THEORETICAL STUDY OF THE EFFECT OF INJECTION CONDITIONS ON THE LAMINAR BOUNDARY LAYER CHARACTERISTICS DEVELOPING ON A FLAT-PLATE

PART 1: Effect of Injection Ratio (λ)

S.M. Weheba[1], B.A.Khlifa[2], N.I.I. Hewedy[3] and M.M.M. Elmayit[4].

Abstract:

The present work is devoted to theoretical study of the effect of injection velocity into laminar boundary layer. The finite difference scheme was used to solve the momentum and continuity equation. Small perturbation theory is used to determine the pressure in the vicinity of injection. The results concerning the effect of injection velocity on the physical and integral characteristics boundary layer showed that, the boundary layer behaviour at injection velocity higher than the free stream is opposite to that at lower injection velocity.

Nomenclature;

- b : Slot height thickness (m).
- $\bar{B}$: Slot height ratio, is the ratio of slot height (b), to the laminar boundary layer (δ_L). (b/δ_L).
- cf : Skin friction coefficient, $(\tau_w / \frac{1}{2} \rho u_e^2)$.
- cp : Pressure coefficient, $\frac{(p - p_\infty)}{\frac{1}{2} \rho u_e^2}$
- $\bar{D}_x$: Step size in $\bar{x}$-direction.
- $\bar{D}_y$: Step size in $\bar{y}$-direction.
- H_{12} : Shape factor parameter, $(\frac{\delta_1}{\delta_2})$

---

(1) Professor and Vice President of Menoufia University, Egypt.

(2) Associate Professor, Faculty of Engineering & Technology, Menoufia University, Egypt.

(3) Lecturer, Faculty of Engineering & Technology Menoufia University, Egypt.

(4) Demonstrator, Faculty of Engineering & Technology Menoufia University, Egypt.

H_{23} : Energy parameter, $(\frac{\delta_2}{\delta_3})$

τ_w : Wall shear stress (N/m^2)

M_a : Mach number.

P : Static pressure at any x-direction $(\frac{N}{m^2})$

Re_1 : Reynolds number, based on a flat- plate length, $(\frac{u_\infty L}{\nu})$

u : Velocity component in x-direction (m/s).

u_e : Free-stream velocity at the edge of the boundary layer ≈ 0.99 u_∞), (m/s).

u_{jet} : The jet velocity (m/s).

$\bar{u}_{max}$: The first maximum velocity, on the profile, measured from the wall, (u_{max}/u_∞).

u_∞ : Outer edge velocity of the boundary layer, at the slot position, (m/s).

v : Velocity component in y-direction (m/s).

x : Distance along a flat-plate measured from the first iteration section (m).

x_o : First iteraction section (m),

y : Distance normal to a flat-plate measured from the plate(m).

$\bar{y}_{max}$: Location of the maximum velocity on the profile, measured from the wall.

$\bar{y}_{min}$: Location of the minimum velocity on the profile, measured from the wall.

: Boundary layer thicknes (m).

δ_1 : Boundary layer displacement thickness, $\int_0^\delta (1 - \frac{u}{u_e})dy.(m)$

δ_2 : Boundary layer momentum thickness $\int_0^\delta \frac{u}{u_e}(1 - \frac{u}{u_e})dy.(m)$

δ_3 : Boundary layer energy thickness, $\int_0^\delta (\frac{u}{u_e})^2(1 - \frac{u}{u_e})dy.(m)$

ρ : Fluid density. (kg/m^3).

μ : Dynamic viscosity. $(N. sec/m^2)$.

ν : Kinematic viscosity (m^2/sec).

α : Angled injection.

λ : Injection ratio, is the ratio of the jet velocity to the outer edge velocity, (u_{jet}/u_∞)

Supscripts:

w	: Condition at the wall:
e	: Condition at the edge of boundary layer.
∞	: Condition at the free-stream of the boundary layer
j	: Designation of mesh point in y-direction.
M	: Maximum value of j.
n	: Designation of mesh point in $\bar{x}$-direction.
N_1	: Value of (n), at the injection section.
N_2	: Value of (n), from the injection section to the end of the iteration.
N_{max}	: Maximum value of (n).
($\bar{}$)	: Nondimensionalized quantities.

1. Introduction:

The behaviour of the boundary layer with injection draw the attention of many investigators. A lot of publications in connection with this problem exists. One of the fundamental references is the Schlickting (9). These publications can be classified according to the method used for the study into, theoretical and experimental works and these can be also, according to the flow regions, classified into, laminar or turbulent flow. It can be noticed that from reviewing the publications that a lot of works deal with turbulent wall jet and injection in turbulent layer. The most famous of these works are those of Schamarz (1) , Kurka et al (2).

Escuaier et al (3), Bradshow (5) and Newman et al (4) All of these works delt with the problem theoritically and showed the effect of several parameters on the boundary layer characteristics and skin friction.

Lee and Clark (6) studied experimentally the angled injection of submerged flat-plate in conditions of laminar flow. They noted that all the length scoles y_{max} and y_{min} vary linearly eith the variation of λ and α^{o}.

Krause, Hanel and Hewedy (8), studied the influence of a tangential slot injection in an attached boundary layer over a flat plate on the surface pressure distribution. In this investigation, the pressure in the vicinity of the injection region, is not prescribed as a pra-prandtl's theory, but is determined in the from of small perturbation theory, for subsonic flow, hence,

$$P(x) = -\frac{2}{\pi\sqrt{1-M_a^2}} \int_{xo}^{x1} \frac{1}{\int e(\xi)} \frac{dA(\xi)}{d\xi} \frac{1}{(x-\xi)} d\xi.$$

Where, $A(\xi)$, is the mass flux displaced by the boundary layer $P(x)$, is the pressure due to viscous displacedment, x_o is the initial iter-action section and (x_1) is the last iteract on section.

They concluded that, if the blowing rate is greater than one $(\lambda > 1)$ the displacement thickness decteases near the injection section and the pressure variation rapidly near the injection section and increase gradually injection section. On the other hand if the blowing ratio is less than one $(\lambda < 1)$ they observed that, the behav-iour is opposite for slot injetion with blowing ration greater than one, the displacement thickness increases continuosly and the press-ure increases slightly upstream from the injection section and after the injection section it undergoes a relatively small dectease and then remains constant.

Hanel (7), studed the effect of the normal injection on the lami-nar boundary layer characteristics. He also used the same pressure equation in (7), in the vicinity of the injection region. Hanel found that, the pressure variaties rapidly in the injection section, after which the rate of pressure decreases almostly constant. The friction coefficient (cf) increasily rapidly in the injection section and then decreases continously in downstream distance.

