

SOS AND HOS PERFORMANCE FOR PROCESSING SATELLITE RELAYED SIGNAL

أداء الطرق الاحصائية من الدرجة الثانية و الدرجات الأعلى في معالجة الإشارات المرتجعة من الأقمار الصناعية

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خلاصة

يقدم البحث تقنيتين لمعالجة اشارات الأقمار الصناعية المرتجعة. و تعتمد الطريقتان على الطرق الاحصائية من الدرجة الثانية (SOS) و الدرجات الأعلى (HOS) باستخدام تقنية التقدير الطيفي البارامتري. و بتطبيق هذه الطرق يمكن ايجاد الضوضاء في القدرة و كشف و تمييز الخواص الرئيسية للإشارات المرسله. و يؤدي استخدام هذه الطرق الى تقليل زمن المعالجة مع جعل الخطأ الترددي أقل ما يمكن

ABSTRACT

Signal detection in the presence of additive Gaussian noise can be performed using higher order statistics. This paper deals with a processing of single and multiple emergency signals that relayed from the low polar orbit satellite using parametric spectral estimation techniques with Second and Higher Order Statistics. The detection algorithm based on the Second Order Statistics (SOS) using the parametric techniques, which are the Maximum Entropy Method (MEM), and MULTiple Signal Classification (MUSIC) is designed and examined in this paper. Simulation results illustrate successful performance of signal detection at low signal to noise ratio. These proposed methods suppress additive Gaussian noise of unknown power spectrum, detect and characterize the main properties of the transmitted signals.

I- INTRODUCTION

During recent years there has been an increasing interest in applying higher order statistics to a wide range of signal processing and system theory problems. These higher order statistics are very useful in problems where non-Gaussianity, non-minimum phase, colored noise, or where nonlinearities are important and must be accounted for [1,2]. These statistics known as cumulants, and their associated Fourier transforms known as polyspectra, not only reveal amplitude information about a process, but also reveal phase information. This is important because, as is well known, second order statistics (i.e. correlation) are phase blind. Higher order spectra (also known as polyspectra) are defined in terms of higher statistics ("cumulants") of a signal do contain such information. Particular cases of higher order spectra are the third order spectrum, also called the bispectrum and the fourth order spectrum, trispectrum. The power spectrum is, in fact a member of the class of higher order spectra, it is a second order spectrum. The main application examined in this paper is nonlinear spectral estimation with higher order spectra [1,2].

II- SECOND and HIGHER ORDER STATISTICS

In this section we describe the processing of the input signal with second order statistics (SOS) and higher order statistics (HOS). The SOS method is based on performing the autocorrelation (ACF) function of the input signal then estimating the power spectral density using the parametric spectral estimation techniques (MEM or MUSIC). The ACF provides a time-domain description of the second order statistics. If the additive noise and signal are uncorrelated, the autocorrelation of the signal in the presence of noise is the individual sum of the ACF of the signal and the ACF of the noise. The ACF tends to cancel the noise component. The second order statistics of a signal extracts useful information from the original signal such that the ACF improves the data adaptive processing methods such as (MEM & MUSIC).

Higher order statistics known as cumulants [1,2], and their associated Fourier Transform known as polyspectra, not only reveal amplitude information about a process, but also reveal phase information. This is important because as it is well known second order statistics (i.e. autocorrelation) are phase blind [1,2]. Cumulants on the other hand are blind to any kind of a Gaussian process, whereas correlation is not. Consequently, cumulant based methods boost signal-to-noise ratio when signals are corrupted by Gaussian noise. Higher order statistics are applicable when we are dealing with non-Gaussian (or, possibly, nonlinear) processes.

The K^{th} order cumulants is defined in terms of the signal joint moment of orders up to K . The second and the third order cumulants of zero-mean signal $x(t)$, are given by [1,2]:

$$C_{2,x}(\tau) = E\{x(t)x(t+\tau)\} \quad (1)$$

$$C_{3,x}(\tau_1, \tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\} \quad (2)$$

Similarly we can define the fourth order cumulants $C_{4,x}(\tau_1, \tau_2, \tau_3)$ [1,2]. Sometimes we need fourth order cumulants because if a random process is symmetrically distributed, then its third-order cumulants is equal zero, hence, for such a process we must use fourth order cumulants. In addition, in some specific applications [1,2]. As indicated from equation (1) it is seen that the second order cumulants $C_{2,x}(\tau)$ is just the autocorrelation of $x(t)$.

