A NEW TRANSIENT STABILITY APPROACH APPLIED TO MULTIMACHINE POWER SYSTEMS

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Abstract:

The work aims at developing a new stability approach used to carry out transient stability studies of a multimachine power system when the generator internal voltage components E'_q and E'_d are changing with the time. Considering the generator mechanical damping and the generator control systems (that is, the voltage regulator and speed governor), each generator is represented by a 6th-order dynamic model, then the mathematical model of the whole system is derived. Applying the decomposition-aggregation method, the system is decomposed by using the triple-wise decomposition. Then a vector Lyapunov function is constructed and used for aggregating the system. An aggregation matrix is obtained, the stability of which implies asymptotic stability of the system equilibrium state.

As an illustrative example, the developed stability approach is used to carry out transient stability studies of a 7-machine, 14-bus power system assuming a 3-phase short circuit (with successful reclosure) near a generator,or a load, bus. The faulted line is tripped out for clearing the fault. An estimate for the system asymptotic stability domain is determined and used to determine directly the critical time for reconnecting the isolated line. It is found that the proposed

Manuscript received from Dr. H. SHAABAN on : 20/2 /2000
Accepted on : 14/3/2000
Engineering Research Bulletin, Vol 23,No 2, 2000 Minufiya University, Faculty of Engineering , Shebien El-Kom , Egypt, ISSN 1110-1180

stability approach is powerful and may be used for stability studies of real power systems.

List of symbols:

P_{mi} , P_{ei} = mechanical power, electrical power of ith machine

P_{gi} , P^{o}_{g} = variation and steady-state variations of ith machine mechanical power

δ = rotor angle, or position of the rotor q-axis from the reference

X_d , X_q = direct-axis, quadrature-axis synchronous reactances

X'_d , X'_q = d-axis, q-axis transient reactances

E_{FD} = exciter voltage referred to the armature circuit

E' = voltage behind d-axis transient reactance

E'_d , E'_q = d-axis, q-axis components of the voltage E'

E_q = armature emf corresponding to the field current

δ^{o}, $\hat{E}_{FD}$, $\hat{E}_q$, $\hat{E}_d$ = steady–state values of δ, E_{FD}, E'_q and E'_d, respectively

V_t , V_T= terminal voltage, terminal voltage variation

V_{tqi}, V_{tdi} = q-axis and d-axis components of the voltage V_t

V^{o}_t , V^{o}_{tq} and V^{o}_{td} = steady–state values of V_t , V_{tq} and V_{td} , respectively

I^{o}_q , I^{o}_d = steady–state q-axis and d-axis current components

K_E , T_E = the exciter gain and time constant

ω = rotor speed with respect to the synchronous speed

T'_{do} , T'_{qo} = direct-axis and quadrature–axis transient open-circuit time constants

D_i , M_i = mechanical damping and inertia coefficient

$\lambda_i = (D_i / M_i)$= mechanical damping coefficient

$(1 / \mu)$= time constant of first-order speed governor

(α / μ)= gain of first-order speed governor

$J_{IN} = \{ iI, iI+1, N \}$= set introduced to denote the Ith subsystem machines

$J_I \subset J_{IN} = \{ iI , iI+1\}$

$\delta_{ij} = \delta_i - \delta_j = \delta_{iN} - \delta_{jN}$; $\sigma_{ij} = \delta_{ij} - \delta^{o}_{ij} = \sigma_{iN} - \sigma_{jN}$

$A_{ij} = A_{ji} = \hat{E}_{qi}\, \hat{E}_{qj} + \hat{E}_{di}\, \hat{E}_{dj}$; $\tilde{A}_{ij} = - \tilde{A}_{ji} = \hat{E}_{qi}\hat{E}_{dj} - \hat{E}_{di}\, \hat{E}_{qj}$

$\mathbf{f}_{ij} = Y_{ij} \cos(\theta_{ij} - \delta_{ij})$, $\mathbf{g}_{ij} = Y_{ij} \sin(\theta_{ij} - \delta_{ij})$

$\mathbf{f}^{o}_{ij} = Y_{ij} \cos(\theta_{ij} - \delta^{o}_{ij})$, $\mathbf{g}^{o}_{ij} = Y_{ij} \sin(\theta_{ij} - \delta^{o}_{ij})$