The Conclusion;

From the previous discussion of the available works on the inj-ection of boundary layer it noticed the following:

The behaviour of the boundary layer characteristics as was shown in case of turbulent, seams to depend on slot heigh (b), and injection angle (α^{o}). For the case of laminar flow the effect of the two parameters is not studied. The effect of this two parameters must be taken in consideration in prediction of laminar boundary layer with injection. For this purposem this work is devoted.

2. Mathematical Formulation:

The following assumptions are concesidered:

1) The flow is two dimensional, laminar, steady and isothermal
2) The fluid is Newtonian and incompressible.
3) The slot height is small compared with the length scale. It is was chosen to be less than the boundary layer thickness. The flat plate thickness is negligible.

The continuity equation for this case is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

The momentum equation is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots\ldots\ldots\ldots (2)$$

4) On the outer edge of the boundary layer the flow is governed by the Euler equation:

$$u_e \frac{due}{dx} = - \frac{1}{\rho} \frac{dp}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The pressure can be throught of as consisting of one part due to the inviscid flow field $P_o(x)$ and one part due to viscous displacement of the inviscid flow P(x), so that

$$P(x) = P_o(x) + P(x) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The pressure due to viscous displacement can be determined in the frame of small perturbation theory for subsonic flow,

$$P(x) = - \frac{2}{\pi \sqrt{1-M_a^2}} \int_{x_o}^{x1} \frac{1}{\rho e(\xi)} \frac{dA(\xi)}{d\xi} \frac{1}{(x-\xi)} d\xi \quad \ldots (5)$$

is the down stream distance from the injection region, x_o is the first it eration, and (x_1) is the last iteration section.

$$A(x) = \int_0^{\delta} (\rho_e u_e - \rho u) \; dy \qquad \ldots\ldots\ldots\ldots (6)$$

Where

A(x) is the mass flux displaced by the boundary layer . The numerical solution can be obtained with an implicit formulation, in which the recursion relation can be written as

$$u(m,n) = E(m,n)\; u(m+1,n) + F(m,n) + G(m,n)\; ue(n) \;\ldots\ldots (7)$$

The indices m and n count the steps in the directions normally and tangentially to the wall. The quantity ue(n) is the unknown external velocity.

the integral of the mass flux can be replaced by a sum in finite-difference formulation, i.e.

$$A(n) = ue(n) + \sum_{J=2}^{J=n-1} u(n,J).\; Dy \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The last two equationa can be combined to yield

$$A(n) = B_1(n) \;\; ue(n) + B_2(n) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The coefficients $B_1(n)$ and $B_2(n)$ are functions of the recursion, coefficients E,F and G.

The viscous pressure distribution P (x), can be replaced by a sum in finite-difference formulation as

$$P_{\delta}(n) = -\frac{2}{\pi\sqrt{1-M_a^2}} \left[\sum_{k=1}^{k=N_1-1} A(k).\; Al(N_1,k) + A(n)\; L_1 + \sum_{N_1+1}^{NMAx} A(k)\; AL(N_1,k) \right] \ldots\ldots\ldots (10)$$

The value of A(k) upstream of the location x = (n-1) Dx are taken from the last iteration and the downatream values from the one. If P(n) is replaced through the Euler equation (3), there is obtained

$$ue(n) = \alpha 1(n) \cdot A(n) \;\; + \alpha 2(n) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

So that from eqs. (9) and (11) the final result is given by

$$U_e(n) = \left[\alpha_1(n)\ B_1(n) + \alpha_2(n)\right] / \left[1- \alpha_1(n).\ B_1(n)\right] \quad .. (12)$$

The equation (12)represents the outer edge velocity distribution through the injection region, so it can be calculated at any section. In all sections after the injection, the convergence is ensured by the condition $\left[\{1-ue(n)\} \leq 0.01\right]$

If the difference is found to be largem the difference equation is solved again. Using the prediction ue(n), as the initial value for calculating the new velocity distribution $\bar{u}(n,J)$. Thus, the velocity components outer edge velocity and the pressure distribution are determined numerically within the prescribed accuracy (0.01).

The Prediction Procedure:

The solution of the difference equation (7) is established on the assumption that the coefficients; E(n,J) and G(n,J) of the derivatives in the momentum equation are known at all points,. The initial conditions which describes velocity profile at the starting of the injection ragion is also known. Since the initial velocity distribution is assumed to be parabolic as shown in fig.(1). At every section the outer edge velicity, ue(n) is computed from equation (12). The computed values of ue(n) as a dimensionless auantities undergo the test, $\left[\{1-ue(n)\} \leq 0.01\right]$. If, the test is satisfied , $\bar{u}(n,J)$ is computed for all points, and consequentally P^J_{n+1} and the calculation proceeds to the next-$\bar{x}$ station. If, the test is not satisfied, the prediction value of ue(n) is taken as the initial value and so on, till the test conditions are satisfied. Finally, at every station the boundary layer parameters, namely $\delta(n)$, $\delta 1(n)$, $\delta 3(n)$, $H_{12}(n)$, $J_w(n)$, and P(n) are calculated numerically.

3. Discussion:

3.1: Effects of Injection on The Velocity Profile:

The computer program designed for the calculation of the boundary layer; physical and itegral charactristics, on flatplate with injection was used to study the effect of several parameters. Among these parameters is, the injection ratio (λ), which was chosen to be

λ = 0.4, 0.8, 1.2 and 1.4 .

In addition, the physical properties of the air , the fluid used in this study, are $\nu = 15.10^{-6} m^2/s$ and $M_a = 0.1$ The (Re) was taken to be constant at $Re_{\infty} \pm 10^5$, which corresponds to stream velocity u_{∞} = 34.4 (m/s) on a plate with L = 220 mm. The velocity is quite small to suit the condition of incompressible fluid flow.

3.1-2: Effects of λ on the general Features of Velocity Profile:

Fig.(3-1) shows the effect of variation- λ on the velocity profile from the injection section, i.e. initial conditions, to the region where the velocity profile becomes typical for the boundary, Layer. The values of $\bar{B}$ and were hold constant at the values (0.5 and 0.0) respectively, while (λ) was given the values o.4, o.8, 1.2 and 1.4 respectively, i.e. was varied from weak to high moderate. The same was repeated for different values of $\bar{B}$ (0.6, 0.7 and 0.8) with different values of α^o ,(4.0, 8.0 and 12.0^o). From this figure it can be seen that:

1- The injected two dimensional-incompressible boundary layer along a flat-plate can be classified into two regions. The first starts from injection section or the initial conditions, and progressing in down-stream direction to the region where the velocity profile becomes similar to the typical boundary layer profiles. This region may be called, the mixing region. The second region begins after the mixing region , the flow is nearly constent, i.e. rater-eduction of local-velocity for any velocity profile is nearly constant along the $\bar{x}$- direction.