This paper deals with processing of single and multiple emergency signals relayed from the low polar orbit satellite using second order statistics (SOS) or second order cumulants $C_{2,x}(\tau)$ which used as a preprocessing method then applying the (MEM or MUSIC) spectral estimation techniques to the data.

III- PARAMETRIC SPECTRAL ESTIMATION TECHNIQUES

The accuracy of the estimation of the input signal is largely dependent on the method employed for processing the input signals. In the following subsections, nonlinear spectral estimation techniques examined in this paper are reviewed [3-7].

III.1 Maximum Entropy Method (MEM)

The Maximum Entropy Method (MEM) is referred to as an autoregressive (AR) or an all pole spectral estimator. The problem can be mathematically expressed as finding the real positive value of the spectral estimate $S(f)$ which maximizes the entropy of the corresponding random process. The power spectral density $S(f)$ calculated by [3-6] is:

$$S(f) = \frac{P(M)}{2B \left[1 + \sum_{m=1}^M a(M, m) e^{-j2\pi m f T_s} \right]^2} \tag{3}$$

Where:

B is the bandwidth of the signal, T_s is the sampling time, M is the order of Prediction Error Filter (PEF), $P(M)$ is output power of PEF, and $\{a(M, m)\}$, $m=0, 1, 2, \dots, M$ represents the PEF coefficients.

There are three main important parameters affecting the performance of an MEM spectral estimator, these are:

- 1 - Number of input samples available for each instantaneous spectral estimate.
- 2 - M : order of Prediction Error Filter.
- 3 - L : number of instantaneous spectral estimate averaged.

The detailed analysis about this algorithm is in [3-6]

III.2- Multiple Signal Classification (MUSIC)

The MUSIC estimate is given by the formula [1, 7]

$$P_{MUSIC}(f) = \frac{1}{e^H(f) \left[\sum_{k=p+1}^N v_k v_k^H \right] e(f)} = \frac{1}{\sum_{k=p+1}^N |v_k^H e(f)|^2} \tag{4}$$

Where N is the dimension of the eigenvectors and v_k is the k -th eigenvector of the correlation matrix of the input signal. The integer p is the dimension of the signal subspace (subspace size), The prefix H means hermitian (complex conjugate transpose) so the eigenvectors v_k used in the sum correspond to the smallest eigenvalues and also span the noise subspace. The vector $e(f)$ consists of complex exponentials, so the inner product $v_k^H e(f)$ amounts to a Fourier transform. The second form is preferred for computation because the FFT is computed for each v_k and then the squared magnitudes are summed.

In the eigenvector method, the summation is weighted by the eigenvalues λ_k of the correlation matrix [1]:

$$P_{ef}(f) = \frac{1}{\sum_{k=p+1}^N |v_k^H e(f)|^2 / \lambda_k} \tag{5}$$

The function relies on the SVD matrix decomposition in the signal case, and it uses the eigen analysis for analyzing the correlation matrix. If SVD is used, the correlation matrix is never explicitly computed, but the singular values are the λ_k for further details see [1]

III.3- Mathematical analysis Of SNR improvement using SOS

The SOS can help in canceling noise from signals as follows:-

Consider $y(n)$ to be a noisy observed complex exponential signal (as Fourier series suggests any signal to be), that is given by [2, 8, 9]:

$$y(n) = s(n) + v(n) \tag{6}$$

$$y(n) = \sum_{i=1}^L a_i e^{j\omega_i n + j\phi_i} + v(n)$$

Where $v(n)$ is white Gaussian noise.

The local SNR, is given by $SNR_i = 10 \log(|a_i|^2 / \sigma_v^2)$

The second order cumulant is given as follows:

$$C_{2,y} = C_{2,s} + C_{2,v} \quad \text{Where:}$$

$$C_{2s}(\tau) = E[s(n)s(n+\tau)]$$

$$= \sum_{i=1}^L |a_i|^2 e^{j\omega_i \tau}$$

$$C_{2v}(\tau) = \sigma_v^2 \delta(\tau)$$

Where σ_v^2 is the noise variance. The new SNR (NSNR) is given by:

$$\begin{aligned} NSNR_i &= 10 \log(|a_i|^4 / \sigma_v^2) \\ &= 10 \log(|a_i|^2) + 10 \log(|a_i|^2 / \sigma_v^2) \\ &= 10 \log(|a_i|^2) + SNR_i \end{aligned} \quad (7)$$

Thus, from Eq.(7) it is clear that applying SOS to the noisy signal enhances its SNR.

