A GOOD APPROXIMATE SOLUTION OF HEAT CONDUCTION EQUATIONS IN CYLINDRICAL COORDINATES COMPOSITE MEDIA.

حل تقريبي جيد لمعادلات إنتقال الحرارة في الأوساط المركبة و الإحداثيات الإسطوانية.

BY:

S. H. BEHIRY[1] , B. ABDEL-HAMID[2] and M. M. HELAL[3]

الخلاصة يتناول هذا البحث دراسة حل تحليلي تقريبي لمسألة إنتقال حرارة معتمدة على الزمن في وسطين إسطوانيين، كما يقدم هذا البحث إعادة تكوين المسألة من جديد حيث يتم الإستعاضة عن الوسط الخارجي (الإسطوانة الخارجية) بما يكافئه من شروط حدية عن طريق دراسة الإتزان الحراري و بالتالي تحولت المسألة الأصلية من إيجاد توزيع درجة الحرارة في طبقتين إلى إيجاد توزيع درجة الحرارة في طبقة واحدة فقط (الوسط المعني بالدراسة) ، و قد قورن هذا الحل الجديد بالحل التحليلي للمسألة الأصلية و أمكن إيجاد معيار لمدى صلاحية هذا الحل التقريبي الجديد

ABSTRACT:

An approximate analysis of transient heat diffusion problem in two composite cylindrical media is investigated. A new formulation of the problem is introduced in which the outer layer of the composite cylinders is lumped by assuming a uniform temperature distribution throughout. The temperature, however, is allowed to vary with time. This assumption reduces the two layers problem to a one region problem with a new set of boundary conditions which compensates the effect of the lumped outer layer. The temperature distribution of the approximate formulation is then compared to the exact distribution obtained analytically. A "breakdown" criterion of the approximation is obtained.

1- INTRODUCTION:

The transient temperature distribution in a composite medium consisting of several layers in contact has numerous applications in engineering [1-4]. Solving the problem of transient heat diffusion in two-layers composite is mathematically involved specially when discontinuities, which appear frequently at the interface, must be dealt with. Because of its difficulty, these problems are mostly treated numerically with some exceptions where analytic solutions were presented.

Heat conduction in cylindrical coordinates system is of interest in many applications such as rocket walls, oil reservoirs, boilers, metal forming processes, nuclear engineering as well as in the food processing industry. This problem has recently been treated in [5] numerically using finite difference and in [6] numerically by first using Laplace transform to remove the time variable followed by a discretization of the spatial variable using a control volume formulation. The problem is analysed for food products in [7]. An inverse heat conduction problem in cylindrical coordinate is presented in [8].

This paper presents a formulation which simplifies the mathematical analysis of the transient heat diffusion problem in composite cylinders by producing a single governing equation with a modified set of auxiliary conditions. In this formulation, the outer layer of a two composite cylinders is lumped by assuming a uniform temperature all over at any given instance. However, the

[1] Assoc. prof. of Math.,Eng.Math.&Phys.Dept.,Faculty of Eng.,Mansoura Univ.

[2] Assoc.Prof.of Math.,Math.&Comp.Sc.Dept.,Faculty of Sc. United Arab Emirates Univ.

[3] Engineer, Eng. Math. &Phys.Dept.,Faculty of Eng., Zagazig Univ.

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temperature is allowed to vary with time. This assumption reduces the two-layer problem to one-layer problem with a new set of boundary conditions which compensates the effect of the outer layer. A limiting criterion, where breakdown of the approximation occurs, is found by comparing the solution of the proposed formulation to the exact solution for different sets of the dimensionless parameters involved.

2- STATEMENT OF THE PROBLEM:

A two-layer solid cylinder contains an inner region $0 \le r \le R_{in}$ and an outer region $R_{in} \le r \le R_{ou}$ which are in perfect thermal contact. k_1 and k_2 are the thermal conductivities, α_1 and α_2 are the thermal diffusivities of the inner and outer region, respectively. Initially, the inner and outer regions are at a uniform temperature T_0. For times $t \succ 0$, heat is dissipated by convection from the outer surface at $r = R_{ou}$ into an environment at a constant temperature T_∞ with a heat transfer coefficient h_∞. Finally constant thermophysical properties of the two-layers composite cylinder are assumed.