Through the mixing region, the velocity profile can be classified into three disstingished layers:

1) Inner-layer, this layer starts from zero velocity at the wall to the first maximum velocity.,
2) Intermediate layer, from the first maximum velocity to the minimum velocity on the profile, and

3) Outer-layer or wake layer, starts from the minimum velocity in the profile to the outer edge velocity ($ue = 0.99\ u_{\infty}$).

The figure indicates clearly that, both inner and intermediate layers increasing alond the down-stream direction, while the shear layer decreases. The distance of the wall, at which the first maximum velocity (u_{max}) exists, y_{max} increase in the down-stream direction, mean while, the point of minimum velocity (u_{min}), y_{min} is displaced off the wall in the down-stream direction. The (u_{max}) decreases in direction of the main flow mean while u_{min} increases. This is explained by the wall-jet behaviour. The maximum velocity is controlled by the semi-bounded jet, which dectates a decreasing character of u_{max} and its further shift from the wall in the down-stream direction. The u_{min} increase is due to accumulation of kinetic energy of both the jet and the main stream, mean while the y_{min} is increasing the down stream direction due to the wake law.

2- An increase of λ leads to an increase of maximum and minimum velocities, since the mass flow rate increases, and increase of the mixing region length; since the kinetic energy of flow increases and needs a longer both before the decay of first maximum velocity occurs. These observations apply to the different values of λ, $\bar{B}$ and $\overset{\circ}{\alpha}$.

3.1-3: Effects of λ on decay-maximum Velocity Profile and Growth of its Location:

The figure (3-2), indicate that, the first maximum velocity decays along the mixing region, but the rate of decay in faster at the beginning, since the velocity at any point is supposed to the resultant of the resultant of the addition of the induced by the jet velocity and the boundary layer velocity. In the beginning of the mixing region the width of the jet is still small, the kinetic energy of the jet is still hight. The stretching and mixing of the jet is accomponied with mixing losses, which are proportional to the velocity of the flow. So the high velocity in the beginning of the mixing region leads to higher losses and hence higher rate of decay. The rate of decay very close to straightline in cose of small ($\lambda \leq 0.4$).

The figure (3-3) shows that the location of maximum velocity (y_{max}) is growing continously, and the growth-rate is nearly constant at the lower injection ratio, i.e. $\lambda \leq 0.4$.

3.1-4: Effects of λ on Growth-Minimum Velocity Profile and Growth of its Location:

Figure (3-4) shows the effects of - λ on growth of minimum velocity, and figure (3-5) indicates the effects of - λ on y_{min}.
From these figures it can be seen that:

In the beginning of the mixing region, the rate of u_{min} increase is very close the straight line in all cases of λ and $\bar{B}$. The increase of - λ leads to higher slope of the line. Downstream the line diverges and the relation between u_{min} and $\bar{x}$-becomes nonlinear.

3.2-2: Effects of Injection Ratio (λ) on that:

3.2.2-1: Effect of injection ratio on δ :

Fig.(3-6) shows the variation of δ at different injection ratios along the flat plate for $\lambda = 0.4$, 0.8, 1.2 and 1.4, mean while the other conditions are kept constant from the figure it can be seen that: Concerning upstream before the injection section in the injection section a sudden increase of (δ) takes place. Begining from the injection section continueous increasing with higher from the upstream rate.

3.2.2-2: Effect of- λ on boundary layer displacement thickness(δ_1):

Fig.(3-7) shows the effect of variation- λ on δ_1 at constant values of $\bar{B} = 0.5$ and $\alpha^{o} = 0.0$. The same was repeated for different values of $\bar{B} = 0.6$, 0.7 and 0.8 as shown in figure (3-7), respectively.
From these figures it can be seen that:
The increase of λ leads to decrease of (δ_1), since the injection flow tangentially enriches the boundary layer. The kinetic energy of the injected flow enriches the inner region of the boundary layer. Keeping in mind that, (δ) is not affected by the injection ratio (λ), the (δ_1) in this conditions must decrease. Accordingly δ_1

δ_1(λ = 0.4) is greater that δ_1(λ < 0.4). Behind the mixing region λ seems to have no effect on δ_1. It can be noticed also that increase of λ leads to increase of the length of the region in which miximing is complited.

3.2.2-3: Effect of - λ on the boundary layer Momentum Thickness(δ_2):

Figure (3-8) shows the effect of on δ_2 variation as a function of x at different slot height ratios, in case of tangential (α° = 0.0) The trend of δ_2-variation at constant (λ) is the same as (δ_1). The variation of B has a different effect of (δ_2). Increase of λ till a certain limit leads to increase of δ_2. Further increase of - λ after the limit leads to decrease of (δ_2). This variation of (δ_2) is limited in the mixing region which is noticed to be function of ($\bar{x}$). The increase of (λ) leads to increase of mixing region length. This can be explained by the kinetic energy increase with λ increase.

3.2.2-4: Effect of λ on the Energy Thickness (δ_3):

Figure (3-9) shows the effect of λ δ_3 variation of δ_3 at different B, with zero angled injection. From this figure it can be observed that, same behaviour of δ_2-variation is repeated for (δ_3) The rate of increase of (δ_3) as a function of $\bar{x}$ is slightly heigher than the case of (δ_2).

3.2.2-5: Effect of - λ on the Shape Factor H_{12}:

Figure (3-10) shows the shape factor H_{12} at different injection ratios at constant $\bar{B}$ = 0.5 and α° = 0.0. The same was repeated for different values of $\bar{B}$ = 0.6, 0.7 and 0.8 as shown in figure (3-10), in case of tangential (α° = 0.0). From these figures it can be noticed that.

