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ON THE DESIGN OF PARAMETRIC ACOUSTIC ARRAY: EFFECT OF THE PROJECTOR SIZE.

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ABSTRACT:

In a previous paper, the authors have presented a study and comparison between two models of the parametric acoustic arrays namely: the parametric array formed by collimated primary beams, suggested by Westervelt, and that formed by diverging primary sound beams as reported by Muir and Willette. A constant value for the radius 'a' of the projector was assumed throughout the mathematical analysis which indicated that the two models under investigation produce very close results at relatively high values of the mean frequency of the two beams 'f_e', whereas for smaller values the results obtained from the two models are significantly different. In this paper, a study of the effect of changing the dimension of the transducer is presented. It can be stated that, in general, a change in the overall results was observed. The value of 'f_e' at which the two models start approaching each other varies according to the value of 'a' as expected, and for the case where 'a' was relatively small some fluctuations in the results obtained from the second model were observed.

NOTATION:

a radius of the piston

$f_{1,2}$	primary beams frequencies
f_e	the mean value of primary frequencies
f_d	the difference frequency
f_{ec}	the critical mean frequency
α	signal attenuation coefficient
β	= 3.5, parameter of non linearity
$P_1 = P_2 = P_0$	peak pressure of the primary waves
P_d	peak of the difference frequency pressure
C_o	sound velocity
ρ_o	static density
ω	angular frequency
k	wave number
q	source strength density
r, R_o, r'	position vectors of the elemental volume, the [illegible] point, and the elemental volume relative field point, respectively.
r_o	$=3a^2/4\lambda_e$, nearfield parameter
δ, Ψ, θ	angular coordinates
Ψ_e	$= \tan^{-1}(a/r_o)$
S_o	projector area

1. INTRODUCTION:

Previously reported analysis by the authors[(1)] discusses two of the well known models for the parametric acoustic array. These models are:

(i) The parametric array formed by collimated primary beams.

(ii) The parametric array formed by diverging primary beams.

The former was originally suggested by westervelt[(2)] who assumed some postulates which facilitate the analysis. His model assumes two high frequency primary plane waves which are collimated in the region of interaction of the two beams. Spreading of the beams is completely neglected and the attenuation coefficient at the difference frequency is negligible.

Westervelt calculated the difference frequency pressure on and off axis of the piston.. The obtained equations are characterised by their simplicity and ease in use for design and analysis calculations.

The second model was reported in a paper by Muir and Willette[3] who extended the work of Westervelt by considering the primary beams to be spherically spreading as shown in Fig. (1) and solved numerically the original integral equation of Westervelt without any approximations. They only assumed that the second order components of the pressure propagate in the medium in a linear manner and their distribution can be predicted by a quasi-linear theory.

The difference frequency pressure obtained is somewhat given by a complicated tripple integral expression which can be reduced to a double integral expression when we consider the on-axis pressure.

The comparison between the two models reported in the previous paper was based on assuming a circular piston of radius a = 0.0381 m (same as reported in reference 3). The value of the absorption coefficients for the difference frequency components were assumed to vary with the square of the corresponding value of the frequencies[4] ($\alpha = 2.5 . 10^{-14} f^2$). The comparison included calculations of the on-axis pressure as well as the 3dB beamwidth as function of the mean frequency of the primary waves 'f_e' keeping the difference frequency 'f_d' and the range R_o constants. A numerical solution was employed to solve the complicated expression describing the difference frequency pressure of the second model. The results obtained there showed clearly that the two models approach each other at a certain value of 'f_e' and that they become closer and closer as 'f_e' increases. However, below the specified (threshold) or (critical) value of 'f_e'

at which the two curves of P_d, obtained from the two models approach each other, the discrepancy between the two models increases. This suggests that when designing (or analysing) parametric array one can determine which of the two models can be used, depending on the operating frequencies and on the size of the transduoer.

It was therefore convenient to extend this work to study the effect of changing the radius of the projector and to recalculate the on-axis difference frequency pressure and notice the change in the pattern of the results which can be obtained.