$K_j = (X_{dj} - X'_{dj}) / T'_{doj}$; $L_j = (X_{qj} - X'_{qj}) / T'_{qoj}$

$\| X_j \| = (X^T_j X_j)^{1/2}$; $\nu_j = 1 / T_{Ej}$

$\Lambda_j = \nu_j K_{Ej} V^0_{tqj} / V^0_{tj}$; $\tilde{\Lambda}_j = \Lambda_j X'_{dj}$

$\overline{\Lambda}_j = [\nu_j K_{Ej} V^0_{tdj} X_{qj} / V^0_{tj} (X'_{dj} - X_{qj})]$; $\chi_j = 1 / T'_{doj}$

$\Gamma_j = [\chi_j - K_j B_{jj}]$, $j \in J_{IN}$

$\sum^0$, $\sum^*$ and $\sum$ are defined and as $\sum_{j=1}^{N}$, $\sum_{j\neq i}^{N}$ and $\sum_{j\neq JI}^{N-1}$,respectively

Z_2, Z_3 and Z_4 = three functions, defined as follows:

$Z_2(\alpha, \phi) = \min \{ \sqrt{2} \max(|\alpha|, |\phi|) ; (|\alpha| + |\phi|) \}$

$Z_3(\alpha, \phi, \gamma) = \min \{ 2 \max(|\alpha|, |\phi|, |\gamma|) ; (|\alpha| + |\phi| + |\gamma|) ; Z_2[Z_2(\alpha, \phi), \gamma] ; Z_2[Z_2(\phi, \gamma), \alpha] ; Z_2[Z_2(\gamma, \alpha), \phi] \}$

$Z_4(\alpha, \phi, \gamma, \psi) = \min \{ Z_2[Z_3(|\alpha|, |\phi|, |\gamma|), |\psi|] ; Z_2[Z_3(|\psi|, |\phi|, |\gamma|) ; |\alpha|] ; Z_2[Z_3(|\alpha|, |\psi|, |\gamma|) ; |\phi|] ; Z_2[Z_3(|\alpha|, |\phi|, |\psi|) ; |\gamma|] \}$

1 Introduction:

The digital-simulation method of determining the power system transient stability gives very accurate results and the method can be used with any degree of modeling sophistication. However, one disadvantage of the digital-simulation method is that, depending on the size, complexity and modeling refinement of the system, it may require huge computational effort, despite the dramatic progress of computer performance in recent years [1]. Another drawback is that it does not provide at a glance the physical insight into the qualitative behavior of the set of differential equations, i.e. of the response for changes in system parameters. Thus sensitivity aspects of the system are not obvious and sensitivity analysis using the digital –simulation method can only be handled via tedious numerical simulation [2]. This leads one to look at some other direct methods of finding the transient stability, and hence the critical clearing time, without explicitly integrating the system dynamic equations.

In the last three decades numerous efforts have been made to apply direct methods of transient stability analysis to multimachine power systems. However, these methods are potentially useful both as off-line tools for planning purposes and for on-line security assessment [3].

The well-known equal-area criterion is an example of such a direct method of determining the critical clearing time. Unfortunately, this criterion is applicable to only a single-machine infinite-bus system. The transient-energy-function method is one of the direct methods of determining the transient stability of a multimachine power system. However, this method has some modeling limitations and for some cases the energy function may not even exist [4].

The Lyapunov's direct method is another type of the direct methods used for determining the transient stability analysis of a multimachine power system. This method has attracted much attention and has proved to be promising tool of analysis for off-line and on-line studies [5]. In addition, it can in principle be faster, and also asses stability margins and sensitivity concerns those numerical-integration methods are unable to tackle.

The scalar (function) Lyapunov method was used, in the last three decades, to carry out transient stability analysis of multimachine power systems. However, this method did not, owing to the continuous increase in the sizes and complexities of real power systems, seem suitable in particular when the problem of the system stability domain estimate is attacked [6]. Furthermore, it is very difficult to derive a valid Lyapunov function for a power system when the automatic voltage regulator (AVR) effect is considered [7].

The Bellman's vector Lyapunov function method has been appeared more suitable than the scalar function method for application to power systems. The vector function method allows more sophisticated mathematical models of generators and transmission. In addition exact estimates of the overall system stability domain may be defined [8].

The vector function Lyapunov method was used for transient stability analysis of an N–machine power system [8-16], considering the generator classical model (i.e. the voltage E' is constant).