The main trend of H_{12}- variation along the plate may be devided into three distinct zones.

a- The pre-injection or the first zone. In this zone the shape factor (H_{12}) is slightly increasing. This means that the flow in the boundary layer is developing.

b- The injection or the second zone. The shape factor in this zone suddenly increases, reaching its maximum value some distance after the injection section and decreases till the end of the end of the mixing region.

c- The third zone. In this zone the effect of injection is completly finished. The shape factor begins to increase slightly down-stream. The variation of (H_{12}) in all the three regions is controlled by the behaviour of the characteristics of the boundary layer . At small injection ratiosm the rate of increase of (H_{12}) is higher than that at higher value of (λ). This is connected with the sudden increase of δ . Along the mixing region (H_{12}) is still varying. At the end of the mixing length the developing character is maintained.

3.2.2-6: Effect of (λ) on energy parameter (H_{23}):

Figure (3-11) shows the energy parameter (H_{23}) at different injection ratios at constant values of B as shown in figure (3-12) in case of tangential ($\overset{\circ}{\alpha}$ = 0.0). From these figures it can be observed that, the rate of increase of (H_{23}) is very small at all the values of λ . The energy factor is mainly affected by the energy losses due to friction. The friction losses will be discused in the following section.

3.2.2-7: Effect of λ on the Friction Coefficient cf:

Figure (3-13) shows the variation of (cf) along the flat-plate at differnt injection ratios λ at $\bar{B}$ $\simeq$ 0.5, 0.6, o.7 and 0.8 respectively and $\overset{\circ}{\alpha}$ = 0.0 From these figures it can be noticed that: In the pre-injection sections (cf) is decreasing, which is connected with the developing character of the boundary layer in this region. In the post injection section (cf) suddenly increases, which is connected with the increase of kinetic energy in the wall region due to injection. The decay of the injected jet along the mixing region leads to gradual decrease of (cf). Further increase of x , behined the mixing region leads to decrease of (cf) for the previously mentioned reason. It can be easly moticed that, an increase of leads to increase of (cf), in the mixing region.

3.2.2-8: Effect of λ on The Pressure Coefficient cp:

Figure (3-14) shows the variation of cp as a function of x at different (λ) at $\bar{B}$ = 0.5, 0.6, 0.7 and 0.8 respectively, in case of tongential α^{o} = 0.0.

From this figure it can be noticed that:

The main features of the curves differies according to either ($\lambda < 1.0$) or ($\lambda > 1.0$).

In case of $\lambda < 1.0$, cp decreases sharply from the pressure at the injection section to lower pressure and then to increase with decreasing rate till the end of the mixing region, after which it continue increasing. The case of $\lambda > 1.0$ differes in that the pressure in the injection section increases sharply to higher values; followed by a sharp fall in cp to minimum value, after which it gets the increasing character as in case of $\lambda < 1.0$. This behaviour of (cp) as a function of (x) is directly connected with the boundary conditions of the injected flow. The static pressure of the injected flow (Pj) was calculated from the injected mass flux A(x) and the velocity distribution by small perturbation method. This pressure (Pj) differs from the local boundary layer pressure. At $\lambda < 1.0$, P(J) is greater than the local pressure, mean while at $\lambda > 1.0$, P_j is smaller. The equilibrium requirement of the static pressure after the injection section leads to decrease of cp in case of $\lambda < 1.0$ and increase of cp in case of $\lambda < 1$. Since the mass flux of the boundary layer is in many times greater than that of the jet, the influence of the jet pressure can not be maintained longer and hence cp decreases some distance after the injection section till a minimum value after which it hegins to increase again. The increasing character of cp after the equilibrium zone is due to the friction losses. The minimum recorded (cp) is function of λ. Increase of (λ) leads to decrease of cp_{min}.

Conclusion:

The mathematical model for calculation of the fluid flow with injection in case of the steady isothermal laminar flow developing on a flat-plate enabled to study the effect of injection ratio, (λ , for constant Re_L) on the characteristics of the boundary layer. The following conclusion can be drawn from the previous descussions.

1- The injection ratio (λ), is the main parameter, affecting on the boundary layer characterisitics.
2- It was noticed that the behaviour of the boundary layer with injection ratio, in case of $\lambda < 1$, differs from that at $\lambda > 1.0$.
3- The momentum thickness (δ_2), increases with increasing the injection ratio for $\lambda < 1$, and decreases for $\lambda > 1$.
4- An increase of injection ratio (λ), for the case of $\lambda < 1$, leads to increase of momentum thickness (δ_2), energy thickness (δ_3), coefficient of friction (cf), and the coefficient of pressure (cp), while the displacement thickness (δ_1), decreases by increasing, λ . In case of $\lambda > 1$, the increase of λ, leads to decrease of δ_2, δ_3, cp, cf and δ_1.
5- The change in the shape factor (H_{12}), due to change of λ is significant, while the energy parameter (H_{23}) varies very slightly varies with λ . The change of whatλ from 0.4 to 1.2, leads to increase of the chape factor by about 5%, for B = 0.5, $Re = 10^5$ and $\alpha^{\circ} = 0.0$.
6- The mixing wone length increases by increasing the injection ratio (λ) as well as by increasing ($\bar{B}$) . The injection angle (α°) partically has no significant effect on the length of the mixing zone length.
7- The first maximum velocity (u_{max}), decays along the mixing region, in the same manner, with what?: distance from the wall which the maximum velocity exists is continuosly departing the wall. The minimum velocity (u_{min}) increases also along the mixing region and its location (y_{min})is departing the wall.
8- The friction coefficient (cf), increases with increasing the injection ratio (λ).

References:

1. SCHARZ, W.H.; COSART, W.P.: The two dimonsional turbulent wall jet. J. Fluid Mech., Vol. 10, (1962).
2. KRAUKA, V.; ESKINAZI, S.: The wall-jet in a moving stream. J. Fluid Mech., Vol. 20, (1964) PP. 555 - 579.

3. ESCUDIER, M.P., NICOLL, W.B.: The entrainnent function in turbulent boundary layer and wall-jet calculations. J.Fluid Mech., Vol.25, Part 2 ,(1966), PP. 337-366.

4. NEWMAN , B.G., IRWIN, A.H.: The prevention of seperation by blowing in two-dimensional flow. AGARD (cp-168, May 1975).