III.4- Proposed Method

This paper suggests two techniques. In this section we are going to summarize these two methods briefly as follows:

1- The first proposed method

The first proposed one is based on:

- Performing the second order statistics of the input signal using Eq.(1). As discussed in section III.3, it is clear that the SOS improves the SNR as given by Eq.(7).
- Applying the MEM or the MUSIC techniques to the obtained data to estimate the power spectral density spectrum. Figure (1) shows the simplified block diagram of this method.

2- The second proposed method

The second proposed method is based on three steps, as shown in Fig.2, which are:

- Performing the coherent time average of the input data. This method consists of the summation of successive repetitions of a signal in such a way that the time signal reinforces itself (is coherent) while the noise, if it is random relative to the occurrence of the signal, tend to cancel out. Thus, coherent time averaging can be most useful for signal to noise ratio improvement.
- Applying the second order cumulant to the coherent time averaged output data improves the SNR as discussed in section III.3.
- Estimating the power spectrum using the MEM or MUSIC techniques.

The detection algorithm of these two proposed techniques are designed and applied for the case of single frequency signal and multiple frequency signals.

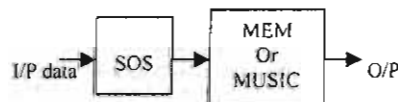


Fig.1.1st proposed method



Fig.1.2nd proposed method

IV- RESULTS AND ANALYSIS

This section presents the results of processing single and multiple frequency signals using the two proposed methods. First, we are going to give a brief description of the input signal.

IV.1 The Input Signal

The problem of search and rescue for aircraft in distress has become a significant concern. The emergency signals can provide both an immediate alert and a homing signal to assist rescue forces in locating the site of distress. The use of a satellite in low polar orbit might greatly enhance the performance of search and rescue facilities. The main advantage of this system is that the satellite has a wide field of view and pass covers many thousands of square kilometers. The emergency signal is detected by an orbiting satellite. A repeater on board the satellite relays the emergency signal to an earth station where the received signal is analyzed to extract the emergency signal position.

An emergency signal is transmitted from a low power emergency radio transmitter radiating about 100 mw with amplitude-modulated signal having a carrier frequency of either 121.5 MHz or optionally 243 MHz. The emergency signals have been processed using a digital band-pass processor implementation. The signal is mixed down to the frequency range from 0 to 25 kHz, which normally covers the vast majority of input signals. [7, 10].

Consider the received signal comprising more than one emergency signal (N_s signals) with different carrier frequencies and different amplitudes, the mathematical representation is given by [7, 10]:

$$s(t) = \sum_{i=1}^N s_i(t) = A_i [1 + m_i(t)] \cos(2\pi f_{ci} t + \theta_i) + n(t) \tag{8}$$

Where:

A_i is the carrier amplitude of the i^{th} signal, f_{ci} is the carrier frequency of i^{th} signal, θ_i is the phase angle of the i^{th} signal and $m_i(t)$: the modulating signal of the i^{th} signal.

The modulating term $m(t)$ can be classified as either sine wave or pulse shaped modulation. In this paper our study is concerned with sinusoidal modulation case. The modulating term $m(t)$ for the sinusoidal modulated signal can be formulated as follows :

$$m_i(t) = \sin(\phi_i(t)) \tag{9}$$

$$\text{where } \phi_i(t) = 2\pi \int f_{in}(t) dt \tag{10}$$

the instantaneous frequency $f_{in}(t)$ is given by :

$$f_{in}(t) = 1400 - 700 \frac{t}{T_r} \tag{11}$$

where T_r is the repetition period of the signal. Solving these equations, the emergency signal is given by [10]:

$$\tilde{s}(t) = [1 + \mu \sin(2\pi(1400t - 1400t^2 + 0.75))] \sin(2\pi f_c t) \tag{12}$$

Figure 3 illustrates the sinusoidal modulated emergency signal waveform with normalized frequency = 0.4 and without noise. While Fig.4 gives the same input signal with $S/N = -5$ dB.