GOVERNING EQUATIONS:

The governing equation in the i-th layer of the composite may be written as [9] :

$$\alpha_i\left(\frac{\partial^2 T_i(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T_i(r,t)}{\partial r}\right)=\frac{\partial T_i(r,t)}{\partial t}, \qquad r_{i-1}\le r\le r_i \ , \ i=1,2 ,$$

$$r_0=0,\ r_1=R_{in},\ r_2=R_{ou},\ t\succ 0 \qquad (1)$$

subject to the boundary conditions:

$$T_1(0,t)=\text{finite}, \qquad t\succ 0 \qquad (2\text{-a})$$

$$T_1(R_{in},t)=T_2(R_{in},t), \qquad t\succ 0 \qquad (2\text{-b})$$

$$k_1\frac{\partial T_1(R_{in},t)}{\partial r}=k_2\frac{\partial T_2(R_{in},t)}{\partial r}, \qquad t\succ 0 \qquad (2\text{-c})$$

$$k_2\frac{\partial T_2(R_{ou},t)}{\partial r}+h_\infty(T_2(R_{ou},t)-T_\infty)=0, \qquad t\succ 0 \qquad (2\text{-d})$$

and the initial condition:

$$T_i(r,0)=T_0, \qquad i=1,2,\ 0\le r\le R_{ou} \qquad (3)$$

where,

$$\alpha_i=\frac{k_i}{\rho_i C_i}, \qquad i=1,2$$

is the thermal diffusivity of the i-th layer, with ρ_i and C_i being the density and the specific heat of the i-th layer, respectivily.

Defining the following dimensionless quantities:

$$\eta_i=\frac{r_i}{R_{in}}, \qquad \tau=\frac{\alpha_1 t}{R_{in}^2}, \qquad \theta_i(\eta,\tau)=\frac{T_i(r,t)-T_\infty}{T_0-T_\infty},$$

$$\xi=\frac{R_{ou}-R_{in}}{R_{in}}, \qquad K_i=\frac{k_i}{k_1}, \qquad \gamma_i=\frac{\alpha_i}{\alpha_1}, \qquad H_2=\frac{h_\infty R_{in}}{k_2}. \qquad (4)$$

The governing equations (1), (2) and (3) may be written in dimensionless form as:

$$\gamma_i\left(\frac{\partial^2\theta_i(\eta,\tau)}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\theta_i(\eta,\tau)}{\partial\eta}\right)=\frac{\partial\theta_i(\eta,\tau)}{\partial\tau},\quad \eta_{i-1}\le\eta\le\eta_i,\quad i=1,2,$$

$$\eta_0=0,\ \ \eta_1=1,\ \ \eta_2=1+\xi,\ \ \tau\succ 0 \qquad (5)$$

subject to:

$$\theta_1(0,\tau)=\text{finite},\qquad \tau\succ 0 \qquad (6\text{-a})$$

$$\theta_1(1,\tau)=\theta_2(1,\tau)\qquad \tau\succ 0 \qquad (6\text{-b})$$

$$\frac{\partial\theta_1(1,\tau)}{\partial\eta}=K_2\frac{\partial\theta_2(1,\tau)}{\partial\eta},\qquad \tau\succ 0 \qquad (6\text{-c})$$

$$\frac{\partial\theta_2(1+\xi,\tau)}{\partial\eta}+H_2\theta_2(1+\xi,\tau)=0,\qquad \tau\succ 0 \qquad (6\text{-d})$$

and the initial condition:

$$\theta_i(\eta,0)=1,\qquad \eta_{i-1}\le\eta\le\eta_i\ ,\ \ i=1,2 \qquad (7)$$

To find the temperature distribution in any layer, one must solve the system of partial differential equation coupled at the interface described above.

