WIND ELECTRIC SYSTEM (WES) DESIGN USING GENERAL AND SIMPLE ALGORITHM

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خلاصة ـ هدف هذا البحث هو حل مشكلة ايجاد التصميم الأمثل لمنظومة رياح كهربية بطريقة بسيطة وعامة يمكن تطبيقها لأية بيانات عن سرعة الرياح ولأية قيود فنية أو اقتصادية معروضة على حل المشكلة . وقد استلزم ذلك ايجاد واستنباط تعبير رياضي لكيفية تغير سعر الكيلووات . ساعة مع كل المتغيرات المؤثرة وهو ما يعرف بدالة التكلفة المستهدفة . وقد تم اجراء تطبيق رقمي أخذت فيه كل القيم المحتملة لسرعة الرياح المتوسطة والنسب العملية للسرعات التوربينية الى سرعتها المقننة . وقد أخذ توزيع وايبول الأحصائي لتمثيل تغير سرعة الرياح على مدى العام. وبعد الحصول على النتائج تم صياغتها رياضيا بنموذجين رياضيين يعطى الأول مباشرة أقل سعر للكيلووات . ساعة المنتج ويعطى الثاني السرعة المقننة المثلى المناظرة وذلك كدوال في السرعات المختلفة كنسبة من سرعة الرياح المقننة بالاضافة الى المواصفات البيانية للموقع وكفاءة المنظومة . وقد أجريت مقارنة بين نتائج حل المشكلة بالطريقة المقترحة ممثلا بالنموذجين الرياضيين ونتائج الحل خطوة ـ خطوة والمنشورة فى المراجع السابقة والتى أثبتت الاتى :

أولا : صلاحية الألجوريثم المقترح لحل هذه المشكلة مهما كان عدد القيود المفروضة .

ثانيا : دقة النماذج الرياضية المستنبطة .

ثالثا : مرونة الطريقة المقترحة فى التعامل مع أية بيانات مختلفة أو اضافية تساعد المصمم فى الحصول على الحل الأمثل بأقصر الطرق وأبسطها .

ABSTRACT - It is intended in this paper to find the optimum design of a wind electric system by a general, but simple algorithm. It is a general since the proposal can be applied to any wind regime, site and design parameters. Its simplicity stems from the direct solution of the optimization problem without hard and numerous calculations. This is obtained although several parameters are involved. Thus, an objective cost function is deduced to express mathematically the energy cost figure against the foregoing variables. It will be minimized subjected to the most effective technical constraints.

Two models are derived also, to describe the optimum normalized rated wind speed and the minimum energy cost figure in explicit forms as functions of wind regime parameters and the wind speeds ratios.

A comparative study has been carried out to validate the proposed algorithm and the deduced models. The results are compared with those attained by applying the previously published methodology which depends on step-by-step calculations for specific conditions. The comparison demonstrates the flexibility of the proposal to any change in data and accuracy in estimating the optimum solution despite of its simplicity.

LIST OF SYMBOLS

V_R	Rated wind speed, m/s
V_C	Cut - in wind speed, m/s
V_F	Furling wind speed, m/s
$V_{R,opt}$	Optimum rated wind speed, m/s
$V_{R,opt/C}$	Optimum normalized rated wind speed, m/s
V_R/C	Normalized rated wind speed, m/s
$P_{e, ave}$	Average electrical power output, W

$P_{e,\ rated}$	Rated electrical power output, W
P_N	Normalized power, W
E	Yearly generated energy, kWh/year
C and K	Wind regime (Weibull) parameters
C_{PR}	Coefficient of performance of the wind turbine at the rated power, p.u.
η_{mR}	Transimission efficiency at the rated power, p.u.
η_{GR}	Generator efficiency at the rated power, p.u.
η_o	Rated overall efficiency, p.u.
C.F.	Capacity factor, p.u.
ρ	Air density, kilopascal
A	Rotor swept area, m^2
A_o	Optimum rotor swept area, m^2
G	Generator rating, K W
L.E.	Egyptian pound
S	Capital investment, L.E.
S_1	Cost per unit swept area of rotor, L.E./m^2
S_2	Cost per KW of generator rating, L.E./KW
s	Energy cost figure, L.E./ kWh
s_{min}	Minimum energy cost figure, L.E/KWh
G_o, G_1, g_o g_1, ψ, B_o, B_1, B_2, b_o b_1, and b_2	Constants

I- INTRODUCTION

The increased search for alternative energy sources has led to renewed interest and studies of the wind electric systems. Their zero fuel cost may act to dampen the cost of other fuels. Today, wind turbines have to compete with many other energy sources. It is therefore, important to meet any load requirements and produce energy at a minimum cost per L.E. of investment.

The selection concept of wind driven generatoros, system optimization, control system design, safety aspects, economic viability on electric utility systems, and potential electric system interfacing problems were emphasized as general guidelines and concepts and not proposed as methodologies or algorithm applicable for the problem in Ref. [1].

On other hand, in Ref. [2] and [3] the most economic design necessitates solving tens of specific problems. So, it could be derived graphically by interpolation. Any changes occurred lead to other iterations and repetition of all previous steps.

So, this paper introduces an algorithm by means of which the optimum WES design has been obtained directly. One of the familiar optimization techniques: "Sequential Search With Three Equally Spaced Experiments" is utilized with the objective cost function subjected to the technical constraints. This algorithm enables to deal with any changeable data without sensible efforts or computation time. The numerous results permit to discuss and evaluate the position and influence of each parameter on the optimum design.