The transient stability analysis of a multimachine power system was carried out in [17] considering the one–axis model (i.e., the voltage component E'_q changes with time), and the two-axis model (i.e., both the two voltages E'_q and E'_d change with time) was considered in [18]. A 10-machine, 11-bus power system was used, in each of the two papers, as an illustrative numerical example.

In the present work, an N-machine power system is considered and the two-axis model represents each machine. Taking into consideration the generator mechanical damping, the automatic voltage regulator (AVR) effect and the speed governor action, the system transient stability analysis is performed using a new decomposition

-aggregation approach. A sixth-order dynamic model represents each machine. The whole system mathematical model is decomposed into (N−1)/2, 17th-order interconnected subsystems. An aggregation matrix of the order (N−1)/2 is obtained, stability of this matrix implies asymptotic stability of the system equilibrium. In a numerical example, a power system, consisting of 7 machines and 14 bus, is considered. The system transient stability studies for a 3-phase short circuit fault are carried out using the developed stability approach. Three case studies are presented.

2 Power system model:

Let us consider an N-machine power system (the transfer conductances are included) with mechanical damping, and assume that each machine is represented by the two-axis model [19]. Considering the speed governor action [15], and the automatic voltage regulator (AVR) effect [20], the absolute motion of each machine is described by the following five nonlinear differential equations (the machine stator resistance is neglected):

$$
\begin{aligned}
&M_i\, \ddot{\delta}_i + D_i\, \dot{\delta}_i = P_{mi} + P_{gi} - P_{ei} \\
&T'_{doi}\, \dot{E}'_{qi} = E_{FDi} - E'_{qi} + (X_{di} - X'_{di})\, I_{di} \\
&T'_{qoi}\, \dot{E}'_{di} = -E'_{di} - (X_{qi} - X'_{qi})\, I_{qi} \\
&T_{Ei}\, \dot{E}_{FDi} = -(E_{FDi} - \hat{E}_{FDi} + K_{Ei}\, \dot{V}_{Ti}) \\
&\dot{P}_{gi} = -\mu_i P_{gi} - \alpha_i \dot{\delta}_i \qquad , i = 1,2, \ldots, N
\end{aligned} \tag{1}
$$

where P_{ei} is given, under the assumption $X'_d = X'_q$ as,

$$
P_{ei} = \Sigma^{o} \{E'_{qi}[E'_{qj}\, f_{ij} - E'_{dj}\, g_{ij}] + E'_{di}[E'_{dj} f_{ij} + E'_{qj}\, g_{ij}]\} \qquad , i = 1,2, \ldots, N \tag{2}
$$

In eqn.1, the terminal voltage variation $\dot{V}_{Ti}$ may be given, for simplicity, in the form [19],

$$
V_{Ti} = (V^{o}_{tqi} / V^{o}_{ti}) V_{Tqi} + (V^{o}_{tdi} / V^{o}_{ti}) V_{Tdi} \qquad , i = 1,2, \ldots, N \tag{3}
$$

where,

$$
\begin{aligned}
&V_{Tqi} = (E'_{qi} - \hat{E}_{qi}) + (I_{di} - I^{o}_{di}) X'_{di} \\
&V_{Tdi} = [-X_{qi} / (X'_{di} - X_{qi})](E'_{di} - \hat{E}_{di}) \qquad , i = 1,2, \ldots, N
\end{aligned} \tag{4}
$$

Now, let the following (6N−1) state variables to be introduced (the Nth- machine is chosen as a comparison machine),

$$
\sigma_{iN} = \delta_{iN} - \delta^{o}_{iN} \qquad , i \neq N
$$

$$\omega_i = \dot{\delta}_i \quad ; \quad E_{Qi} = E'_{qi} - \hat{E}_{qi} \quad ; \quad E_{Di} = E'_{di} - \hat{E}_{di}$$

$$E_{fi} = E_{FDi} - \hat{E}_{FDi} \quad ; \quad P_i = P_{gi} - P^o_{gi} \quad , i = 1,2,\ldots,N \tag{5}$$