3- EXACT SOLUTION:

Although lengthy and mathematically involved, the analytic solution of the problem described above is needed for validating the proposed simplified analysis. We follow the procedures of [9] to obtain the analytic solution by employing the finite integral transform technique. The integral transform pair needed for the solution may be developed by considering representation of an arbitrary function in terms of eigenfunctions corresponding to the eigenvalue problem associated with the system described by equations (5) and (6). Such eigenfunctions, $\psi_{i,n}(\eta)$ are found to be:

$$\psi_{1,n}(\eta)=A_{1,n}J_0\left(\frac{\lambda_n}{\sqrt{\gamma_1}}\eta\right),\qquad 0\le\eta\le 1,\quad n=1,2,3,\ldots \qquad (8\text{-a})$$

$$\psi_{2,n}(\eta)=A_{2,n}J_0\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\eta\right)+B_{2,n}Y_0\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\eta\right),\quad 1\le\eta\le 1+\xi,\quad n=1,2,3,\ldots \qquad (8\text{-b})$$

where J_0 and Y_0 are Bessel functions of order zero of the first and second kind, respectively.

The first step in the analysis is the determination of the three coefficients $A_{1,n}$, $A_{2,n}$ and $B_{2,n}$. Without loss of generality, we set one of the coefficients, say $A_{1,n}$, equals to unity. The eigenfunctions given by equations (8), with $A_{1,n}=1$, are then introduced into the conditions complementing the eigenvalue problem associated with the system given by (5) and (6). The resulting system of equations, expressed in matrix form, is:

$$\begin{bmatrix} J_0\left(\frac{\lambda_n}{\sqrt{\gamma_1}}\right) & -J_0\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\right) & -Y_0\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\right) \\ \sqrt{\frac{\gamma_2}{\gamma_1}}\frac{1}{k_2}J_1\left(\frac{\lambda_n}{\sqrt{\gamma_1}}\right) & -J_1\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\right) & -Y_1\left(\frac{\lambda_n}{\sqrt{\gamma_2}}\right) \\ 0 & [H_2J_0\left(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}}\right)-\frac{\lambda_n}{\sqrt{\gamma_2}}J_1\left(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}}\right)] & [H_2Y_0\left(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}}\right)-\frac{\lambda_n}{\sqrt{\gamma_2}}Y_1\left(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}}\right)] \end{bmatrix}\begin{bmatrix}1\\ A_{2,n}\\ B_{2,n}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad (9)$$

where J_1 and Y_1 being Bessel functions of order one of the first and second kind, respectively. Any two of these equations (9) can be used to determine the coefficients $A_{2,n}$ and $B_{2,n}$. Choosing the first two equations, one gets:

$$A_{2,n}=\frac{1}{\Delta}\left[J_0(\frac{\lambda_n}{\sqrt{\gamma_1}})Y_1(\frac{\lambda_n}{\sqrt{\gamma_2}})-\sqrt{\frac{\gamma_2}{\gamma_1}}\frac{1}{k_2}Y_0(\frac{\lambda_n}{\sqrt{\gamma_2}})J_1(\frac{\lambda_n}{\sqrt{\gamma_1}})\right], \tag{10-a}$$

$$B_{2,n}=\frac{1}{\Delta}\left[\sqrt{\frac{\gamma_2}{\gamma_1}}\frac{1}{k_2}J_0(\frac{\lambda_n}{\sqrt{\gamma_2}})J_1(\frac{\lambda_n}{\sqrt{\gamma_1}})-J_0(\frac{\lambda_n}{\sqrt{\gamma_1}})J_1(\frac{\lambda_n}{\sqrt{\gamma_2}})\right], \tag{10-b}$$