II- PROPOSED ALGORITHM

a- WES Energy

The average power output from a wind turbine is the power produced at each wind speed times the fraction of the time that wind speed is experienced, integrated over all possible wind speed. In an integral form, it is formulated as [2]:

$$P_{e,\,ave} = \int_{o}^{\infty} P_e f(v)\, dv \quad W \qquad \ldots (1)$$

where f(v) is a probability density function of wind speed. The Weibull distribution will be used, thus

$$\int f(v) = \frac{K}{C} [\frac{v}{C}]^{K-1} \exp [-(\frac{v}{C})^K] \qquad \ldots (2)$$

This is a two parameters distribution where C and K are the scale and the shape parameters, respectively. The electrical power output of a model wind turbine is:

$$\begin{aligned} P_e &= o && (V<V_C) \\ P_e &= a + b\, v^K && (V_C \leqslant V \leqslant V_R) \\ P_e &= P_{e,\,rated} && (V_R \leqslant V \leqslant V_F) \\ P_e &= o && (V > V_F) \end{aligned} \qquad \ldots (3)$$

Substituting in Eq n. (1) from Eq n. (2) yields:

$$P_{e,\,ave} = \int_{V_C}^{V_R} (a + bv^K)\, f(v)\, dv + P_{e,rated} \int_{V_R}^{V_F} f(v)\, dv \qquad \ldots (4)$$

Applying the limits of integration (into) Eq n. (4), and reducing the terms, this results in:

$$P_{e,\,ave} = (C_{pR}\, \eta_{mR}\, \eta_{GR})\, (\rho/2)\, A\, V_R^3 \left[\frac{\exp[-(V_C/C)^K] - \exp[-(V_R/C)^K]}{(V_R/C)^K - (V_C/c)^K} - \exp[-(V_F/C)^K] \right] \quad W \qquad \ldots (5)$$

This equation shows the effects of the cut-in, rated and furling wind speeds on the average power production of a turbine. Also, its energy output, E is given by

$$E = (\text{average power}).\ \text{time} = P_{e,\,ave} \times 8760 \ \ \text{k Wh/year} \qquad \ldots (7)$$

b- Development of the Objective Cost Function

The capital investment (S) required for installing a wind electric system involves two essential items. The first is the cost of the turbine rotor, tower and land area denoted by S_1 per unit of rotor swept area. The second is the cost of the electrical generator, switch-gears and interconnections symbolized by S_2 per kW of the generator rating. So, the capital investment of the wind electric system can be expressed for a certain wind regime parameters and load demand as follows:

$$S = S_1\, A + S_2\, G \qquad \ldots (8)$$

It is well-known that, the generator rating is selected according to the supplied rating electrical power, hence

$$G = P_{e,\,rated} = (\rho/2)\, \eta_o\, A\, V_R^3 \quad W \qquad \ldots (9)$$

$$\text{Thus, } A = P_{e,\ rated} \,/\, (\rho/2)\, \eta_o\, V_R^3 \quad m^2 \qquad \ldots (10)$$

Substituting Eq n. (9) and Eq n. (10) into Eq n. (8) yields:

$$S = S_1\, (P_{e,\ rated} \,/\, [\, (\rho/2)\, \eta_o\, V_R^3]\,) + S_2\, (P_{e,\ rated} \times 10^{-3}) \qquad \ldots (11)$$

The energy cost figure,s is the most important indicator that illustrates the appropriateness of installing a WES. Its value can be estimated by dividing Eq n. (11) by Eq n (7) resulting in a general equation of s expressing its change with the most deciding parameters.

$$s = S/E = [S_1 \,/\, (\, (\rho/2)\, \eta_o\, V_R^3) + S_2 \times 10^{-3}] \Big/ 0.8760 \left[\frac{\exp[-(V_C/C)^K] - \exp[-(V_R/C)^K]}{(V_R/C)^K - (V_C/C)^K} - \exp[-(V_F/C)^K] \right] \qquad \ldots (12)$$

This is the objective cost function to be minimized subjected to suitable technical and economic constraints.

c- Optimization

Sequential Search With Three Equally Spaced Experiments optimization technique is applied to minimizing the energy cost figure [4]. With this technique, the three terms: $s(V_{RI}/4)$, $s(V_{RI}/2)$, and $s(3V_{RI}/4)$ as shown in Fig. 1, are initially estimated for the first interval of search, V_{RI}. Three possibilities are displayed in this figure.

The new search interval is half the length of the initial search region, V_{RI}. Thus for three calculations, the new search interval is half the length of the initial one. Treating the new interval in the same way, it will be seen that, while three experiments are still used in the search in the new cycle, the center of them coincides with one of the experiments of the previous cycle. Therefore, in the second cycle, only two new experiments need to be made, placed at the quarter and the three-quarter positions in the interval. So, with m iterations, the extreme value will be located within $(1/2)^m$ of the original length, V_{RI} but involves only N calculations, where $N = 1 + 2m$. With m repetitions of this procedure, the position of the extreme value may be located within a length V_{RN}. The accuracy, α, of search is

$$\alpha = V_{RN} \,/\, V_{RI} \qquad \ldots (13)$$

Thus,

$$\alpha = V_{RN} \,/\, V_{RI} = (1/2)^m = (1/2)^{(N-1)/2} \qquad \ldots (14)$$

Hence, the number of experiments required for the objective accuracy α is given by

$$N \geqslant 1 + 2\, (\,\log \alpha \,/\, \log 1/2) \text{ for odd } N \qquad \ldots (15)$$

For example, 7 iterations or 15 experiments are required to get an accuracy defined by $\alpha \leqslant 0.01$.