Then, the whole system motion can be derived in the form (see Notation),

$$\dot{\sigma}_{iN} = \omega_i - \omega_N \quad , i \neq N$$

$$\dot{\omega}_i = -\lambda_i \omega_i + (P_i / M_i) - (1/M_i)\Big[(E_{Qi} + \hat{E}_{qi}) \Sigma^o \{(E_{Qj} + \hat{E}_{qj}) \mathbf{f}_{ij} - (E_{Dj} + \hat{E}_{dj}) \mathbf{g}_{ij}\} + (E_{Di} + \hat{E}_{di}) \Sigma^o \{(E_{Dj} + \hat{E}_{dj}) \mathbf{f}_{ij} + (E_{Qj} + \hat{E}_{qj}) \mathbf{g}_{ij}\} - \Sigma^o \{A_{ij} \mathbf{f}^o_{ij} - \tilde{A}_{ij} \mathbf{g}^o_{ij}\}\Big]$$

$$\dot{E}_{Qi} = -\Gamma_i E_{Qi} + \chi_i E_{fi} + K_i \Sigma^* \{(E_{Dj} + \hat{E}_{dj}) \mathbf{f}_{ij} + (E_{Qj} + \hat{E}_{qj}) \mathbf{g}_{ij} - \hat{E}_{dj} \mathbf{f}^o_{ij} - \hat{E}_{qj} \mathbf{g}^o_{ij}\}$$

$$\dot{E}_{Di} = -(1/T'_{qoi}) E_{Di} - L_i \Sigma^o \{(E_{Qj} + \hat{E}_{qj}) \mathbf{f}_{ij} - (E_{Dj} + \hat{E}_{dj}) \mathbf{g}_{ij} - \hat{E}_{qj} \mathbf{f}^o_{ij} + \hat{E}_{dj} \mathbf{g}^o_{ij}\}$$

$$\dot{E}_{fi} = -\Lambda_i E_{Qi} + \overline{\Lambda}_i E_{Di} - \nu_i E_{fi} - \tilde{\Lambda}_i \Sigma^* \{(E_{Dj} + \hat{E}_{dj}) \mathbf{f}_{ij} + (E_{Qj} + \hat{E}_{qj}) \mathbf{g}_{ij} - \hat{E}_{dj} \mathbf{f}^o_{ij} - \hat{E}_{qj} \mathbf{g}^o_{ij}\}$$

$$\dot{P}_i = -\mu_i P_i - \alpha_i \omega_i \quad , i = 1,2,\ldots,N \tag{6}$$

3 Power system decomposition :

The first step in the system decomposition is performed by determining the system Nth-order reduced admittance matrix Y, where the system loads are represented by constant shunt impedances, and the system buses are eliminated, retaining only the generator internal nodes. Then, using the triple-wise decomposition [13,14, 16], the system is decomposed into (N–1)/2 interconnected subsystems, each has a state vector X_I in the form:

$$X_I = [\sigma_{iI,N}, \sigma_{iI+1,N}, \omega_{iI}, \omega_{iI+1}, \omega_N, E_{QiI}, E_{QiI+1}, E_{QN}, E_{DiI}, E_{DiI+1}, E_{DN}, E_{fiI}, E_{fiI+1}, E_{fN}, P_{iI}, P_{iI+1}, P_N]^T = [X_{I1}, X_{I2}, X_{I3}, \ldots, X_{I17}]^T \tag{7}$$

Finally, the system mathematical model (eqn.6) is decomposed into $S = (N-1)/2$, **17th-order interconnected subsystems, each of them can be written in the general form**

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) + h_I(X) \quad , \quad \sigma_I = C^T_I X_I \quad , I = 1,2,\ldots,S \tag{8}$$

where P_I, B_I and C_I are constant matrices with appropriate dimensions. In eqn. 8, F_I is a nonlinear vector function whose elements are chosen such that

they satisfy the Lure's sector conditions, and h_I is an interconnection (vector) matrix.
Now, defining each free (disconnected) subsystem as,

$$\dot{X}_I = P_I\, X_I + B_I F_I(\sigma_I) \quad , \quad \sigma_I = C_I^T\, X_I \qquad , I = 1,2,\ldots\ldots, S \quad (9)$$

it is found that the vector F_I contains, at most, the following three nonlinearities,