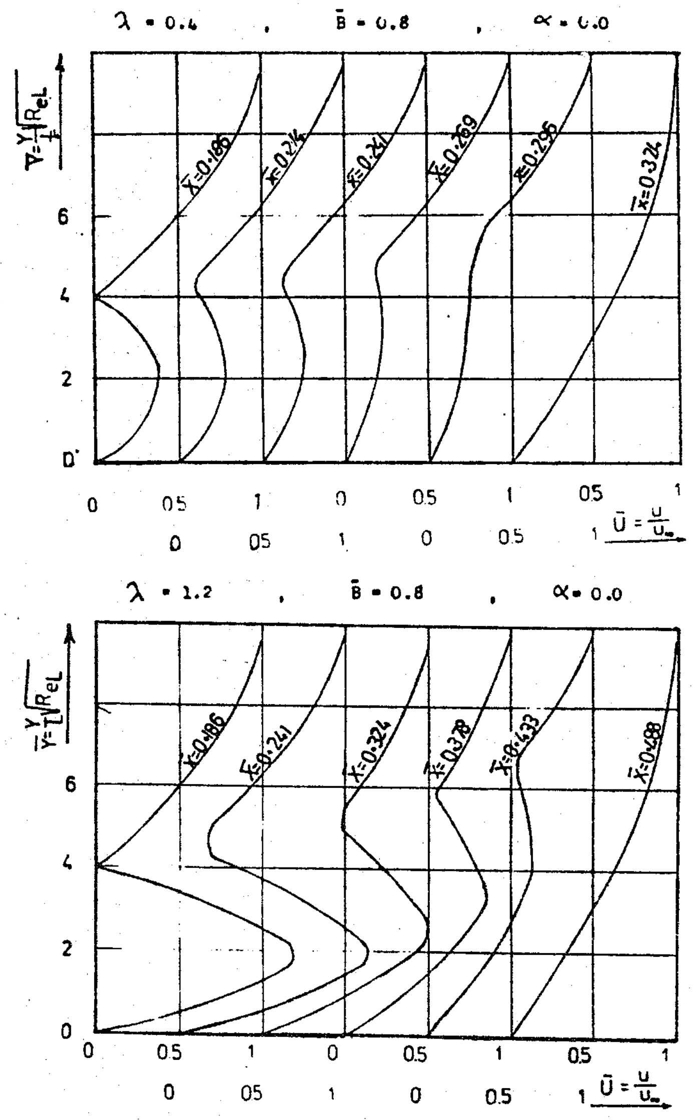
5. BRADSHAW, P.: Effect of external disturbences on the spreading rate of a plane turbulent jet. J. Fluid Mech., Vol. 80, Part 4, (1977), PP. 395 - 797.

6. LEE, L.H.; CLARK, J.A.: Angled injection of jets into a turbulent boundary layer, J. Fluid Eng., Vol. 102/211, June, (1980).

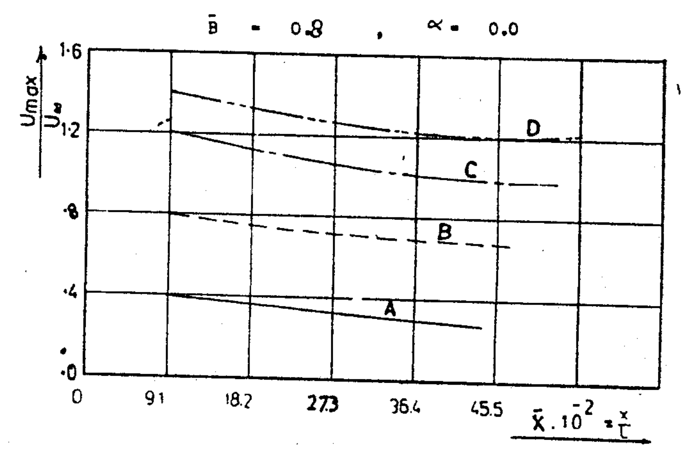
7. HANEL, D.: Laminar Grenzechichtstromung mit Anlosung. Aachen, 12; May (1981).

8. KRAUSE, E.; HANEL, D. HEWEDY, N.I.I.: Investigation of wall jets. AGARD - cp - 308,30-1, Lisbon, Partngal, 2-5 November (1981).

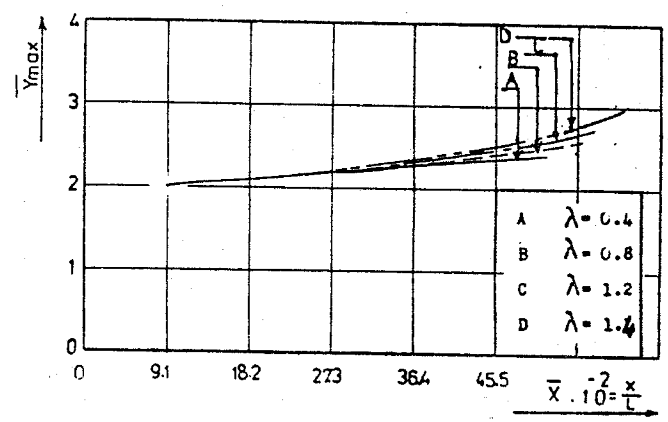
9. SCHLICHTING, H.: Boundary layer theory, MCGRAW - HILL, New-Youk (1968).



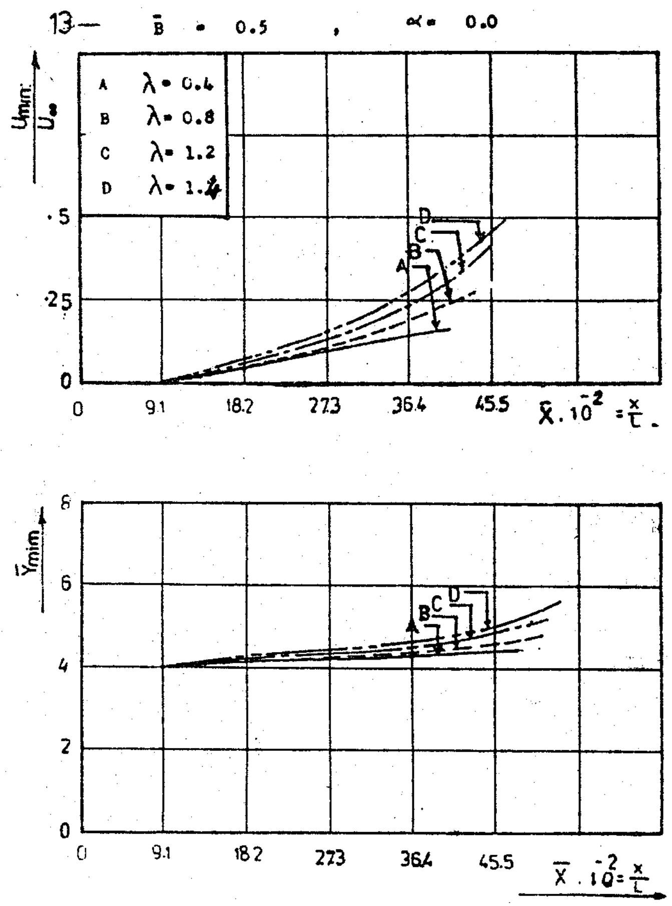
FIG(1): EFFECT OF INJECTION RATIO (λ) ON VELOCITY PROFILE.