Note: Normalized frequency is defined as: (Signal frequency / 25000).

V.2 Processing Results using the Maximum Entropy Method (MEM)

1) Single frequency Signal

Figures (5) and (6) illustrate the MEM spectrum results using MEM filter order 20 and 40 respectively for the input signal with normalized frequency 0.4 at $S/N = -5$ dB. From these results it is seen that the MEM filter order does have an influence on the spectral performances. At a low MEM filter order, the spectrum resolution is not good. By using a high MEM filter order, the width of the main peak can be reduced considerably but giving rise of spurious peaks as indicated in Fig.(6), which causes false alarms.

Now, we are going to examine different methods to improve the detection of the input signal at low S/N .

– First, we consider the preprocessing method based on coherent time average of the input signal. This method consists of the summation of successive repetitions of the signal, (averaging 50 continuous blocks of 256 points in each block of the input signal) and then estimating the output averaged data using MEM technique. Figure (7) gives the MEM spectrum estimate using this method with MEM filter order equal 20. Figure (8) illustrates the averaged MEM spectrum with filter order equal 20 also. Thus a significant enhancement on signal detection can be noted using the coherent time averaging method as indicated in Fig.(8).

– The second method tested in this paper (first proposed method) is used to improve the detection of the input signal at low S/N based on applying the second order statistics. Figure (9) illustrates the spectrum result using second order statistics and MEM with MEM filter order 20. From these obtained results we note that a great improvement in the frequency resolution is obtained, such as a sharp peak is located at the frequency of the transmitted signal.

– The third method studied in this paper (second proposed method) is used to improve the detection of the input signal at low S/N . Figure (10) illustrates the spectrum result using the second proposed method with MEM filter order 20. From this obtained result we note that a great improvement in the frequency resolution is obtained, such as a very sharp peak is located at the frequency of the transmitted signal.

(2) Multiple frequency Signals

In this section, we discuss the processing of multiple signals using the parametric techniques (MEM) with higher order statistics. The study, which is given here, is for four signals. The frequencies of these signals are selected randomly. In this case the signals with continuous phase and all signals contained in the input signal have the same amplitude. The spectral estimation performance degrades as a result of increasing the number of signals.

Four signals having normalized frequency 0.2, 0.4, 0.5 and 0.8. Figure(11) illustrates the time domain representation of the four input signals without noise, while Fig.(12) depicts the input four signals with noise at S/N equal -5dB. The MEM spectrum with MEM filter order equal 20 and 50 are given in Fig.13 and Fig.14 respectively.

From these results it is clear that the main peak for each signal is immersed in the noise, which make the detection of the transmitted signals difficult as shown in Fig.13, while the spectrum shown in Fig.14 congested with many undesirable peaks.

Now, we are going to examine different methods to enhance the spectrum and improve the detection of these multiple signals at low S/N; (S/N=-5 dB).

1- Averaging the time series data of the input four signals (averaging 50 continuous blocks of 256 points in each block) and then estimating the output averaged data using the MEM with MEM filter order equal 30 gives the spectrum shown in Fig.(15). Averaging MEM spectrum with filter order equal 20 gives the spectrum shown in Fig.(16). As indicated in these figures, a significant enhancement on signal detection can be noted as compared to the results using the above method. It is seen that we can identify and detect the main peak for every transmitted emergency signals.

2 - Applying the (first proposed method) based on second order statistics SOS with MEM gives the estimate spectrum shown in Fig. (17) using MEM filter order equal 20. By using this technique it is seen that, this proposed method improves the detection of the main signal peak in noise, and reduces the undesired sideband peaks. A very sharp peak for each transmitted signal can be detected easily with very minimum frequency error.

3 - Applying the (second proposed method) to improve the detection of the input signal at low S/N. Figure (18) illustrates the power spectrum estimate result using this proposed method with MEM filter order equal 20. By using this technique it is seen that, this proposed method improves the detection of the main signal peak in noise, and reduces the undesired sideband peaks. A very sharp peak for each transmitted signals can be detected easily with very minimum frequency error. In addition it reduces the required filter order for proper detection of the input signals.