$$\begin{aligned} f_{I1}(\sigma_{I1}) &= \sin(\sigma_{iI,N} + \delta^{o}_{iI,N}) - \sin\delta^{o}_{iI,N} \\ f_{I2}(\sigma_{I2}) &= \sin(\sigma_{iI+1,N} + \delta^{o}_{iI+1,N}) - \sin\delta^{o}_{iI+1,N} \\ f_{I3}(\sigma_{I3}) &= \sin(\sigma_{iI,iI+1} + \delta^{o}_{iI,iI+1}) - \sin\delta^{o}_{iI,iI+1} \end{aligned} \qquad (10)$$

which satisfy the conditions:

$$\sigma_{Ik} f_{Ik}(\sigma_{Ik}) \geq \varepsilon_{Ik}\, \sigma^2_{Ik} \qquad , k = 1, 2, 3 \qquad (11)$$

where $\varepsilon_{Ik} \in (0, \xi_{Ik})$ and ξ_{Ik} is positive number and may be determined as,

$$\xi_{Ik} = \left(\partial f_{Ik}(\sigma_{Ik}) / \partial\sigma_{Ik} \right) \Big|_{\sigma_{Ik}=0} \qquad , k = 1, 2, 3 \qquad (12)$$

Referring to eqns.6-8, the following matrices are defined (see Notation):

$$P_I = \left[\begin{array}{c|c|ccc|c} & I_2 \;\; -a & & O_{2\times 11} & & \\ \hline & -P_{I1} & -P_{I2} & -P_{I3} & O_{3\times 3} & P_{I4} \\ \hline & & -P_{I5} & P_{I6} & P_{I7} & \\ O_{17\times 2} & O_{9\times 3} & -P_{I8} & -P_{I9} & O_{3\times 3} & O_{9\times 3} \\ & & -P_{I10} & P_{I11} & -P_{I12} & \\ \hline & -P_{I13} & & O_{3\times 9} & & -P_{I14} \end{array} \right] \qquad (13)$$

where O and I are zero and identity (square) matrices, respectively, of the indicated dimensions, and a is a second-order unit vector and where,

$P_{I1} = \text{diag}[\lambda_{iI}, \lambda_{iI+1}, \lambda_N]$; $P_{I2} = \text{diag}[\Theta_{iI}, \Theta_{iI+1}, \Theta_N]$

$P_{I3} = \text{diag}[\Theta^*_{iI}, \Theta^*_{iI+1}, \Theta^*_N]$; $P_{I4} = \text{diag}[1/M_{iI}; 1/M_{iI+1}; 1/M_N]$

$P_{I5} = \text{diag}[\Gamma_{iI}, \Gamma_{iI+1}, \Gamma_N]$; $P_{I6} = \text{diag}[\hat{c}_{iI}, \hat{c}_{iI+1}, \hat{c}_N]$

$P_{I7} = \text{diag}[\chi_{iI}, \chi_{iI+1}, \chi_N]$; $P_{I8} = \text{diag}[\hat{d}_{iI}, \hat{d}_{iI+1}, \hat{d}_N]$

$P_{I9} = \text{diag}[\beta_{iI}, \beta_{iI+1}, \beta_N]$; $P_{I10} = \text{diag}[\eta_{iI}, \eta_{iI+1}, \eta_N]$

$$\mathbf{P}_{I11} = \mathrm{diag}\,[\phi_{iI}, \phi_{iI+1}, \phi_N] \quad ; \quad \mathbf{P}_{I12} = \mathrm{diag}\,[\nu_{iI}, \nu_{iI+1}, \nu_N]$$

$$\mathbf{P}_{I13} = \mathrm{diag}\,[\alpha_{iI}, \alpha_{iI+1}, \alpha_N] \quad ; \quad \mathbf{P}_{I14} = \mathrm{diag}\,[\mu_{iI}, \mu_{iI+1}, \mu_N]$$

$$B_I = \left[\begin{array}{c|c|c} \multicolumn{3}{c}{\mathbf{O}_{2\times 3}} \\ \hline B_{I1} & B_{I2} & B_{I3} \\ \hline \multicolumn{3}{c}{\mathbf{O}_{3\times 3}} \end{array}\right] \tag{14}$$

where

$$B_{I1} = [-q_{iI,N}, 0, q_{N,iI}, u_{iI,N}, 0, -u_{N,iI}, -\bar{u}_{iI,N}, 0, \bar{u}_{N,iI}, -v_{iI,N}, 0, v_{N,iI}]^T$$