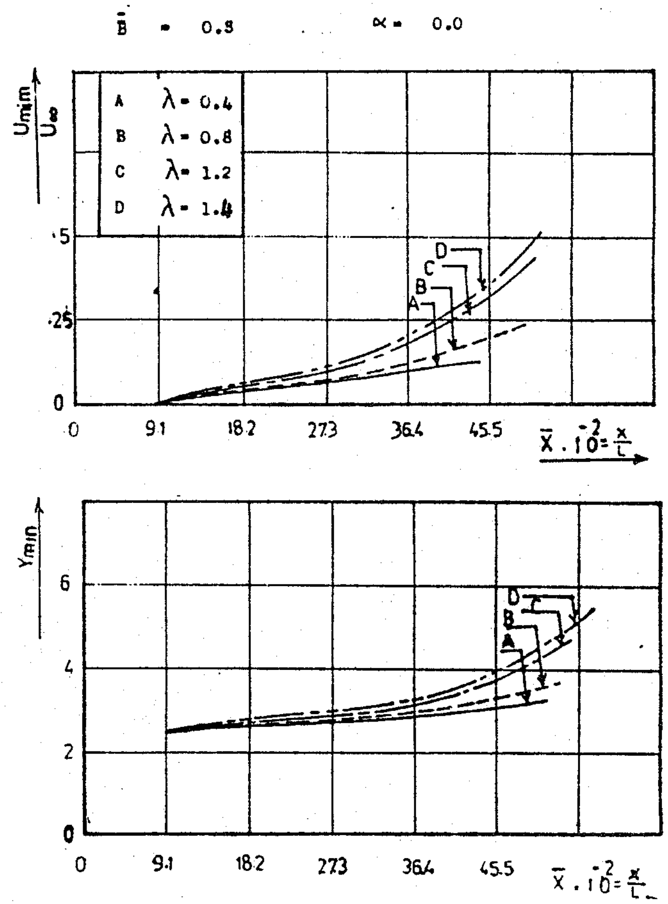
FIG(2): EFFECT OF (λ) ON DECAY MAXIMUM VELOCITY PROFILE.



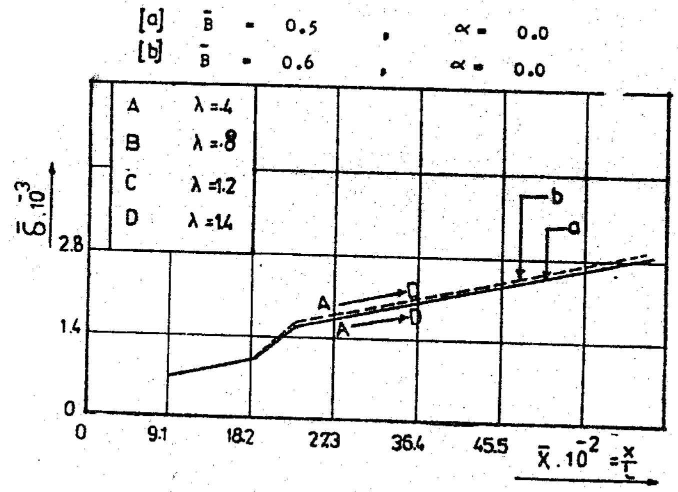
FIG(3): EFFECT OF (λ) ON LOCATION OF MAXIMUM VELOCITY PROFILE.



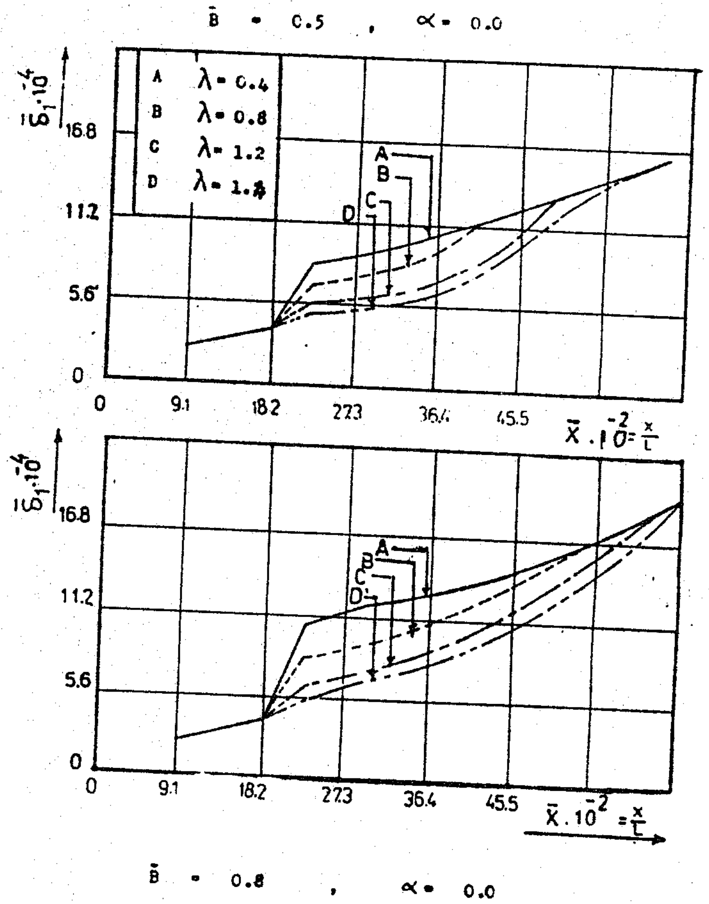
FIG(5): EFFECT OF (λ) ON-LOCATION OF MINIMUM VELOCITY PROFILE.