IV.3 Processing results using MUSIC technique

1 - Single frequency signal

the main important parameter affect the MUSIC technique performance is the subspace size, parameter [1]. To examine this effect, first, applying the MUSIC technique with varying the subspace size on the input signal at S/N = -5 dB. Figures (19) and (20) illustrate the MUSIC spectrum results using subspace size of 10 and 15 respectively. From these results it is clear that as subspace size increases, the spectrum is enhanced and the input signal can be detected easily.

To improve the signal detectability at low S/N using smaller subspace size, we examine the following methods:

1 - Using the coherent time domain averaging of (50 times X 256 point in each block) and processing the obtained data using MUSIC with subspace size of 10 give the spectrum shown in Fig.(21). From this result, it is clear that the spectrum is enhanced such as the undesired side peaks levels are reduced and gives sharp peak located at the transmitted input signal frequency.

2 - The second method given here which is based on applying the first proposed method gives the spectrum illustrated in Fig.(22) for subspace of 10. It is noted from these results that a very sharp peak is detected and located at the signal frequency of the input signal and a very high frequency resolution spectrum is obtained with minimum frequency error, approximately tending to zero. In addition the processing time is reduced.

(2) Multiple frequency Signals

In most practical situations, the received signal comprises more than one signal due to false alarms, which may mask the signal from a platform in distress. With multiple sidebands, it is obvious that the signal band is extremely congested. Figures (23) and (24) illustrate the MUSIC processing results for subspace size of 10 and 15 respectively. To improve the multiple signal detectability, we consider:

1- Processing the multiple input signal data using the coherent time average of (50 times X 256 points in each block) with subspace size of 15 gives the spectrum depicted in Fig.(25). As indicated in this figure, we note that the width of the main peak of each transmitted signal is broadened and located at the signal frequency component for each signal.

2. Applying the second proposed method gives the spectrum illustrated in Fig. (26) for subspace size of 15. Thus, it is seen from this result that the detection of multiple signals at low S/N is improved very much as indicated in Fig.(26).

In addition, the second proposed method produces; high frequency resolution spectrum, very sharp peak with minimum frequency error. The transmitted emergency signal can be detected easily with very small processing time.

IV.4 Mathematical analysis of SNR improvement using HOS

For the noisy signal $y(n)$ given by Eq.(6), where $y(n)$ and $v(n)$ are independent; then from the main properties of cumulants [2, 8, 9]:

$$C_{k,y}(\tau_1, \tau_2, \dots, \tau_{k-1}) = C_{k,s}(\tau_1, \tau_2, \dots, \tau_{k-1}) + C_{k,v}(\tau_1, \tau_2, \dots, \tau_{k-1}) \quad (13)$$

If $v(n)$ is Gaussian (colored or white) and $k \geq 3$, then

$$C_{k,y}(\tau_1, \tau_2, \dots, \tau_{k-1}) = C_{k,s}(\tau_1, \tau_2, \dots, \tau_{k-1}) \quad (14)$$

This makes the higher order statistics more robust to additive measurements noise than correlation, even if that noise is colored. In addition, cumulants can draw non-Gaussian signals out of Gaussian noise, thereby boosting their signal to noise ratio.

1. The bispectrum of a single emergency signal with normalized frequency (0.4) with noise (SNR= -5dB) is indicated in Fig.(27), while the bispectrum of the output signal is depicted in Fig. (28).
2. The bispectrum of four emergency signals with normalized frequencies (0.2, 0.4, 0.5, 0.8) with noise (SNR= -5dB) in depicted in Fig.(29) while the bispectrum of the output signal is depicted in Fig. (30). From these figures and equations.(13)and(14), it is seen that the HOS cancel out the additive noise which consequently improve the signal detectability.

Thus from the above study we conclude that:

- Low MEM order of PEF produces a broad peak with large frequency error for the analysis of the multiple emergency signals.
- The sharpness of the spectral peak increases with the prediction error filter order M . While for large value of PEF order M , the estimated spectrum is much more accurate.
- With small subspace size, MUSIC gives not accurate spectrum, while large values of subspace size produce good spectrum but with more computational time.
- Averaging the time series data of input signals improves the signal to noise ratio that consequently improves the signal detectability.
- The proposed methods based on using second order statistics or applying the second order statistics of the coherent time averaged signal data and estimating the power spectrum using the (MEM & MUSIC) suppress the Gaussian noise of power spectrum. A sharp peak is detected and located at the signal-normalized frequency of the transmitted signal with minimum frequency error, using lower MEM filter order and small subspace size.