Such routine can be programmed for operating a computer without difficulty, since it uses the same basic formula for each cycle of the calculation. Since the curves of s are gently rounded near their minimum values, therfore, this makes the technique a good and effective means in assigning the optimum rated wind speed.

III- MODELLING OF OPTIMUM DESIGN CHARACTERISTICS

The prementioned sequential search optimization technique is applied for minimizing the objective cost function. It is carried out for various values of wind regime parameters C and K covering a wide range of cut-in to rated speed ratios. They represent different

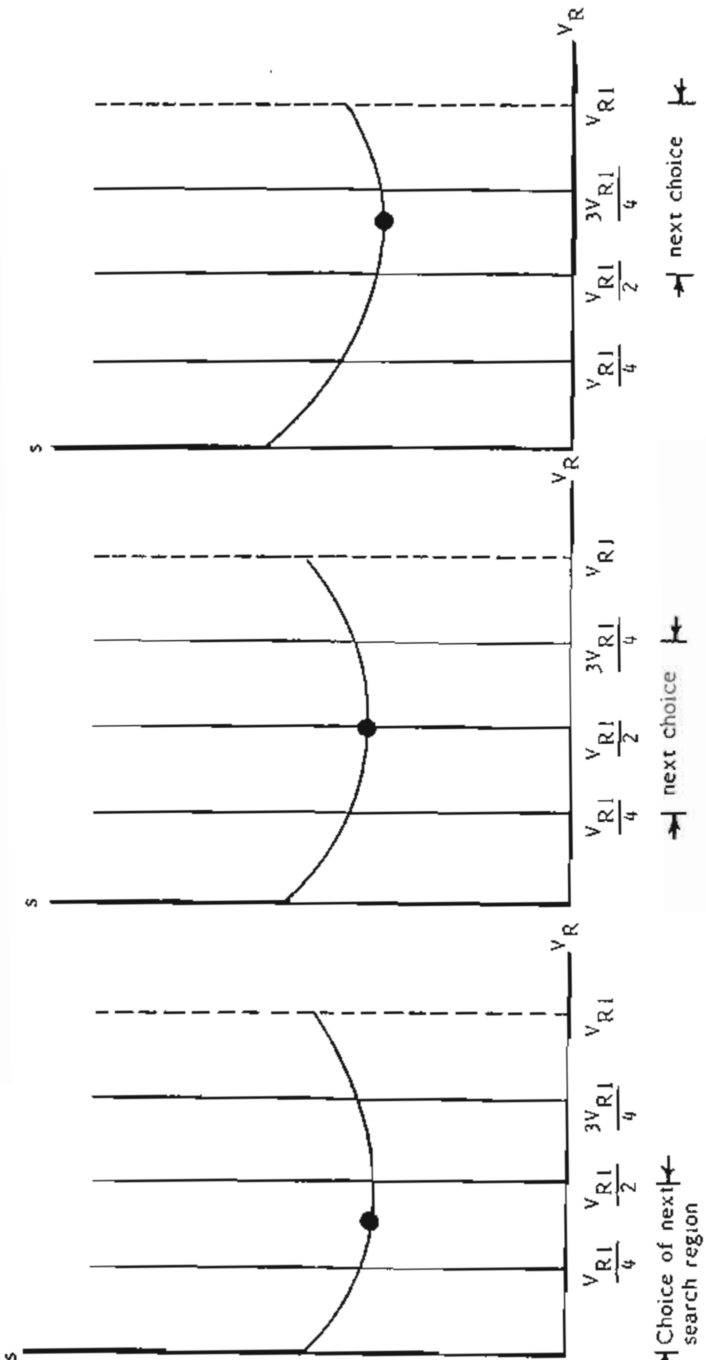

Fig. 1 Search using three experiments per cycle

WES designs. So, the optimum rated wind speed has been obtained resulting in the optimum system design. Two simple and general models are developed out of the optimization process resulting in the following forms:

1- Optimum Normalized Rated Wind Speed

$$V_{R,\,opt} / C = d_o + d_1 K \qquad \ldots (16)$$

where :

$$\log d_o = G_o + G_1 (V_C / V_R) \qquad \ldots (17)$$

, and

$$d_1 = g_o + g_1 \log (V_C / V_R) \qquad \ldots (18)$$

It is evident that the parameters d_o and d_1 are principally dependent upon the cut-in to rated speed ratio. Therefore, this model brackets all the ratios of practical and hopeful designs. Practically, it is recommended to choose the furling speed to be twice the rated one. This results in challenge, since the power regulation within the 1-8 times the rated power range is a difficult task.

2- Minimum Energy Cost Figure

The energy cost figure is a significant indicator to experiment the appropriateness of producing energy from WES's at certain wind regime parameters. The behavior of this figure is deduced and proposed to have the form of:

$$s_{min} = \psi \left[S_1 / [(\rho/2)\, \eta_o\, C^3] + S_2 (V_{R,\,opt} / C^3) \times 10^{-3} \right] / \exp (a_o + a_1 K) \qquad \ldots (19)$$

where,

$$a_o = B_o + B_1 (V_C / V_R) + B_2 (V_C / V_R)^2 \qquad \ldots (20)$$

, and

$$a_1 = b_o + b_1 (V_C / V_R) + b_2 (V_C / V_R)^2 \qquad \ldots (21)$$

This model shows the explicit change of the minimum energy cost figure with:

a- Wind regime parameters, C and K,
b- Cut-in to rated wind speed ratio, and
c- Price marketing of WES equipment: S_1 and S_2

Eq n. (19) enables the designer to compromize between different available designs and deciding the optimum and grading the others.

