جامعة المنصورة
كلية الهندسة (الفرقة الثانية قسم هندسة الإنتاج والتصميم حديث + قديم)
قسم الرياضيات والفيزياء الهندسية

التاريخ: ٢٠٠٩/٥/١٦
المادة: رياضيات ٥
الزمن: ٣ ساعات

الورقة الأولى : التحليل العددي [الأسئلة (1)، (2)، (5)، (6)] تحل في صفحات متتالية من الجهة اليسرى من كراس الإجابة

أجب على جميع الأسئلة

1] (a) Derive the convergence condition of the Newton-Raphson method to solve the equation $f(x)=0$.

(b) Use the Newton-Raphson method to find the smallest real root of the equation:

$$x^3+3x^2-16x+2=0$$

Correct to 4 decimal places.

2] (a) What is the condition for which the following system of equations has no solution

$$\begin{bmatrix} d-4 & d-3 & d-2 \\ d-2 & d+1 & d-1 \\ d-4 & d-3 & d-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -5 \\ 3 \\ -1 \end{bmatrix}$$

(b) Solve the following system of equations using the SOR method.

$$\begin{aligned} 10x_1 + 2x_2 + 2x_3 &= -10 \\ 4x_1 + 10x_2 - 5x_3 &= 5 \\ -2x_1 + 2x_2 - 5x_3 &= -8 \end{aligned}$$

Correct to 3 decimal places with $\omega=1.25$.

5] From the following readings:

x	0.4	0.6	0.8	1.0
y	0.3799	0.5370	0.6640	0.7616

(a) find $y(0.77)$ using Stirling formula.

(b) find $y(0.7)$ using Bessel's formula.

6] Solve the following initial value problem by the use of Runge-Kutta method of order 4,

$$y'=4x-2y,\ 0\le x\le 0.6,\ y(0)=2 \text{ with } h=0.2$$

second Part: Statistics

☒ من فضلك إبدأ إجابة هذا الجزء من الجهة الأخرى لورقة الإجابة

(3) a) Stores A, B and C have 75, 50, 100 employees and respectively 60, 50, 70 percent of these are women. One employee resigns. **(a)** What is the probability that this employee is a woman. **(b)** If this employee is a woman, What is the probability that she works in store C.

b) A committee of 5 persons is to be selected randomly from a group of 10 men and 5 women.

(i) Find the probability that the committee consists of all women.

(ii) Find the probability that the committee consists of 2 men and 3 women.

(iii) Find the probability that the committee consists of at most one men.

c) A box contains 6 white balls, 7 red balls, 4 blue balls and 8 green balls. One ball is drawn at random from the box and its color is observed then the ball is replaced. The process is repeated 12 times independently. Find the probability of obtaining:

(i) 4 white, 3 red, 2 blue and 3 green balls. **(ii)** 5 red balls.

(4) a) If the probability density function of a continuous random variable is given by

$$f(x)=\begin{cases} e^{2x} & x \le 0 \\ k\,x & 0 < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find: **(i)** the value of k, **(ii)** the cumulative distribution function,
(iii) $P(-0.5 < x < 0.5)$.

b) Prove that, if the random variable X has an exponential distribution with parameter λ, then $\mu = E(X) = \frac{1}{\lambda}$ and $\sigma^2 = V(X) = \frac{1}{\lambda^2}$.

c) The strengths of individual bars made by a certain manufacturing process are approximately normally distributed with mean 28.26 and standard deviation 3.0 units. To ensure safety, a customer requires at least 95 % of the bars to be stronger than 24.

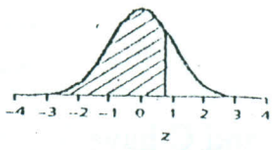
(i) Do the bars meet the specification?

(ii) By improved manufacturing techniques the manufacturer can decrease the standard deviation. What value of the standard deviation will just meet the specification if the mean stays the same?