where, $\Delta=-\frac{2\sqrt{\gamma_2}}{\pi\lambda_n}$

The transcendental equation of the eigenvalues λ_n , $n=1,2,3,\ldots$ is obtained from the requirement that the determinant of the coefficient matrix in equation (9) should vanish. The eigenfunctions given by equations (8) satisfy the following orthogonality relation:

$$\sum_{i=1}^{2}\frac{K_i}{\gamma_i}\int_{D_i}\eta\psi_{i,m}(\eta)\psi_{i,n}(\eta)d\eta=\begin{cases}0 , & m\neq n\\ N(\lambda_n) , & m=n\end{cases} \tag{11}$$

where D_1 from 0 to 1, and D_2 from 1 to $1+\xi$. The normalization integral $N(\lambda_n)$ is given by:

$$N(\lambda_n)=\frac{k_1}{\gamma_1}M_1(\lambda_n,1)+\frac{k_2}{\gamma_2}A_{2,n}^2[M_2(\lambda_n,1+\xi)-M_2(\lambda_n,1)]+$$
$$2\frac{k_2}{\gamma_2}A_{2,n}B_{2,n}[M_3(\lambda_n,1+\xi)-M_3(\lambda_n,1)]+\frac{k_2}{\gamma_2}B_{2,n}^2[M_4(\lambda_n,1+\xi)-M_4(\lambda_n,1)],$$
$$n=1,2,3,\ldots \tag{12}$$

with,

$$M_1(\lambda_n,\eta)=\frac{\eta^2}{2}\left[J_0^2(\frac{\lambda_n\eta}{\sqrt{\gamma_1}})+J_1^2(\frac{\lambda_n\eta}{\sqrt{\gamma_1}})\right],$$

$$M_2(\lambda_n,\eta)=\frac{\eta^2}{2}\left[J_0^2(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})+J_1^2(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})\right],$$

$$M_3(\lambda_n,\eta)=\frac{\eta^2}{2}\left[J_0(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})Y_0(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})+J_1(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})Y_1(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})\right],$$

$$M_4(\lambda_n,\eta)=\frac{\eta^2}{2}\left[Y_0^2(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})+Y_1^2(\frac{\lambda_n\eta}{\sqrt{\gamma_2}})\right], \quad \text{and}$$

$A_{1,n}$, and $B_{2,n}$ are given by (10).

Having obtained expressions for the eigenfunctions, the eigenvalues and the normalization integral, we now proceed to find the general solution by following the usual procedures of the finite integral transform technique. We begin by defining the transform pair as follows [9]:

Integral Transform

$$\Phi_n(\tau)=\sum_{i=1}^{2}\frac{k_i}{\gamma_i}\int_{D_i}\eta\psi_{i,n}(\eta)\theta_i(\eta,\tau)d\eta , \quad n=1,2,3,\ldots \tag{13-a}$$

Inversion Formula

$$\theta_i(\eta,\tau)=\sum_{n=1}^{\infty}\frac{\psi_{i,n}(\eta)}{N(\lambda_n)}\Phi_n(\tau) , \quad i=1,2. \tag{13-b}$$

Using integral transform above, the system of governing equations given by equations (5) subject to the boundary conditions given in equations (6) is reduced to a system of first order differential equations in the dimensionless transform variable, namely;

$$\frac{d\Phi_n(\tau)}{d\tau} + \lambda_n^2 \Phi_n(\tau) = 0 \ , \qquad n = 1,2,3,\ldots \tag{14}$$

subject to the following transformed dimensionless initial conditions:

$$\Phi_n(0) = \frac{k_1}{\sqrt{\gamma_1}\lambda_n} J_1(\frac{\lambda_n}{\sqrt{\gamma_1}}) + \frac{k_2(1+\xi)}{\sqrt{\gamma_2}\lambda_n}\left[A_{2,n}J_1(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}}) + B_{2,n}Y_1(\frac{\lambda_n(1+\xi)}{\sqrt{\gamma_2}})\right] - \frac{k_2}{\sqrt{\gamma_2}\lambda_n}\left[A_{2,n}J_1(\frac{\lambda_n}{\sqrt{\gamma_2}}) + B_{2,n}Y_1(\frac{\lambda_n}{\sqrt{\gamma_2}})\right], \qquad n = 1,2,3,\ldots \tag{15}$$

