رياضة (٣) وجه واحد — قسم الكهرباء — د. إبراهيم العوضي — د. محمد حطل

Mansoura University	**First Semester**
Faculty of Engineering	**25-12-2010**
Department of Engg. Math. and Phys.	**Time: 3 hr**
First year — **Math(3)**	**Full mark(110)**

[1]-(a)[8 pts] Find Laplace transform of the following functions

$$f(t) = e^{3t} \sinh 4t, \qquad h(t) = \int_0^t (t-u)\, e^{3u} du.$$

(b) [4 pts] If $L\{f(t)\} = F(s)$, prove that

$$L\{f'''(t)\} = S^3 F(s) - \sum_{k=0}^{2} S^k f^{(2-k)}(0).$$

(c)[8 pts] Find the inverse laplace transform of the following functions

$$F_1(s) = \frac{s}{s^2 + 13s + 9}, \qquad F_2(s) = \frac{s\, e^{-\pi s}}{s^2 + 4}.$$

(d) [10 pts] Use the method of Laplace transforms to solve the given boundary value problem

$$y''(t) + 2y'(t) + y(t) = f(t), \qquad x(0) = 0, \qquad x'(0) = 0.$$

[2]-(a) [6 pts] Set up the appropriate form of a particular solution y_p (**Undetermined Coefficients**), but **DO NOT** determine the values of the coefficients

$$y'' + 2y' - 3y = f(x)$$

where 1. $f(x) = (5+x)\, e^{-3x} + 333\, e^x$, 2. $f(x) = (\sin 3x)^{-1}$

(c) [4 pts] Write a general solution for the homogeneous differential equation with constant coefficients whose auxiliary equations is

$$(r+5)^3 (r-2)^2 \left(r^2+1\right)^2 = 0.$$

(d) [15 pts] Solve by any method

1. $y' = \dfrac{y^2 x^2}{1+x}$
2. $x y' = -y + \sqrt{x y + 1}$
3. $y''' + 3y'' - y' - 3y = 0$

3. Given the function $\varphi(x, y) = e^{x^2 y}$ and the point $p = (2,0)$.

(a) [5 pts] Show that:

$$x\varphi_x + y\varphi_y = 3x^2 y\varphi,$$
$$x^2\varphi_{xx} + 2xy\varphi_{xy} + y^2\varphi_{yy} = 3x^2 y(3x^2 y + 2)\varphi \ .$$

(b) [5 pts] Expand $\varphi(x, y)$ in a Taylor series about the point p.

(c) [5 pts] Use the results obtained in part (b) to get the following at the point p:

- **i.** The equation of the normal line to $\varphi = constant$.
- **ii.** The equation of the Tangent plane to $\varphi = constant$.
- **iii.** The gradient of φ.
- **iv.** The maximum rate of change and its direction.
- **v.** $\frac{dy}{dx}$ by applying the implicit differentiation rule to $\varphi(x, y) = 1$.

(d) [5 pts] Let $x = t$, $y = s^2 - t$. Applying the chain rule to evaluate φ_s, φ_t and φ_{st}. Use your results to show that $(2s^2 - 3t)\varphi_s = 2st\varphi_t$.

(e) [5 pts] Find the extreme values of φ on the region $R: x^2 + \frac{y^2}{3} \leq 1$.

(f) [5 pts] Evaluate $\int_0^\infty \int_{1/y}^{2/y} \frac{x^2}{\varphi(x,y)} dxdy$.

(g) [5 pts] Given that $I = \int_1^\infty \frac{e^{-x^2}}{x} dy = c$, where c is known constant.
Find $I = \int_{\frac{1}{\sqrt{y}}}^{\infty} \frac{1}{x\varphi(x,y)} dx$.

4. Consider the vector field $F = yzi - xzj + 3z^2 k$ and the volume R: $z \leq 5 - x^2 - y^2$, $z \geq 1$.

(a) [5 pts] Verify Green's theorem for the vector field F and the lower surface of R.

(b) [10 pts] Verify Stokes' theorem for the vector field F and the upper surface of R.

(c) [10 pts] Verify Gauss' theorem for the vector field F and the volume R.

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