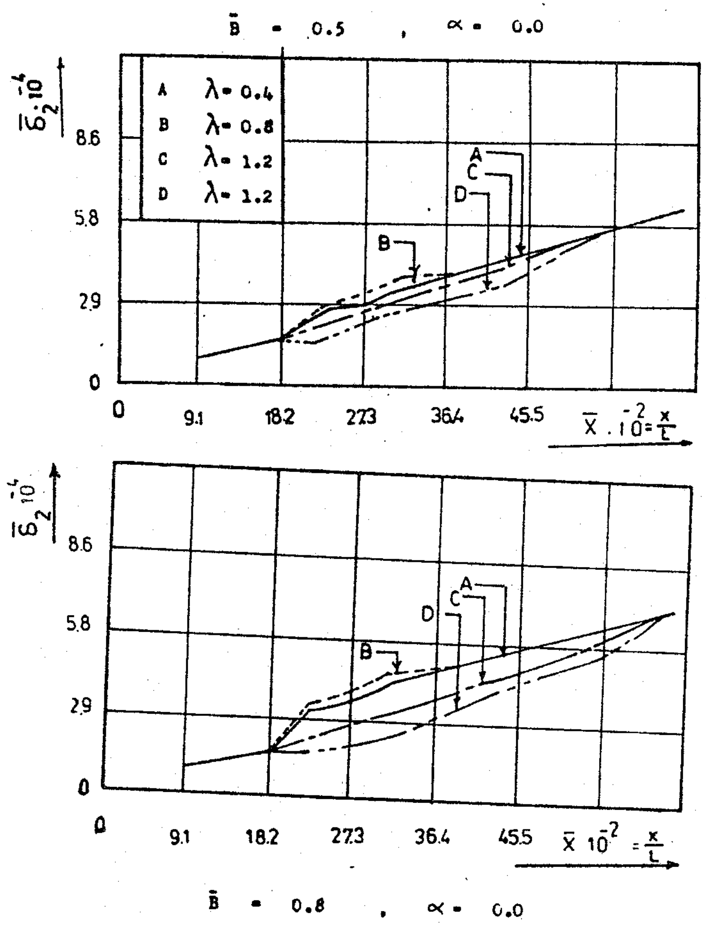
FIG(4): EFFECT OF (λ) ON-GROWTH MINIMUM VELOCITY PROFILE.



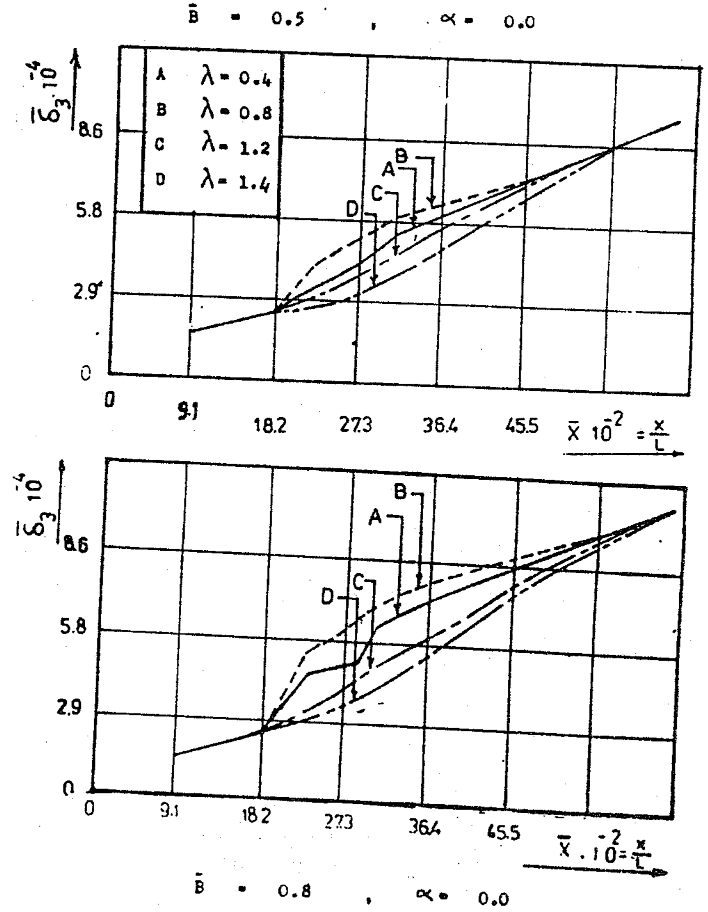
FIG(6) : EFFECT OF INJECTION RATIO (λ) ON GROWTH-BOUNDARY LAYER THICKNESS (δ).



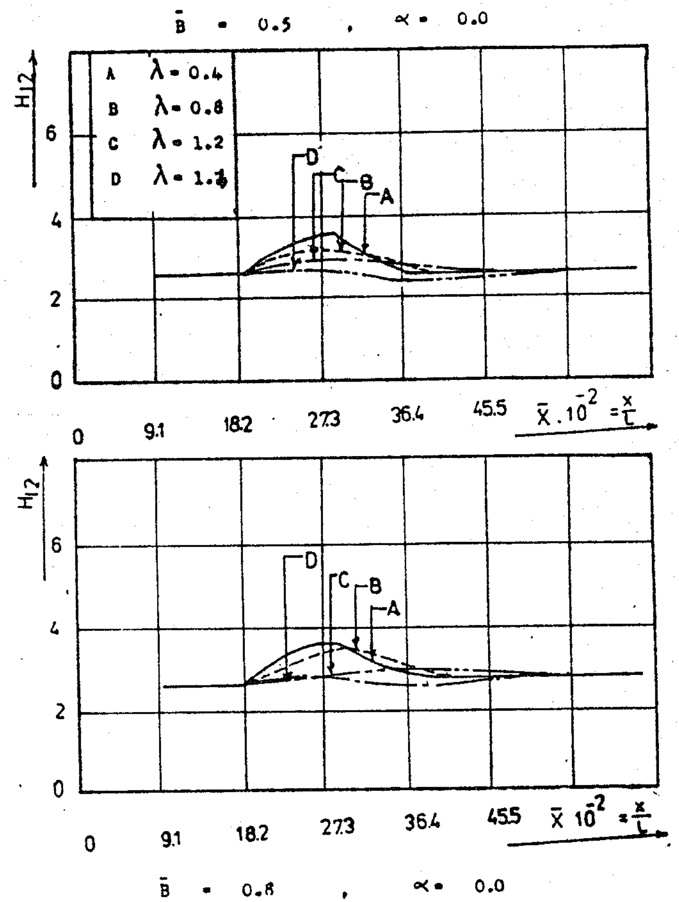
FIG(7): EFFECT OF INJECTION RATIO (λ) ON GROWTH OF DISPLACEMENT THICKNESS (δ_1).