CONCLUSIONS

Two proposed techniques are given in this paper. These two methods are based on using second order statistics (second order cumulants) and on using the parametric spectral estimation techniques (MEM and MUSIC). By using these proposed methods we can suppress the additive Gaussian noise of the power spectrum and characterize the main properties of the transmitted signals. It produces a very high frequency resolution of the estimated power spectrum using lower order of MEM and small subspace size of MUSIC techniques. These methods reduce the processing time with minimum frequency error. HOS cancel out the additive noise, which consequently improve the signal detectability.

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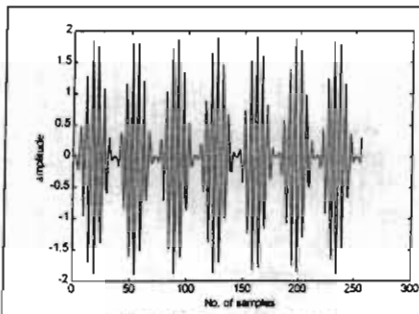


Fig.3(ELT signal in time domain)

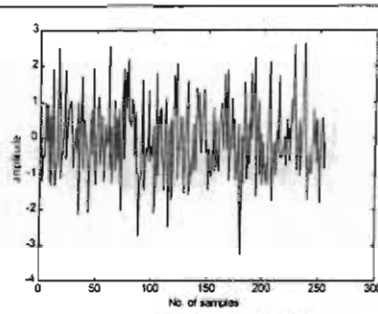


Fig.4(ELT signal with S/N = -5 dB)

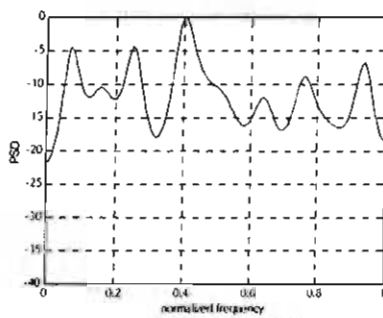


Fig.5(MEM spectrum, M=20)

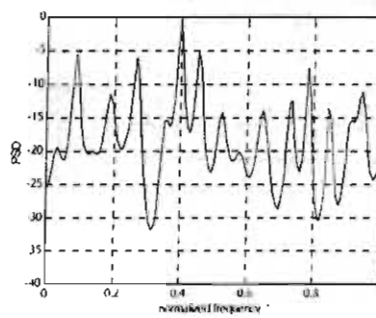


Fig.6(MEM spectrum, M=10)

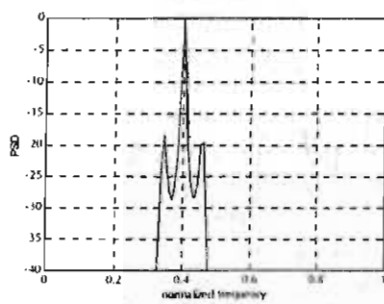


Fig.7(Coherent averaging, MEM, M=20)

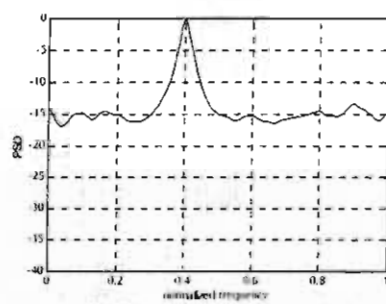


Fig.8(Average MEM spectrum, M=20)

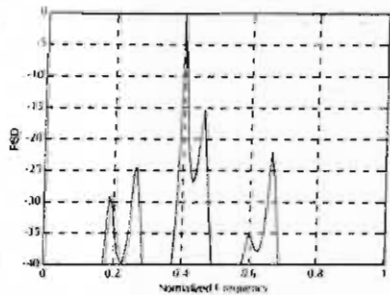


Fig.9(SOS, MEM, M=20)

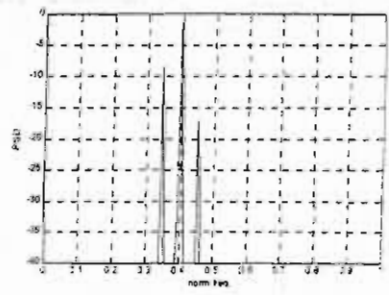


Fig.10(Coherent time Avg., SOS, MEM, M=20)