With all best wishes
Dr.\ Waleed El-Beshbeeshy

Cumulative Normal Probability

$\Phi(z) = \Pr[Z < z]$



$\Delta z =$ / z_0	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	/ z_0
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.0
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.1
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.2
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.3
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.4
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.5
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.6
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.9
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	1.0
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	1.1
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	1.2
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	1.3
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1.4
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1.5
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1.6
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1.7
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1.8
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1.9
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	2.0

$\Delta z =$ / z_0	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	/ z_0
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	2.0
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	2.1
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989	2.2
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	2.3
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	2.4
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	2.5
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	2.6
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	2.7
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	2.8
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	2.9
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	3.0
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	3.1
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	3.2
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	3.3
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	3.4
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	3.5
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.6
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.7
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.8

Normal Distribution Table:

$\Delta z =$ / z_0	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z_0
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.0
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.1
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.2
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1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1.4
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2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	2.0

t- Distribution Table:

F(t)	0.9	0.95	0.975	0.99	0.995	0.999	
df							df
17	1.333	1.740	2.110	2.567	2.898	3.646	17
18	1.330	1.734	2.101	2.552	2.878	3.610	18
19	1.328	1.729	2.093	2.539	2.861	3.579	19
20	1.325	1.725	2.086	2.528	2.845	3.552	20
21	1.323	1.721	2.080	2.518	2.831	3.527	21
22	1.321	1.717	2.074	2.508	2.819	3.505	22
23	1.319	1.714	2.069	2.500	2.807	3.485	23
24	1.318	1.711	2.064	2.492	2.797	3.467	24
25	1.316	1.708	2.060	2.485	2.787	3.450	25
26	1.315	1.706	2.056	2.479	2.779	3.435	26

Part(II): Probability

Question (3):

(a) Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

(b) Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 70% are cotton and 30% are nylon.
(i) What is the probability that a randomly selected roll used contains flaws.
(ii) If the selected roll contains flaws, what is the probability that it is nylon fabric.

(c) A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.)
(i) What is the probability of getting an even number in one throw?
(ii) What is the probability that the third six will occur at the 18^{th} roll of this die?
(iii) What is the probability that the first six will occur at the 10^{th} roll of this die?

(d) The joint probability mass function of X and Y is given by:

$$p(x_i, y_j) = \begin{cases} k\left(x_i^2 + y_j\right), & x_i = 1,2 \; ; \; y_j = 0,1,2 \\ 0 & otherwise \end{cases}$$

(i) Find the value of constant k. **(ii)** Evaluate the correlation coefficient of X and Y.

Question (4):

(a) State and prove Markov's inequality.

(b) The lifetime of a special type of battery is a exponential distribution with $\lambda = 0.025$.
(i) What is the probability that the lifetime will be more 50 hours.
(ii) A battery is used until it fails and then replaced by a new one. A stock of 25 independent batteries, approximate the probabilities that over 1100 hours of use can be obtained.
(iii) Find the probability that a random sample of 100 batteries chosen will have an average lifetime between 35 and 44 hours.

(c) A material engineering test describes the results of tensile adhesion on 20 alloy specimens. The load at specimen failure (in megapascals) in that sample has a mean of 13.7 and Standard deviation of 3.5. Find a 95% confidence interval on the mean.

(d) A machine has produced ball bearings with a mean diameter of 15.00 mm and a standard deviation of 0.6 mm. To determine whether the process is in control a sample of 40 bearings was checked and their mean was found to be 15.12 mm. Is the process in control under 5% level of significance.

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(c) Apply the finite difference method to solve Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ at the interior points of the square $0 \le x \le 4, \quad 0 \le y \le 4$. Take the mesh sizes $h = k = 1$ and the unknown u is given to be $\left(x^3+2y^2\right)$ on the boundaries. Use the relaxation method to solve the resulting system of algebraic equations with the initial values

$$u_{11}^{(0)} = 10, \quad u_{21}^{(0)} = 20, \quad u_{31}^{(0)} = 30,$$

$$u_{12}^{(0)} = 20, \quad u_{22}^{(0)} = 30, \quad u_{32}^{(0)} = 40,$$

$$u_{13}^{(0)} = 30, \quad u_{23}^{(0)} = 40, \quad u_{33}^{(0)} = 50.$$