IV-APPLICATION AND ARGUMENT

This section involves the finding of the optimum WES design parameters that result in minimum energy cost figure by applying two methodologies:

a- The first is to solve the problem step-by-step as mentioned in the previous literature [1 , 2 , and 3]. It is applied only to certain and unique conditions of wind speed data. For other conditions, the computations should be repeated again to get the corresponding cost figure and the minimum value would, then, be attained.

Its steps of computation can be stated-briefly-in the following way:

1- According to the choosen WES turbine, the ratios of the cut-in and the furling speeds to the rated value are determined.

2- Find the wind regime parameters (Weibull density functions parameters) C and K from the long-term recorded wind data of the site under consideration [1].

3- Deduce the relation between the normalized power, P_N, and the rated wind speed, V_R where:

$$P_N = P_{e,ave} / [(\rho/2) \eta_o A V_R^3] \quad \ldots (22)$$

So, substituting for $P_{e,ave}$ as given by Eq n. (5), we have:

$$P_N = (V_R/C)^3 \text{ C.F.} \quad \ldots (23)$$

where C.F. is given by:

$$\text{C.F.} = P_{e,ave}/ P_{e,rated} = \left[\frac{\exp[-(V_C/C)^K] - \exp[-(V_R/C)^K]}{(V_R/C)^K - (V_C/C)^K} - \exp[-(V_F/C)^K] \right] \quad \ldots (24)$$

4- Several V_R / C ratios are taken and the respective P_N are estimated. So, Fig. 2 can be deduced with K as a parameter. Thus, the optimum normalized rated wind speed, $V_{R,opt}/C$ at which P_N has its maximum value can be determined for various values of K.

5- Compute the required swept area, A_o, of the rotor according to the rated load demand, $P_{e,rated}$ and the previously computed optimum normalized rated wind speed, where

$$A_o = P_{e,rated} / [(\rho/2) \eta_o V_{R,opt}^3] \quad \ldots (25)$$

and the corresponding generator rating, G, has been estimated by the aid of Eq n. (9).

6- Compute the capital investment needed to install the WES ,S, from Eq n. (8).

7- Estimate the annual generated energy, E, as expressed by Eq n. (7)

8- Find the minimum energy cost figure, L.E./ kWh, taken into consideration the rate of cost return.

Several discriminative conditions would be taken to deduce the optimum design parameters and the corresponding minimum energy cost figure. Of course this methodology needs and consumes considerable effort and computation time and distinguished by no flexibility to any changes in the input wind speed data.

b- The second methodology is to solve the optimization problem directly by applying the "Sequential Search With Three Equally Spaced Experiments" optimization technique on a new and proposed objective function developed in this paper as given by Eq n. (12). It is intended, here, to minimize s where:

$$s = [S_1 / ((\rho/2) \eta_o V_R^3) + S_2 \times 10^{-3}] \Big/ 0.8760 \left[\frac{\exp[-(V_C/C)^K] - \exp[-(V_R/C)^K]}{(V_R/C)^K - (V_C/C)^K} - \exp[-(V_F/C)^K] \right] \quad \ldots (12)$$

or

$$s = [S_1 / ((\rho/2) \eta_o V_R^3) + S_2 \times 10^{-3}] \Big/ 0.876 \text{ (C.F.)} \quad \ldots (12)$$

Subjected to equality constraints :