2. THEORETICAL:

In Westervelts model[2] the axial difference frequency pressure is given by:

$$P_d\ (R_o, 0) = \frac{\omega_d^2\ \rho_o^{-1}\ C_o^{-4} \beta\ \ P_o^2\ S_o}{8\ R_o\ \pi\ \alpha_e} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The directivity function is given by[2]:

$$D(\theta) = 1/(1+(k_d/\ \alpha_e)^2 .\ \sin^4\theta/2)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Many authors[3-10] modified Westervelt's results by taking into account the effect that arises from the spreading of the primary beams to form several models.

The generality of Muir and Willette model[3] makes it quite preferable to represent a useful discussion for the diverging beams case. The equation of difference frequency pressure is of the form,

$$P(R,0) = \frac{2P_1P_2\ r_o^2\omega_d^2\beta}{\rho_o C_o^2 k_1 k_2\ a^2} \int_{r_o}^{R_o}\int_0^{\Phi_e} \frac{J_1(z_1)J_1(z_2)}{z} \frac{\text{EXP} - [(\chi_1+\chi_2)-jK_d]\ r-(jK_d-\alpha_d)r'\}}{r'}\, d\Phi\, dr \quad \ldots (3)$$

where,

$z_1 = k_1\ a \sin\Phi$

$z_2 = k_2\ a \sin\Phi$

$z = \sin\Phi$

for the other parameters see Fig. (1).

3. RESULTS AND DISCUSSION:

The results obtained in this case indicate, that the two curves obtained using Eqs. (1) and (3) behave in the same manner as was previously observed in reference (1). For the sake of comparison, Fig. (2) of ref. (1) is replotted where 'a' was taken to be equal to 0.0381 m. From this Figure it can be shown that the value of 'f_e' at which the difference between the two curves is 1.9 dB is about 1MHz. As mentioned above, the two curves approach each other assymptotically as 'f_e' increases, and give more or less the same values at large value of 'f_e' whereas the difference between the two curves increases significantly as 'f_e' is decreased. Let us denote the value of 'f_e' at which the difference is 1.9 dB by "f_{ec}".

If this criterion is used for the other results at the different values of (a) it can be said that as (a) increases "f_{ec}" decreases and vice versa, which is an expected results, since as the size of the transducer increases the beams beams become more and more collimated and therefore the two

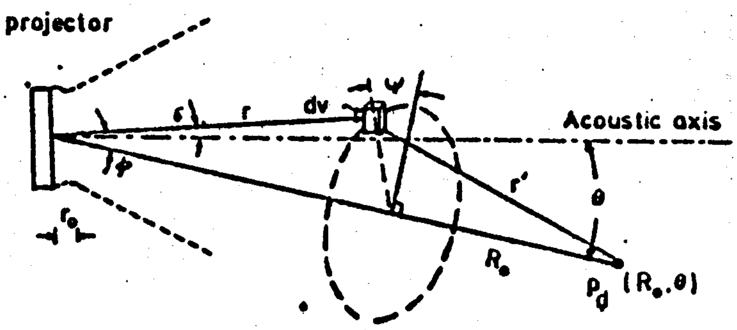

Fig.1-Geometry.