The solution of the system given by equations (14) and initial conditions (15) can be written as:

$$\Phi_n(\tau) = \Phi_n(0)\, e^{-\lambda_n^2 \tau} \quad , \qquad n = 1,2,3,\ldots$$

Utilizing this solution into the inversion formula (13-b), one obtains the unsteady dimensionless temperature in each region as:

$$\theta_i(\eta,\tau) = \sum_{n=1}^{\infty} \frac{\psi_{i,n}(\eta)}{N(\lambda_n)} \Phi_n(0)\, e^{-\lambda_n^2 \tau} \quad , \qquad i = 1,2. \tag{16}$$

where $\psi_{i,n}(\eta)$ and $N(\lambda_n)$ are given by equations (8) and (12), respectively.
This expression will be used in the validation of the alternative formulation presented next.

4- THE ALTERNATIVE FORMULATIOM:

The alternative formulation suggests a way for finding the temperature distribution in the region of interest directing without having to solve for the temperature in all regions. In this alternative formulation the layer of interest is considered and other layers are lumped by assuming uniform temperature at any instance but allowing the temperature to vary with time. The mathematical problem is thus reduced to a single partial differential equation subject to some modified boundary conditions. We demonstrate this method on the two-layer problem presented above. The outer layer (outer cylinder) is lumped by assuming a uniform temperature which is allowed to vary with time. A boundary condition which compensates the effect of the outer layer is derived by assuming the quantity of heat due to the average temperature $T_2^*(t)$ is equal to the quantity of heat into the outer layer at any given instance or, briefly;

$$\int_{R_{in}}^{R_{ou}} r T_2(r,t)\, dr = \frac{1}{2}(R_{ou}^2 - R_{in}^2) T_2^*(t)$$

By this assumption the governing field equation of the lumped layer problem can be easily derived.

Governing Equation:

A suitable form for the governing heat conduction equation of the lumped layer problem may be written as:

$$\frac{\partial^2 T_1(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r,t)}{\partial r} = \frac{1}{\alpha_1}\frac{\partial T_1(r,t)}{\partial t} \ , \qquad 0 \le r \le R_{in} \ , \ t \succ 0 \tag{17}$$

subject to the modified boundary condition:

$$R_{in}k_1 \frac{\partial T_1(R_{in},t)}{\partial r} + R_{ou}h_\infty[T_1(R_{in},t) - T_\infty] = -\frac{\rho_2 C_2}{2}(R_{ou}^2 - R_{in}^2)\frac{\partial T_1(R_{in},t)}{\partial t}, \quad t \succ 0 \quad (18)$$

and the initial condition:

$$T_1(r,0) = T_0, \qquad 0 \le r \le R_{in} \quad (19)$$

The above system can be transformed into the dimensionless form

$$\frac{\partial^2\theta_1(\eta,\tau)}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\theta_1(\eta,\tau)}{\partial\eta} = \frac{\partial\theta_1(\eta,\tau)}{\partial\tau}, \qquad 0 \le \eta \le 1,\ \tau \succ 0 \quad (20)$$

subject to the modified dimensionless boundary condition:

$$\frac{\partial\theta_1(1,\tau)}{\partial\eta} + H_1\theta_1(1,\tau) = -\frac{1}{2}\Gamma\frac{\partial\theta_1(1,\tau)}{\partial\tau}, \qquad \tau \succ 0 \quad (21)$$

and the dimensionless initial condition:

$$\theta_1(\eta,0) = 1, \qquad 0 \le \eta \le 1 \quad (22)$$

Where, besides the dimensionless quantities in (4), we defined the following dimensionless quantities:

$$\theta_1(\eta,\tau) = \frac{T_1(r,t) - T_\infty}{T_0 - T_\infty}, \quad H_1 = \frac{h_\infty R_{ou}}{k_1}, \quad \Gamma = \frac{\rho_2 C_2}{\rho_1 C_1}\frac{(R_{ou}^2 - R_{in}^2)}{R_{in}^2}.$$