$$V_F / V_R = 2.0 \quad \ldots (12.1)$$

$$\rho / 2 = 0.647 \quad \ldots (12.2)$$

$$\eta_o = 0.25 \quad \ldots (12.3)$$

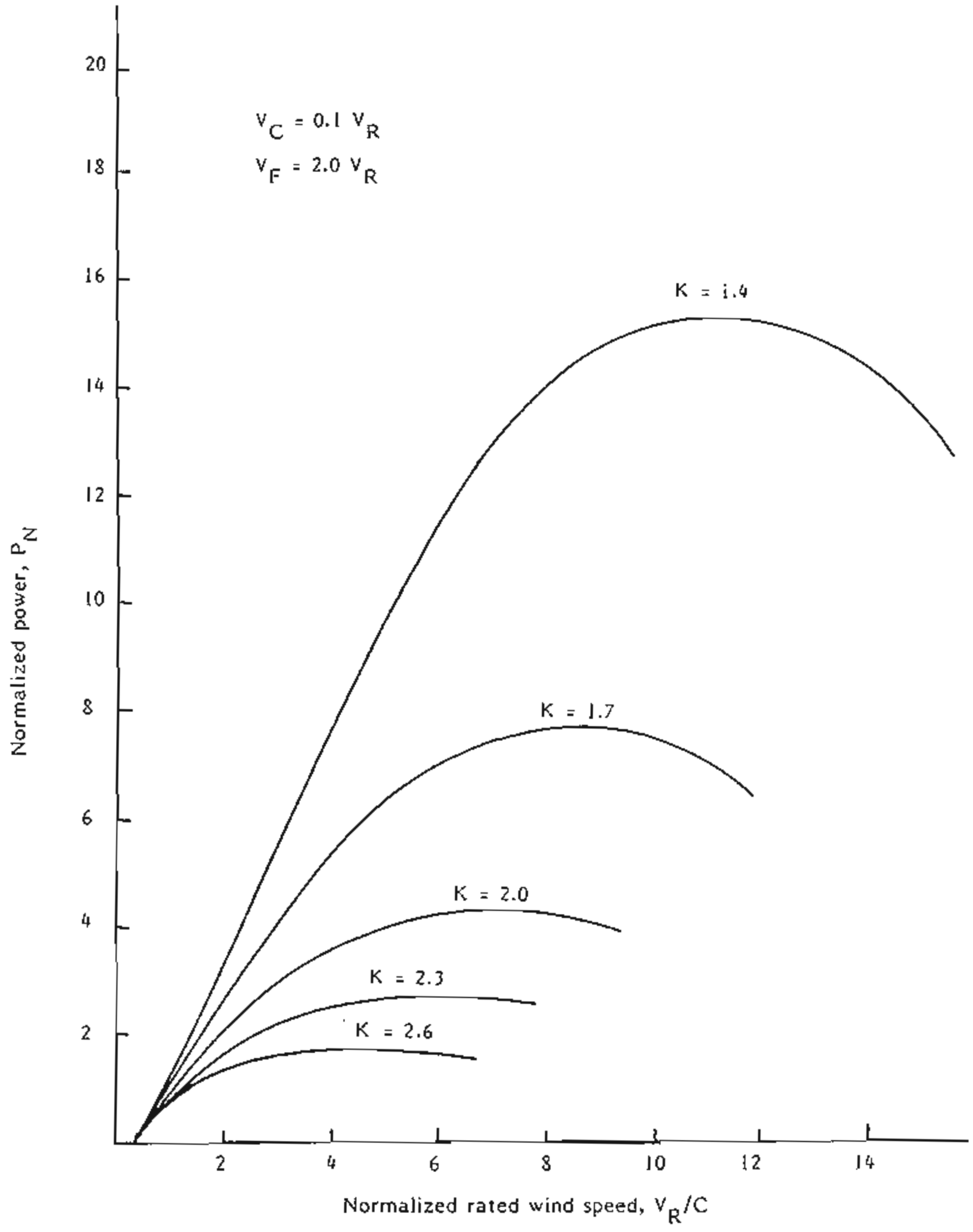

Fig. 2 Normalized power versus rated speed

and inequality constraints :

$$4 \text{ m/s} \leqslant \bar{V} \leqslant 12 \text{ m/s} \quad \ldots (12.4)$$

$$0.4 \leqslant V_C/V_R \leqslant 0.5 \quad \ldots (12.5)$$

$$1.3 \leqslant K \leqslant 3.0 \quad \ldots (12.6)$$

$$1.11\ \bar{v} \leqslant C \leqslant 1.13\ \bar{v} \quad \ldots (12.7)$$

In contrast to the first methodology, the optimization method has distinctive advantages and features. It can be applied directly for any wind regime, site, and design parameters without additional effort and computation time, only changing the input data for the new case. Thus, it enables to investigate in an easy and simple manner the influence of any of the above parameters on the minimum energy cost figure. Moreover, this figure and the optimum normalized rated wind speed are modelled to describe in explicit forms their behavior againts all the effective parameters. These behaviors and the optimum rotor swept area versus the wind regime parameters are illustrated in Fig. 3, 4 and 5.

From the results described by the proposed models, the optimum rated wind speed, optimum rotor.turbine swept area, maximum average electrical power, and minimum energy cost figure of a wind electric system of a rated power of 200 KW are illustrated in Table (1). The WES is assumed to be installed in alternative twenty sites each has its own optimum. They can be classified into five main categories A,B,C,D, and E covering practical values of K ranging from 1.4 - 2.7. Each of these categories has been again classified into four secondary sites to supply the same load according to its C ranging from 4.032 to 12.096. Thus, a site of secondary category of C.3 means that it has wind regime parameters (C and K) of 7.56 and 2, respectively. From the sites of category A.1 and E.1, it has been noticed that the minimum energy cost figure increases by an amount of 95.69% when the Weibull shape parameter increases from 1.4 to 2.6 keeping the Weibull scale parameter be the same 4.032. Also, sites A.1 and A.4 reveal that the minimum energy cost figure reaches a value of one-ninth when the Weibull scale parameter increases from 4.032 to 12.096 (3 times) having the same Weibull shape parameter (1.4). In addition, the maximum average electrical power and the corresponding optimum swept area differ according to the site of installation. This is ascribed by the fact that the WES is essentially designed to produce a specified electrical rated power of 200 - KW.

A plot of the optimum normalized rated wind speed, $V_{R,opt}/C$ versus the cut-in to rated wind speed ratio is given in Fig. 6. This characteristic is deduced for three different values of K (1.4, 2.0, and 2.6). The family of curves labelled (Models) is found by applying the proposed models and that labelled (Calculated) is computed from applying the first methodology. Both families agree reasonably well where the maximum percentage deviation is 6.37. This demonstrates rational accuracy of modelling the optimum normalized rated wind speed.

