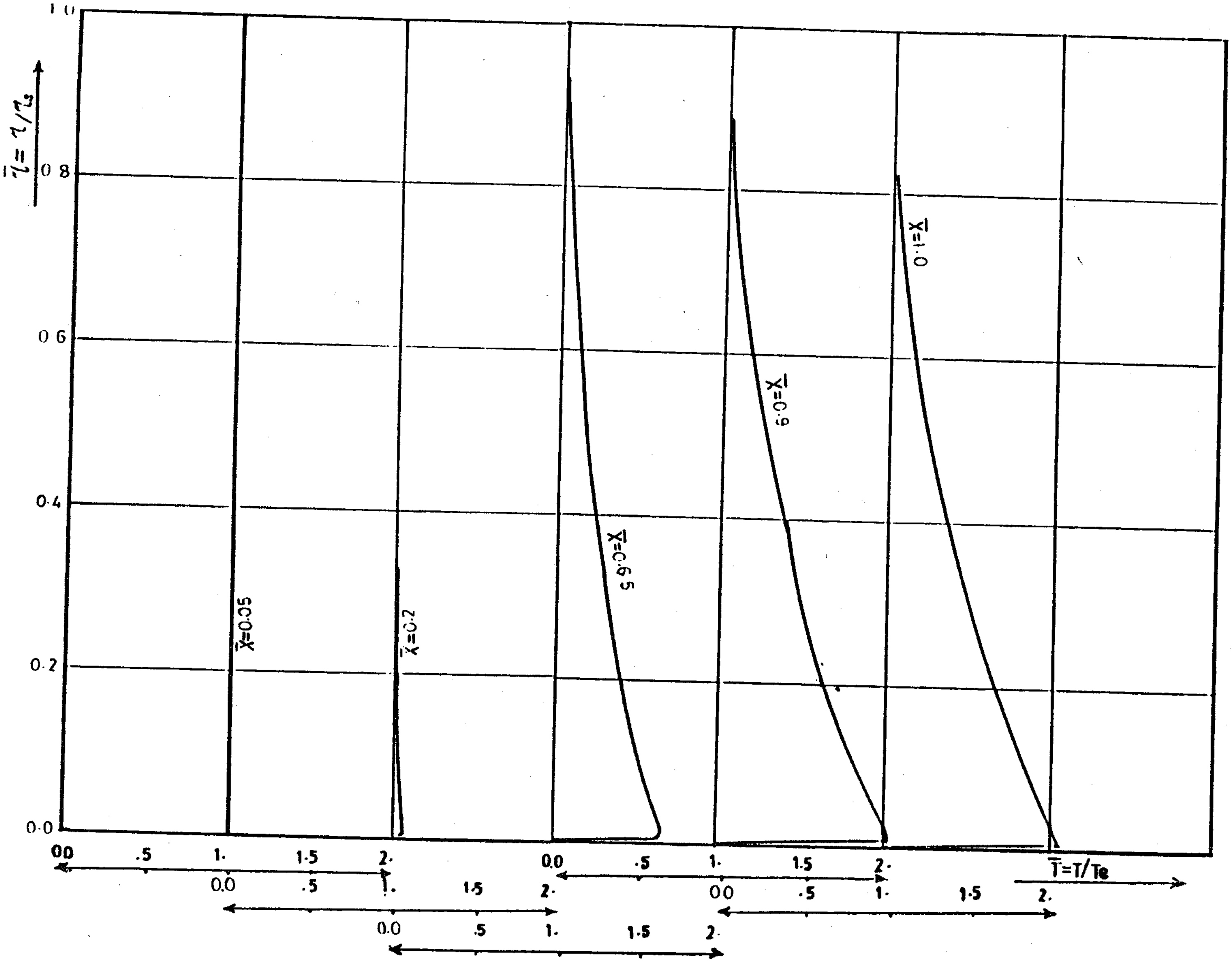

FIG.(12): DISTRIBUTION OF THE TEMPERATURE PROFILES ALONG THE SURFACE THROUGH THE INJECTION REGION.
(RE = 8.0 x 10^5 , t_∞ = 50°C , M_∞ = 0.35 , MI = 0.01).