Solution:

Solution of the problem described by equations (20-22) is obtained using the finite integral transform technique. The eigenfunction corresponding to the n-th eigenvalue λ_n is found to be:

$$\psi_{1,n}(\eta) = J_0(\lambda_n\eta) \quad (23)$$

The eigenvalues are the roots of the transcendental equation:

$$\lambda_n J_1(\lambda_n) - (H_1 - \frac{1}{2}\Gamma\lambda_n^2)J_0(\lambda_n) = 0 \quad (24)$$

The orthogonality property with respect to a weight function $w(\eta)$ may be established as:

$$\int_0^1 w(\eta)\psi_{1,n}(\eta)\psi_{1,m}(\eta)d\eta = \begin{cases} 0, & m \ne n \\ N(\lambda_n), & m = n \end{cases} \quad (25)$$

where the weight function $w(\eta)$ is found to be [10]:

$$w(\eta) = [1 + \frac{1}{2}\Gamma\delta(\eta - 1)]\eta \quad (26)$$

where $\delta(\eta - 1)$ is the Dirac delta-function.
The normalization integral is evaluated to be:

$$N(\lambda_n) = \frac{1}{2}(1+\Gamma)J_0^2(\lambda_n) + \frac{1}{2}J_1^2(\lambda_n) \quad (27)$$

The appropriate transform pair can now be defined as:

Integral Transform:

$$\Phi_n(\tau) = \int_0^1 (1 + \frac{1}{2}\Gamma\delta(\eta - 1))\eta\psi_{1,n}(\eta)\theta_1(\eta,\tau)d\eta \quad (28.a)$$

Inversion Formula:

$$\theta_1(\eta,\tau) = \sum_{n=1}^{\infty} \frac{\psi_{1,n}(\eta)}{N(\lambda_n)} \Phi_n(\tau) \tag{28.b}$$

The usual procedure of the finite integral transform technique leads to the following dimensionless temperature distribution throughout the inner cylinder:

$$\theta_1(\eta,\tau) = \sum_{n=1}^{\infty} \frac{J_0(\lambda_n\eta)}{N(\lambda_n)} \left[\frac{1}{\lambda_n} J_1(\lambda_n) + \frac{1}{2}\Gamma J_0(\lambda_n) \right] e^{-\lambda_n^2 \tau} \tag{29}$$

where $N(\lambda_n)$ is given by (27).

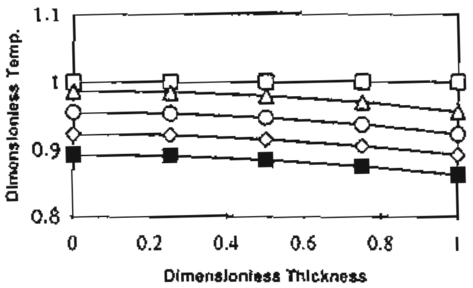
5- COMPARISON BETWEEN EXACT AND APPROXIMATE SOLUTIONS:

The exact and approximate solutions can now be compared in order to establish a "breakdown" criterion by which the validity and accuracy of the lumped assumption is determined. From the dimensionless parameters introduced in the analysis, it can be shown that the Biot number of the inner cylinder ($\beta_{in} = h_\infty R_{in} / k_1$), the thickness ratio($\xi$), the thermal conductivity ratio (K_2) and thermal diffusivity ratio (γ_2) which are given by relations in (4)are the dimensionless parameters required for a valid comparison.