On other hand, the minimum energy cost figure versus K is plotted in Fig 7-a with 4.032 as a scale parameter. It is also drawn against the Weibull scale parameter, C, as shown in Fig. 7-b with shape parameter of 1.4. The maximum percentage deviation between both families is 7.34 which illustrates the accurateness of the proposed models although it incorporates several parameters that affect appreciably the results. These parameters are listed in the following table:

Table 2: Models Coefficients

Coefficient	G_o	G_1	g_o	g_1	ψ
Its Value	1.287	1.587	0.284	3.63	1.1419

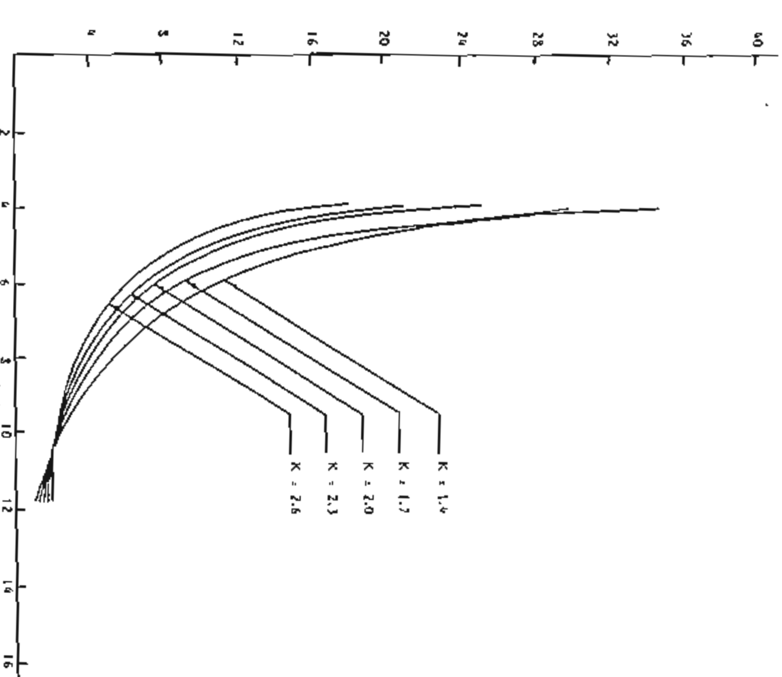

Fig. 4 Minimum energy cost figure versus C for various values of K (V_C/V_R = 0.5 and V_F/V_R = 2.0)

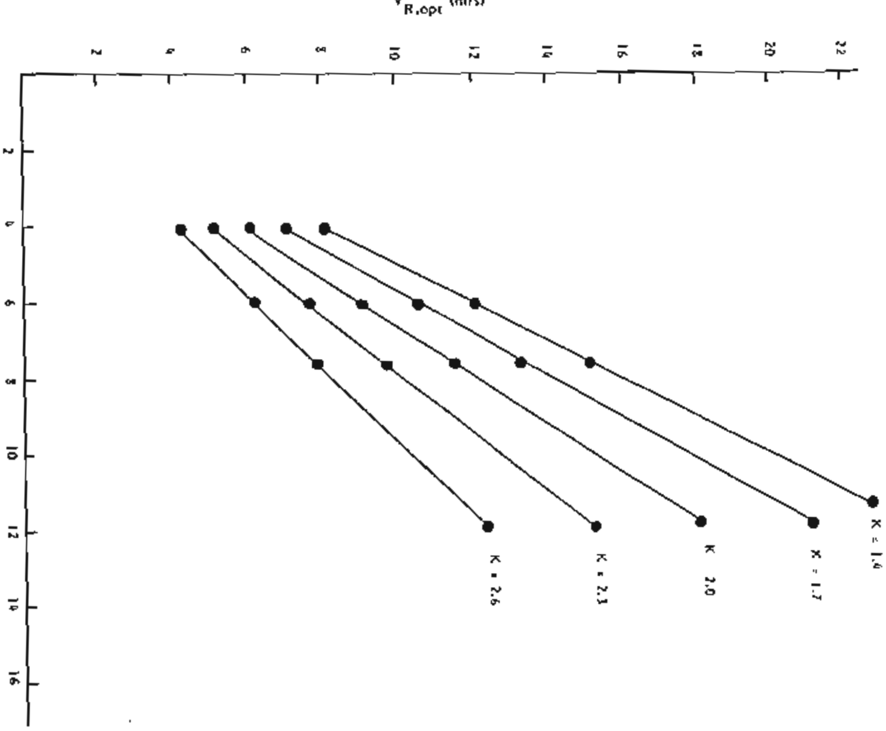

Fig. 3 Optimum normalized rated wind speed versus C for various values of K (V_C/V_R = 0.5 and V_F/V_R = 2.0)