$$B_{I2} = [0, -q_{iI+1,N}, q_{N,iI+1}, 0, u_{iI+1,N}, -u_{N,iI+1}, 0, -\bar{u}_{iI+1,N}, \bar{u}_{N,iI+1}, 0, -v_{iI+1,N}, v_{N,iI+1}]^T$$

$$B_{I3} = [-q_{iI,iI+1}, q_{iI+1,iI}, 0, u_{iI,iI+1}, -u_{iI+1,iI}, 0, -\bar{u}_{iI,iI+1}, \bar{u}_{iI+1,iI}, 0, -v_{iI,iI+1}, v_{iI+1,iI}, 0]^T$$

$$F_I(\sigma_I) = [\, f_{I1}(\sigma_{I1}), f_{I2}(\sigma_{I2}), f_{I3}(\sigma_{I3})]^T \tag{15}$$

$$C^T_I = \left[\begin{array}{cc|c} 1.0 & 0 & \\ 0 & 1.0 & O_{3\times 15} \\ 1.0 & -1.0 & \end{array}\right] \tag{16}$$

Using eqns.13–16, the free subsystem of eqn.9, is completely defined, and hence the interconnection matrix $h_I(X)$ is obtained as,

$$\mathbf{h}_I(X) = [\,\mathbf{0}, \mathbf{0}, \mathbf{h}_{I3}(X), \mathbf{h}_{I4}(X), \ldots\ldots\ldots, \mathbf{h}_{I14}(X), \mathbf{0}, \mathbf{0}, \mathbf{0}]^T \tag{17}$$

where

$$\mathbf{h}_{I3}(X) = -(1/M_{iI})\Big[\,(X^2_{I6} + X^2_{I9})\,G_{iI,iI} + C_{iI,N}\,\tilde{f}_{I1}(\sigma_{I1}) + C_{iI,iI+1}\,\tilde{f}_{I3}(\sigma_{I3}) + \Sigma\, S_{iI,j} + \Sigma^*\{L_{iI,j} + D_{iI,j} - \bar{L}_{iI,j} + \bar{D}_{iI,j}\}\Big]$$

$$\mathbf{h}_{I4}(X) = -(1/M_{iI+1})\Big[(X^2_{I7} + X^2_{I10})\,G_{iI+1,iI+1} + C_{iI+1,N}\,\tilde{f}_{I2}(\sigma_{I2}) + C_{iI+1,iI}\,\tilde{f}_{I3}(\sigma_{I3}) + \Sigma\, S_{iI+1,j} + \Sigma^*\{L_{iI+1,j} + D_{iI+1,j} - \bar{L}_{iI+1,j} + \bar{D}_{iI+1,j}\}\Big]$$

$$\mathbf{h}_{I5}(X) = -(1/M_N)\Big[(X^2_{I8} + X^2_{I11})\,G_{N,N} + C_{N,iI}\,\tilde{f}_{I1}(\sigma_{I1}) + C_{N,iI+1}\,\tilde{f}_{I2}(\sigma_{I2}) + \Sigma\, S_{N,j} + \Sigma^*\{L_{N,j} + D_{N,j} - \bar{L}_{N,j} + \bar{D}_{N,j}\}\Big]$$

$$\mathbf{h}_{I6}(X) = K_{iI}\Big[\bar{C}_{iI,N}\,\tilde{f}_{I1}(\sigma_{I1}) + \bar{C}_{iI,iI+1}\,\tilde{f}_{I3}(\sigma_{I3}) + \Sigma\,\hat{S}_{iI,j} - \Sigma^*\,\tilde{L}_{iI,j}\Big]$$

$$\mathbf{h}_{I7}(X) = K_{il+1}\Big[\overline{C}_{il+1,N}\ \tilde{f}_{I2}(\sigma_{I2}) + \overline{C}_{il+1,il}\ \tilde{f}_{I3}(\sigma_{I3}) + \Sigma\ \hat{S}_{il+1,j} - \Sigma^{*}\ \tilde{L}_{il+1,j}\Big]$$

$$\mathbf{h}_{I8}(X) = K_{N}\Big[\overline{C}_{N,il}\ \tilde{f}_{I1}(\sigma_{I1}) + \overline{C}_{N,il+1}\ \tilde{f}_{I2}(\sigma_{I2}) + \Sigma\ \hat{S}_{N,j} - \Sigma^{*}\ \tilde{L}_{N,j}\Big]$$