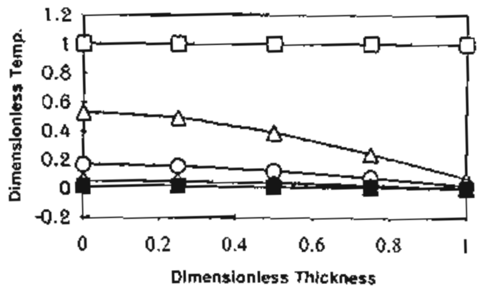
5.1- RESULTS AND DISCUSSION:

A parametric study investigating the role of the four parameters mentioned above, namely β_{in}, K_2, γ_2 and ξ has been carried out. The effect of each parameter is examined by allowing one parameter to vary, at a time, while the other parameters remain fixed.
The study was conducted for a wide range of variables of each parameter. however, only representative results are displayed in Figs.1-4 and in Table 1. In all figures, solid lines denote results from the solution of the two-region problem, and dotted lines reprsent the solution of the one-region approximation (the alternative formulation).

The effect of Biot number on the establishment of the temperature distribution is shown in Fig. 1. A maximum difference of 6% occurs when β_{in}=20, ξ=0.05 and K_2=10 at τ=0.5.



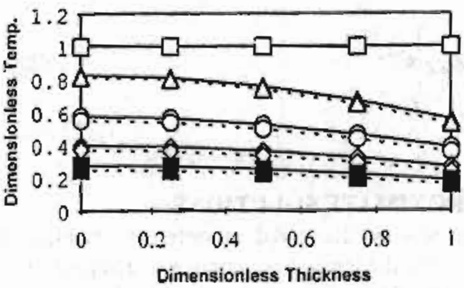
β_{in} = 0.1 K_2 = 10 γ_2 = 2 ξ = 0.05
Fig.1.a



β_{in} = 10 K_2 = 10 γ_2 = 2 ξ = 0.05
Fig.1.b

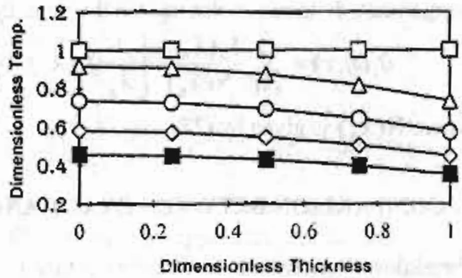
Symbol	□	△	○	◇	■
τ	0	0.25	0.5	0.75	1

The effect of the thermal conductivity ratio K_2 is shown when β_{in}, γ_2 and ξ are kept at fixed values is indicated in Fig. 2. One observes that the difference between the two formultions decreases as K_2 increases. The maximum difference between the exact and approximate formulations for the given results is about 10% at K_2=0.5, ξ=0.05, β_{in}=1 and τ=1.



β_{in} = 1 K_2 = 0.5 γ_2 = 1 ξ = 0.05

Fig.2.a

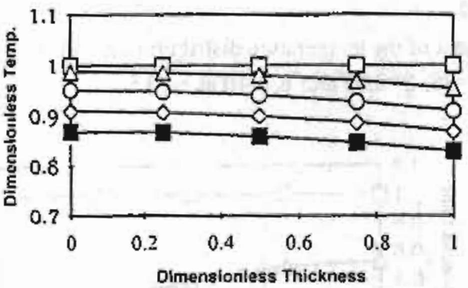


β_{in} = 1 K_2 = 10 γ_2 = 1 ξ = 0.05

Fig.2.b

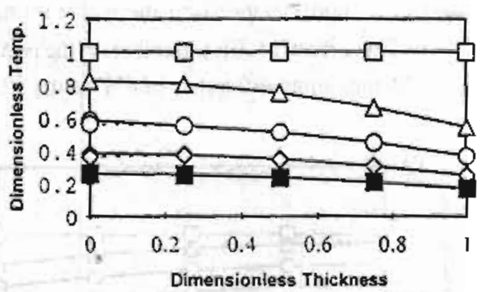
Symbol	□	△	○	◇	■
τ	0	0.25	0.5	0.75	1

The effect of the thermal diffusivity ratio γ_2 is displayed in Fig. 3. Results show that for a given set of values β_{in}, γ_2 and ξ, the effect of the thermal diffusivity results in little deviation between the two solutions.