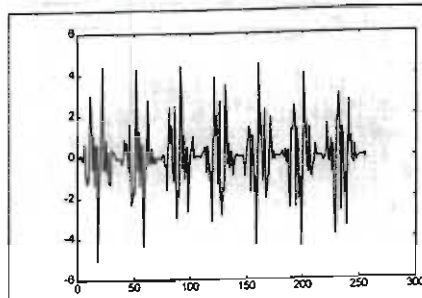


Fig.11(4 ELT signals)

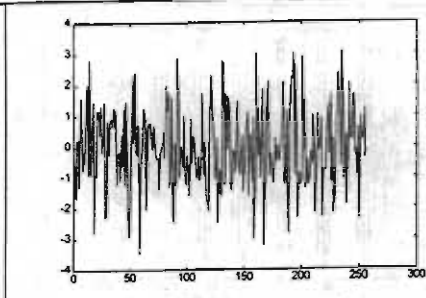


Fig.12(4 ELT signals, S/N =-3 dB)

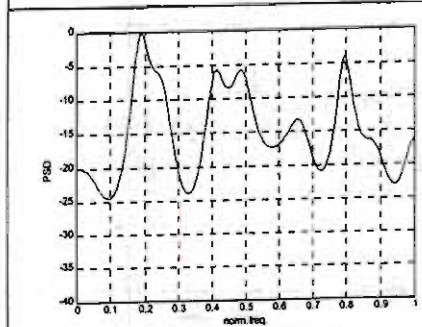


Fig.13(MEM spectrum ,M=20)

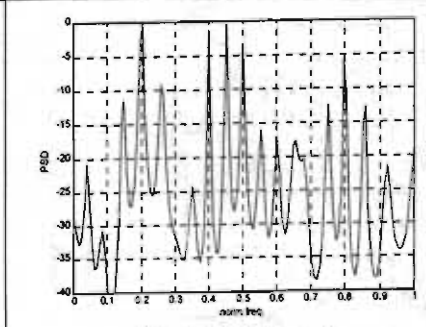


Fig.14(MEM spectrum ,M=50)

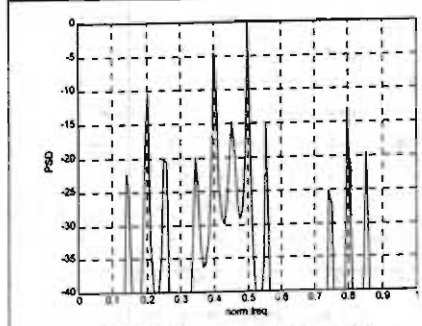


Fig.15(Coherent averaging, MEM ,M=20)

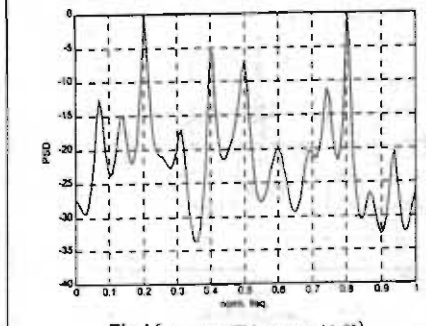


Fig.16(Average MEM spectrum, M=20)

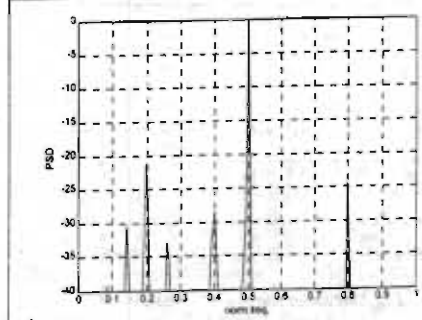


Fig.17(SOS, MEM, M=20)

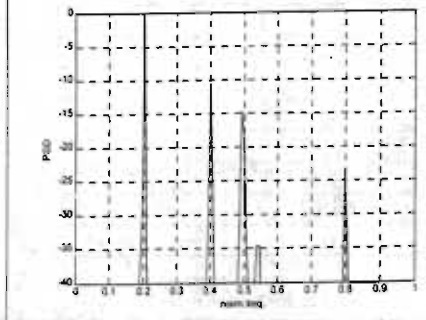


Fig.18(Coherent averaging SOS, MEM, M=20)