$$\mathbf{h}_{I9}(X) = -L_{il}\Big[\tilde{C}_{il,N}\ \tilde{f}_{I1}(\sigma_{I1}) + \tilde{C}_{il,il+1}\ \tilde{f}_{I3}(\sigma_{I3}) + \Sigma\ \tilde{S}_{il,j} + \Sigma^{*}\ \hat{L}_{il,j}\Big]$$

$$\mathbf{h}_{I10}(X) = -L_{il+1}\Big[\tilde{C}_{il+1,N}\ \tilde{f}_{I2}(\sigma_{I2}) + \tilde{C}_{il+1,il}\ \tilde{f}_{I3}(\sigma_{I3}) + \Sigma\ \tilde{S}_{il+1,j} + \Sigma^{*}\ \hat{L}_{il+1,j}\Big]$$

$$\mathbf{h}_{I11}(X) = -L_{N}\Big[\tilde{C}_{N,il}\ \tilde{f}_{I1}(\sigma_{I1}) + \tilde{C}_{N,il+1}\ \tilde{f}_{I2}(\sigma_{I2}) + \Sigma\ \tilde{S}_{N,j} + \Sigma^{*}\ \hat{L}_{N,j}\Big]$$

$$\mathbf{h}_{I12}(X) = -\mathbf{h}_{I6}(X)\,[\tilde{\Lambda}_{il} / K_{il}]$$

$$\mathbf{h}_{I13}(X) = -\mathbf{h}_{I7}(X)\,[\tilde{\Lambda}_{il+1} / K_{il+1}]$$

$$\mathbf{h}_{I14}(X) = -\mathbf{h}_{I8}(X)\,[\tilde{\Lambda}_{N} / K_{N}]$$

In eqns.13,15 and 17, the following constants and functions are defined:

$$\Theta_j = 2\,\hat{E}_{qj}\,G_{jj}/M_j \quad ; \quad \Theta^{*}_j = 2\,\hat{E}_{dj}\,G_{jj}/M_j \quad ; \quad \hat{c}_j = K_j\,G_{jj}$$

$$\hat{d}_j = L_j\,G_{jj} \quad ; \quad \eta_j = \Lambda_j\,[1.0 + X'_{dj}\,B_{jj}]$$

$$\beta_j = [(1.0/T'_{qoj}) - L_j B_{jj}] \quad ; \quad \phi_j = \overline{\Lambda}_j - \tilde{\Lambda}_j G_{jj} \qquad , j \varepsilon J_{IN}$$

$$q_{kj} = (A_{kj} B_{kj} + \tilde{A}_{kj} G_{kj}) / M_k \quad ; \quad C_{kj} = (A_{kj} G_{kj} - \tilde{A}_{kj} B_{kj})$$

$$\overline{C}_{kj} = (\hat{E}_{dj}\,G_{kj} + \hat{E}_{qj}\,B_{kj}) \quad ; \quad \tilde{C}_{kj} = (\hat{E}_{qj}\,G_{kj} - \hat{E}_{dj}\,B_{kj})$$

$$u_{kj} = K_k(\hat{E}_{dj}\,B_{kj} - \hat{E}_{qj}\,G_{kj})$$

$$\overline{u}_{kj} = L_k(\hat{E}_{qj}\,B_{kj} + \hat{E}_{dj}\,G_{kj})$$

$$V_{kj} = \tilde{\Lambda}_k(\hat{E}_{dj}\,B_{kj} - \hat{E}_{qj}\,G_{kj}) \qquad , k \neq j,\ k, j \in J_{IN}$$

$$L_{ij} = \{(E_{Qi} + \hat{E}_{qi})\,\mathbf{f}_{ij} + \hat{E}_{di}\,\mathbf{g}_{ij}\}\,E_{Qj}$$

$$\overline{L}_{ij} = \{(E_{Qi} + \hat{E}_{qi})\,\mathbf{g}_{ij} - \hat{E}_{di}\,\mathbf{f}_{ij}\}\,E_{Dj}$$

$$D_{ij} = \{\hat{E}_{qj}\,\mathbf{f}_{ij} - \hat{E}_{dj}\,\mathbf{g}_{ij}\}\,E_{Qi}$$

$$\overline{D}_{ij} = \{(E_{Dj} + \hat{E}_{dj})\,\mathbf{f}_{ij} + (E_{Qj} + \hat{E}_{qj})\,\mathbf{g}_{ij}\}\,E_{Di}$$