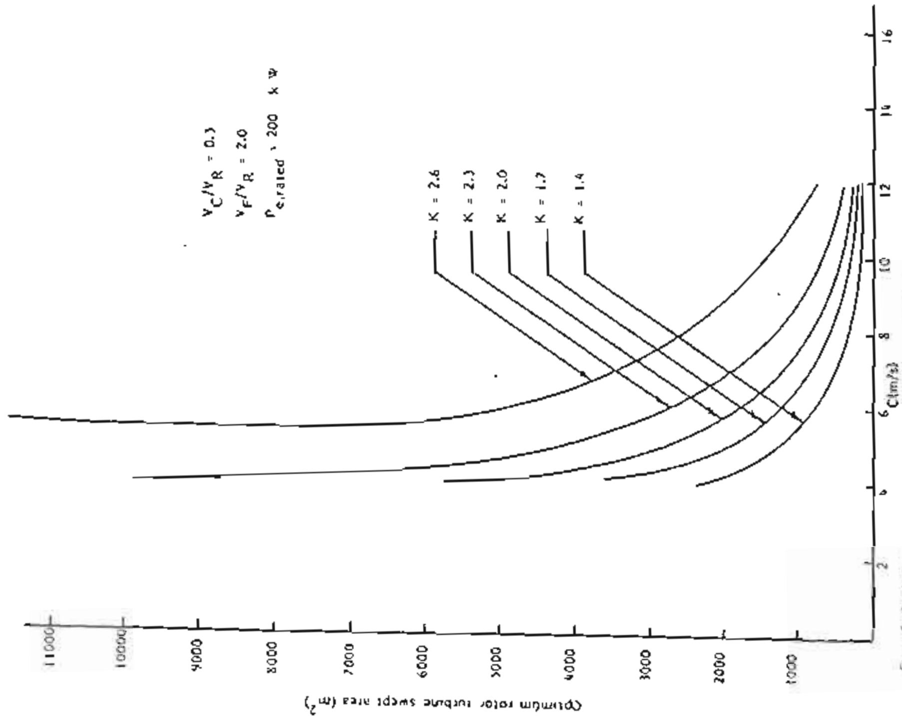

Fig. 5 Optimum rotor turbine swept area required for various wind regime parameters.

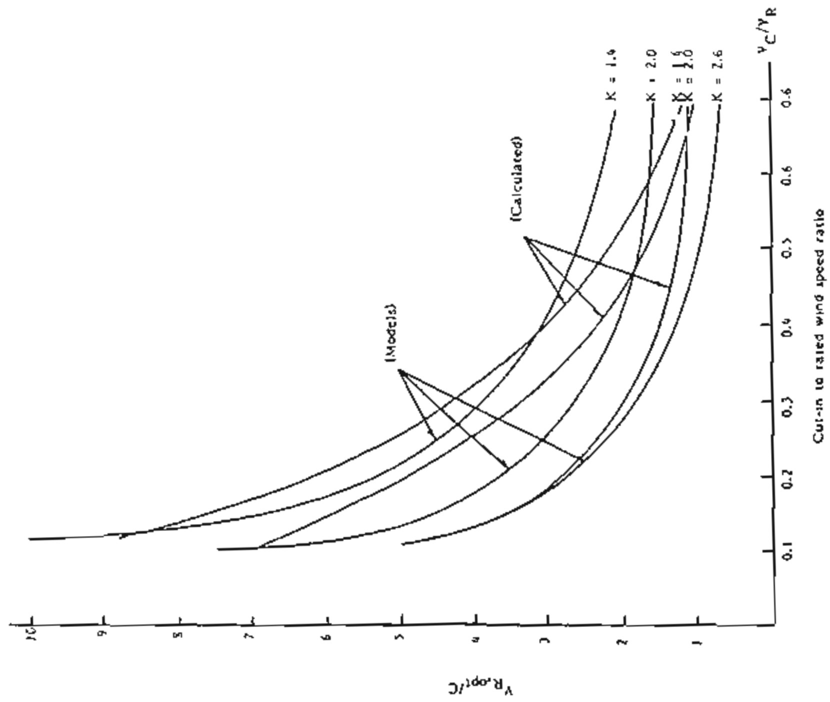

Fig. 6 Normalized rated wind speed versus the cut-in to rated wind speed.

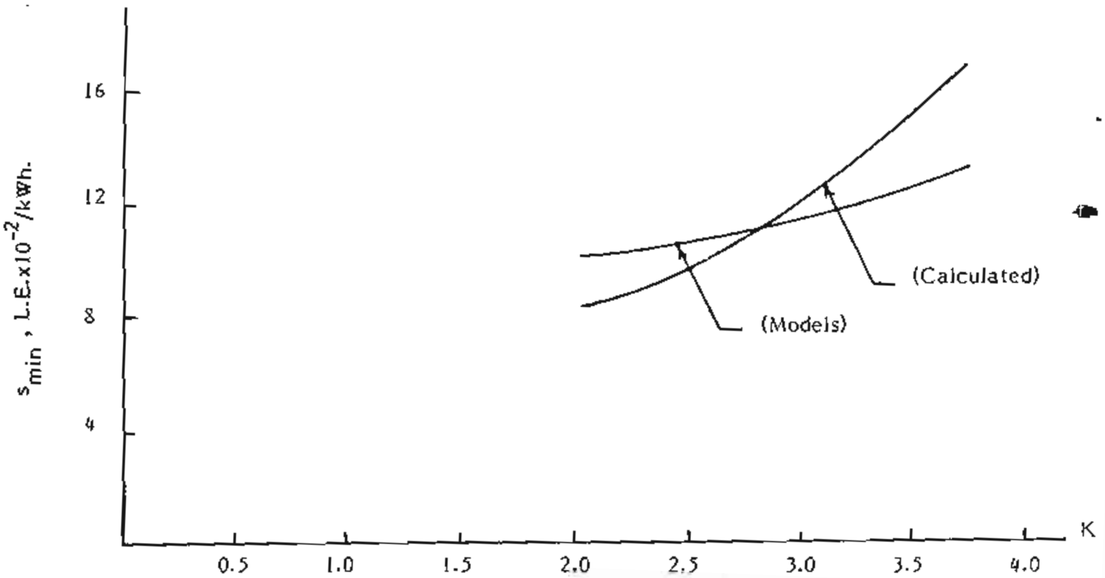

Fig. 7-a Minimum energy cost figure as a function of K (C = 4.032)