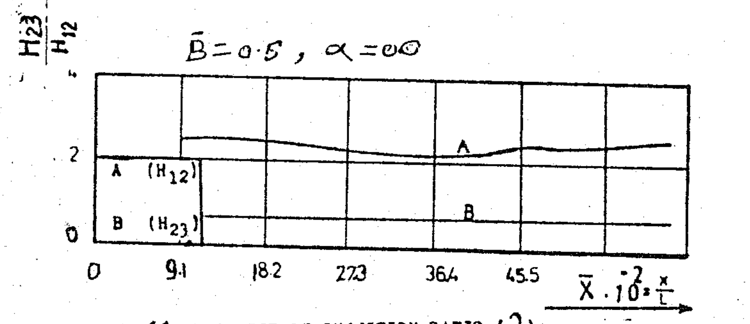
FIG(. 8 ,: EFFECT OF INJECTION RATIO (λ) ON GROWTH OF MOMENTUM THICKNESS (δ_2).



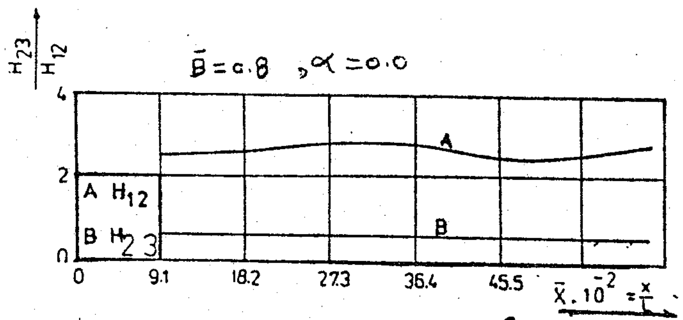
FIG(3- 9): EFFECT OF (λ) O N (δ_3)



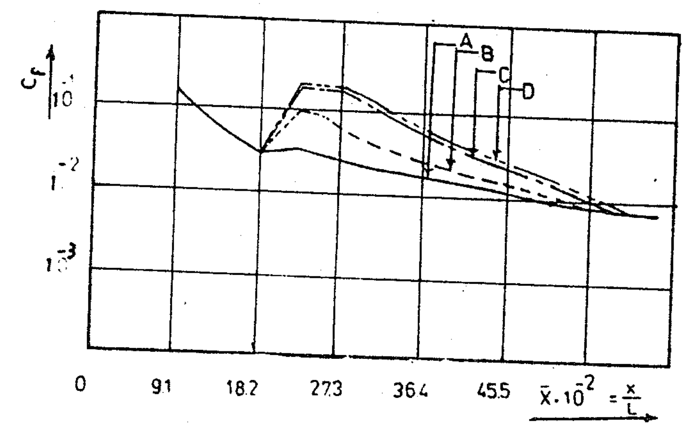
FIG(·10): EFFECT OF INJECTION RATIO (λ) ON SHAPE FACTOR PARAMETER (H_{12}).



FIG(·11): EFFECT OF INJECTION RATIO (λ) ON (H_{12}) AND (H_{23})

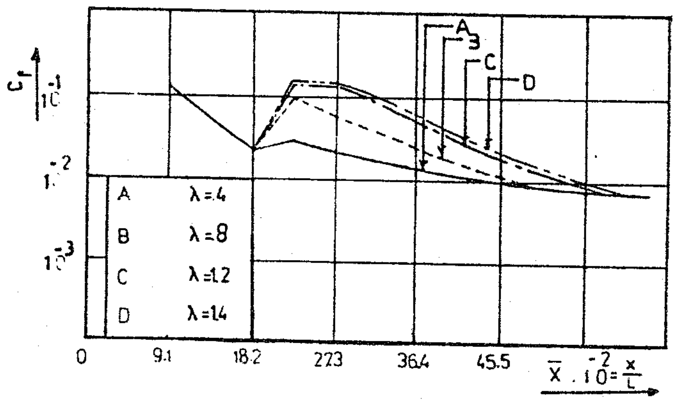


FIG(·12): EFFECT OF INJECTION RATIO (λ) ON (H_{12}) AND (H_{23})



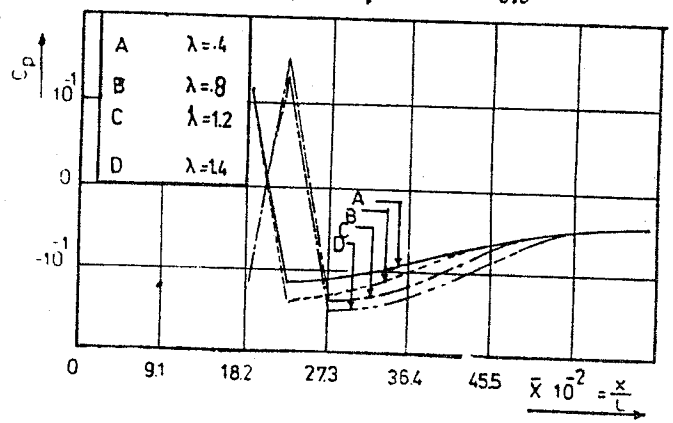
$\bar{B} = 0.8 \quad , \quad \alpha = 0.0$

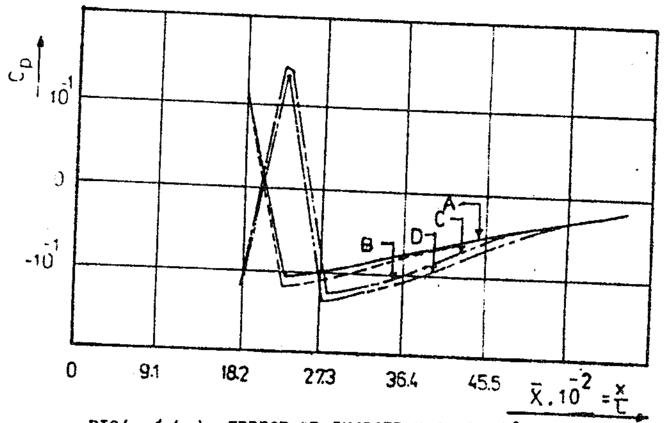
$\bar{B} = 0.7 \quad , \quad \alpha = 0.0$



FIG(13): EFFECT OF INJECTION RATIO (λ) ON SKIN FRICTION COEFFICIENT ($c_f = \frac{\tau_w}{\frac{1}{2}\rho u^2}$).

$\bar{B} = 0.5 \quad , \quad \alpha = 0.0$





FIG(14): EFFECT OF INJECTION RATIO (λ) ON PRESSURE COEFFICIENT ($c_p = \frac{p - p_\infty}{\frac{1}{2}\rho u_e^2}$).

$\bar{B} = 0.8 \quad , \quad \alpha = 0.0$