β_{in} = 1 K_2 = 10 γ_2 = 0.1 ξ = 0.05

Fig.3.a

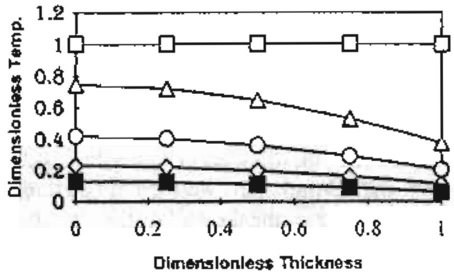


β_{in} = 1 K_2 = 10 γ_2 = 10 ξ = 0.05

Fig.3.b

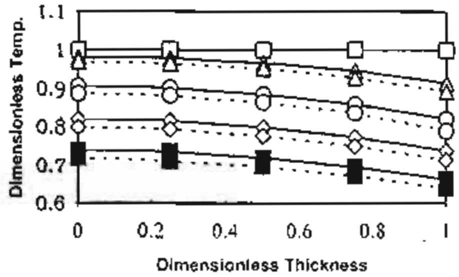
Symbol	□	△	○	◇	■
τ	0	0.25	0.5	0.75	1

The effect of the thickness ratio ξ on the two formulations is shown in Fig. 4. Results Show that the difference between the solutions of the two-regions problem and the simplified one-region problem increases as ξ inceases. For thickness ratio ξ=0.5 and when β_{in}=2, K_2=10 a maximum difference of about 3% is calculated.



$\beta_{in} = 2$ $K_2 = 10$ $\gamma_2 = 1$ $\xi = 0.01$

Fig.4.a



$\beta_{in} = 2$ $K_2 = 10$ $\gamma_2 = 1$ $\xi = 0.5$

Fig.4.b

Symbol	□	△	○	◇	■
τ	0	0.25	0.5	0.75	1

It is interesting to observe that a common factor among all cases discussed above appears. That is, in spite of the different combination of values considered for the four parameters, an acceptable approximation is always achieved whenever the ratio between β_{in}, ξ and K_2 is less than 0.1, i.e.

$$\frac{\beta_{in}\xi}{K_2} \prec 0.1 \tag{30}$$

This value is verified by computing the above ratio for all cases studied in Table. 1. If the dimensionless quantities is introduced in the left hand side of inquality (30), one can get

$$\frac{h_\infty (R_{ow} - R_{in})}{k_2} \prec 0.1 \tag{31}$$

Table. 1.

β_{in}	γ_2	K_2	ξ	$\beta_{in}\xi/K_2$	Max.error
1	0.1	10	0.05	0.005	0.2%
1	1	10	0.05	0.005	0.5%
1	10	10	0.05	0.005	0.8%
1	20	10	0.05	0.005	0.9%
1	1	0.5	0.05	0.1	10%
1	1	1	0.05	0.05	6%
1	1	10	0.05	0.005	0.5%
1	1	100	0.05	0.0005	0.01%
1	1	10	0.05	0.005	0.5%
1	1	10	0.1	0.01	0.8%
1	1	10	1	0.1	3%
1	1	10	2	0.2	5.2%
1	1	10	0.05	0.005	1%
5	1	10	0.05	0.025	3%
10	1	10	0.05	0.05	5%
20	1	10	0.05	0.1	6%

6- CONCLUSIONS:

A simplified one-region formulation for the transient heat conduction problem in a two-layer composite in cylindrical coordinates was considered. The problem is reformulated by lumping the outer layer (cylinder) and treating it as a thin film, thus introducing its effect in a modified boundary condition. Solutions of the two-regions and the simplified formulations were examined through a parametric study of the Biot number, the thermal conductivity ratio, the thermal diffusivity ratio and the thickness ratio of the two layers. It was found that a valid approximation can be achieved if the relation given by the inequality (31) is satisfied.

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