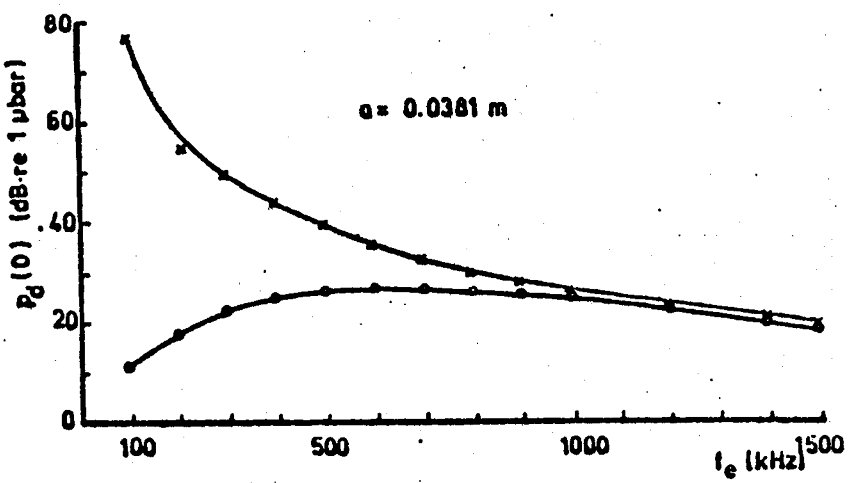

Fig. 2 - Dependence of p_d (0) on the mean value of the primary frequencies.
xx case of Westervelt
•• case of divergence.
f_d = 30 kHz, R_o = 48.5 m

models approach one another at relatively, low values of 'f_e' this is clear from Fig. (3.g).

However as "a" takes small values the value of "f_{ec}", defined in the same manner as above, is increased. This is shown in Figs. (3.a-f).

To summarize the results obtained in these figures, a plot of "f_{ec}" against "a" is shown in Fig. (4).

One remaining point to discuss is the observed fluctuations in the values of P_d at low values of 'f_e' especially for small values of "a". The proposed reason for these is the nature and behaviour of the Bessel function which appear in the expression, at small arguments.

4. CONCLUSIONS:

The foregoing analysis shows clearly that the size of the projector affects the agreement of the two models (Westervelt's model and the general model). For relatively small projector size the frequency at which the two models approach each other "f_{ec}" is consequently large. However, as the projector size increases "f_{ec}" decreases rapidly at first and then approaches slowly a constant value.

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Figs.(3.a,b,c,d,e,f,g) illustrate the variation of $p_d(0)$ with f_e for different values of "a"