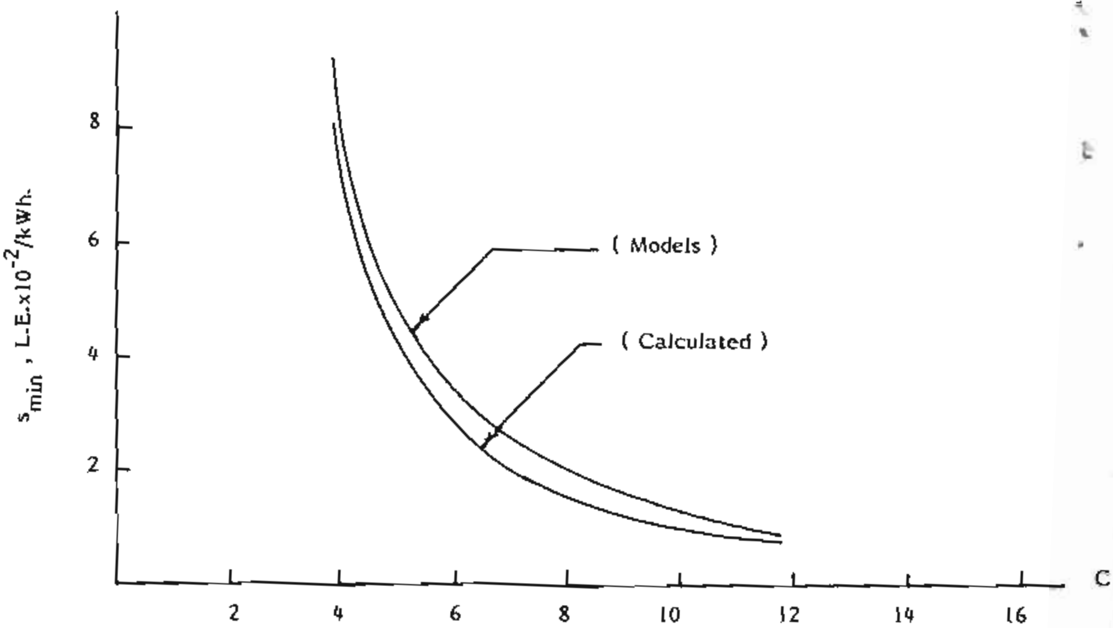

Fig. 7-b Minimum energy cost rigure as a function of C (K = 1.4).

Table (1): Optimum Normalized Rated Wind Speed Optimum Rotor Swept Area, Electrical Average power, and Energy Cost Figure for Various Wind Regime Parameters ($V_C/V_R = 0.5$).

Sites of Main Category	K	Sites of Secondary Category	V (m/s)	C(1.12V) (m/s)	$V_{R,opt}$ (m/s)	A_o (m^2)	($P_{e,ave}$) (KW)	MECF* (Mill./kWh)
A	1.4	A.1	3.6	4.032	8.0	2420	36.40	180.8
		A.2	5.4	6.048	12.0	720	36.55	63.4
		A.3	6.75	7.56	15.0	366	36.32	39.2
		A.4	10.8	12.096	23.99	89	36.32	20.0
B	1.7	B.1	3.6	4.032	7.0	3605	45.06	212.1
		B.2	5.4	6.048	10.53	1059	44.69	70.8
		B.3	6.75	7.56	13.16	542	44.72	41.8
		B.4	10.8	12.096	21.0	134	45.08	18.5
C	2.0	C.1	3.6	4.032	6.0	5724	59.48	25.01
		C.2	5.4	6.048	9.0	1696	59.50	80.1
		C.3	6.75	7.56	11.33	850	58.25	45.3
		C.4	10.8	12.096	18.12	208	58.33	17.6
D	2.3	D.1	3.6	4.032	5.0	9892	85.47	296.6
		D.2	5.4	6.048	7.6	2817	82.14	92.2
		D.3	6.75	7.56	9.49	1447	82.42	50.2
		D.4	10.8	12.096	15.19	353	82.36	16.9
E	2.6	E.1	3.6	4.032	4.09	18072	129.76	353.8
		E.2	5.4	6.048	6.13	5368	130.12	107.4
		E.3	6.75	7.56	7.3	2606	123.39	57.2
		E.4	10.8	12.096	12.26	671	130.14	16.72

* MECF Minimum Energy Cost Figure, Millims = 10^{-3} L.E., and \$ 1.0 = 2.20 L.E.

Table 2: (Cont.) Models Coefficients

Coefficients	B_o	B_1	B_2	b_o	b_1	b_2
Its Value	1.263	-3.029	1.921	-0.438	0.973	-0.658

V- CONCLUSIONS

A proposed general and simple algorithm has been presented in this paper for optimizing the WES. It can be applied for any wind regime, site and design parameters. It gives the direct solution of the optimization problem without difficulty and numerous calculations. Thus, it enables to investigate in an easy and simple manner the influence of any of the above parameters on the minimum energy cost figure.

Moreover, this cost figure and the optimum normalized rated wind speed are modelled describing in explicit forms their behavior against all the effective parameters. The numerical application introduced demonstrates the accurateness of the proposed models where the maximum percentage deviation in estimating the minimum energy cost figure and the optimum normalized rated wind speed are 7.34 and 6.37, respectively.

The influence of the wind regime parameters on the minimum energy cost figure is studied which can be used as an index showing the appropriateness of installing a WES in a site having wind regime parameters.

The introduced algorithm can be used as quite useful tool in making the economic decisions on installing a WES.

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