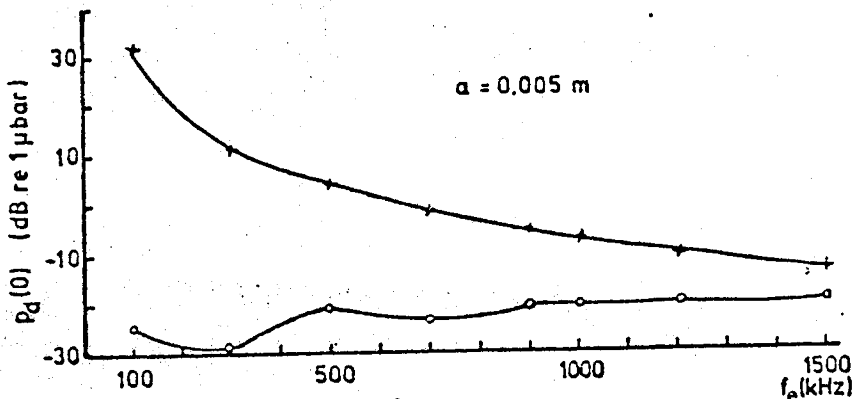

Fig. 3-a

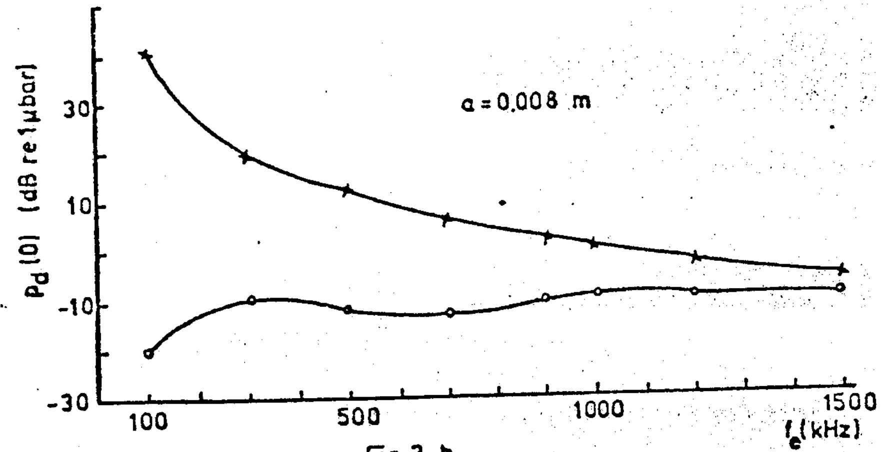

Fig. 3-b

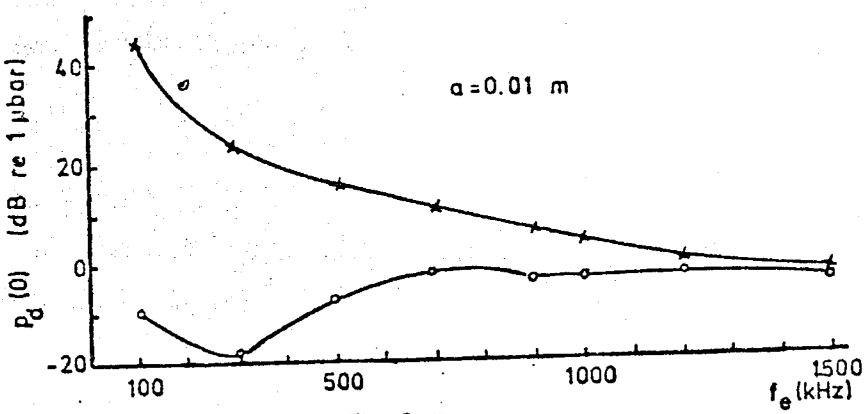

Fig. 3-c

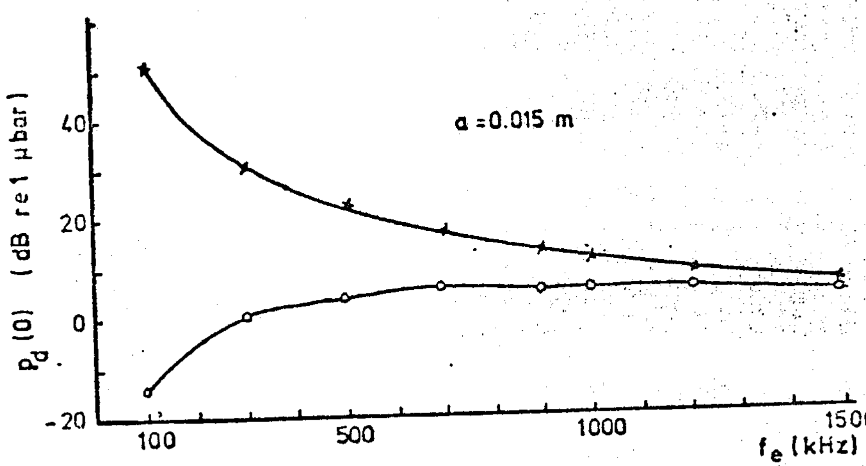

Fig. 3-d

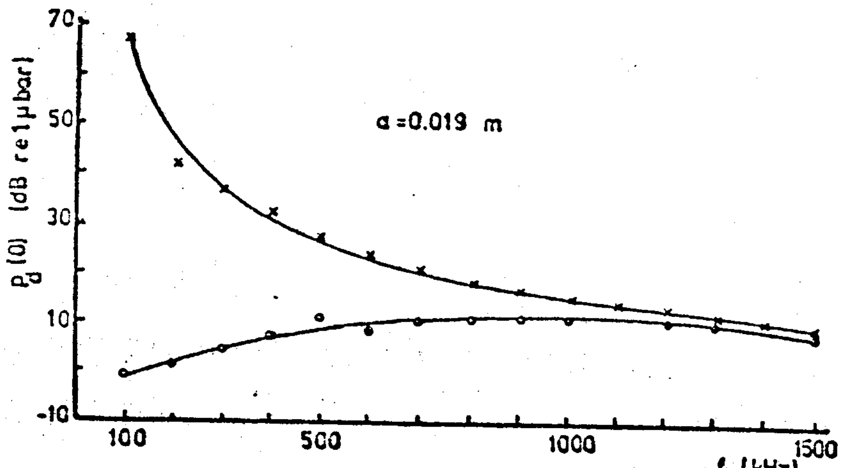

Fig. 3-e

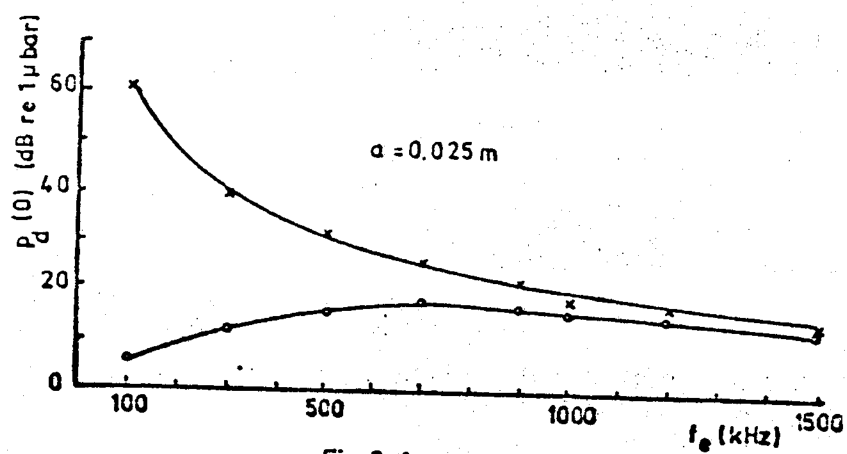

Fig. 3-f

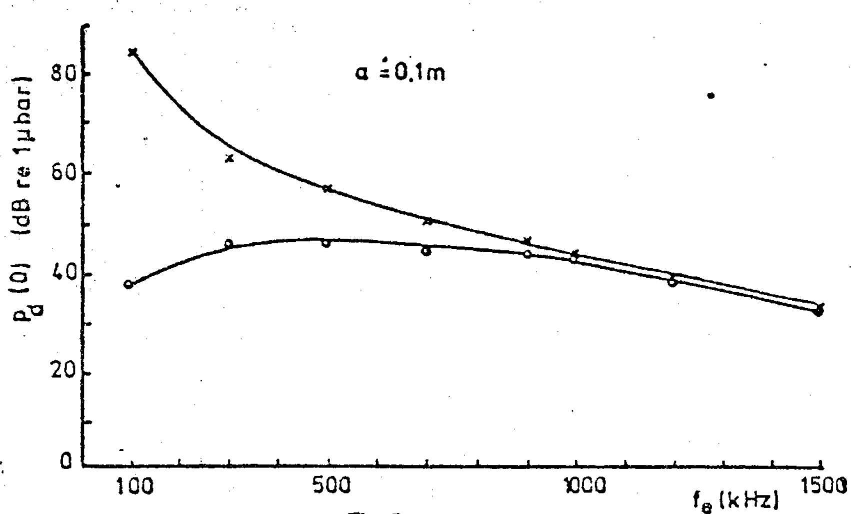

Fig. 3-g

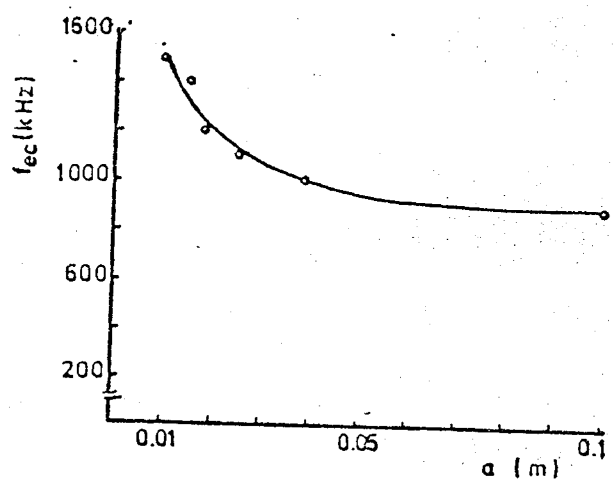

Fig. 4- Dependence of f_{ec} on the value of the projector radius.

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