$$\tilde{L}_{ij} = \hat{E}_{Qj} g_{ij} + \hat{E}_{Dj} f_{ij} \quad ; \quad \hat{L}_{ij} = \hat{E}_{Qj} f_{ij} - \hat{E}_{Dj} g_{ij}$$

$$S_{ij} = \{A_{ij} f_{ij}(\sigma_{ij}) + \tilde{A}_{ij} g_{ij}(\sigma_{ij})\} Y_{ij}$$

$$\hat{S}_{ij} = \{\hat{E}_{dj} f_{ij}(\sigma_{ij}) - \hat{E}_{qj} g_{ij}(\sigma_{ij})\} Y_{ij}$$

$$\tilde{S}_{ij} = \{\hat{E}_{dj} g_{ij}(\sigma_{ij}) + \hat{E}_{qj} f_{ij}(\sigma_{ij})\} Y_{ij}$$

$$f_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \delta^{o}_{ij} - \theta_{ij}) - \cos(\delta^{o}_{ij} - \theta_{ij})$$

$$g_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \delta^{o}_{ij} - \theta_{ij}) - \sin(\delta^{o}_{ij} - \theta_{ij}) \quad , i \neq j \, , \, i \in J_{IN}$$

$$\tilde{f}_{I1}(\sigma_{I1}) = \cos(\sigma_{iI,N} + \delta^{o}_{iI,N}) - \cos\delta^{o}_{iI,N}$$

$$\tilde{f}_{I2}(\sigma_{I2}) = \cos(\sigma_{iI+1,N} + \delta^{o}_{iI+1,N}) - \cos\delta^{o}_{iI+1,N}$$

$$\tilde{f}_{I3}(\sigma_{I3}) = \cos(\sigma_{iI,iI+1} + \delta^{o}_{iI,iI+1}) - \cos\delta^{o}_{iI,iI+1} \qquad (18)$$

4 Power system aggregation

Following the aggregation procedure in ref [21], we construct an aggregation (square) matrix, $A = [\alpha_{IJ}]$, whose elements (real numbers) obey the inequality

$$\dot{V}_I(X_I) \leq \sum_{J=1}^{S} \alpha_{IJ} \|X_I\| \|X_J\| \quad , I = 1,2,\ldots\ldots\ldots,S \qquad (19)$$

where $V_I(X_I)$ is a free subsystem Lyapunov function, and $\|X\|$ is chosen to be a comparison function.

In this work, a Lyapunov function in the following form [8-10, 13-18], is adopted for each free subsystem:

$$V_I(X_I) = X^T_I H_I X_I + \sum_{m=1}^{3} \gamma_{Im} \int_0^{\sigma I} f_{Im}(\sigma_{Im}) \, d\sigma_{Im} \quad , I = 1,2,\ldots\ldots,S \quad (20)$$

where, H_I is a 17th-order symmetric positive-definite matrix, the functions f'_{Im} are given by eqn. 10, and γ_{Im} are arbitrary positive numbers.

Now, as a first step for determining the system aggregation matrix, we compute $\dot{V}_I$ along the motion of the interconnected subsystem of eqn.8. Then, using a number of introduced majorizations, the left-hand side of inequality 19 is completely majorized. Finally, the matrix elements α_{IK} are obtained and defined as,

$$\alpha_{IK} = \begin{cases} -\lambda^{*}_I & , K = I \\ 2 Z_{IK} & , K \neq I \quad , K, I = 1,2,\ldots\ldots\ldots, S = (N-1)/2 \end{cases} \qquad (21)$$

In eqn.21, λ^* is the minimal (positive) eigenvalue of the 18th-order symmetric matrix R_I, whose elements are given by eqn. (A-1), and Z_{IK} is defined by eqn. (A-2). It should be noted that, stability of the obtained aggregation matrix A, implies asymptotic stability of the system equilibrium [21].

5 Numerical example:

The 7-machine, 14-bus power system shown in Fig.1, is considered, in this example, as a simple power system for an application of the developed stability approach. The first step of the stability computations is performed by determining the system reduced 7th-order (symmetric) matrix Y, whose elements are given in Table 1 (see Appendix). Then, the system is decomposed (machine 7 is selected as the comparison machine) into three "3-machine" interconnected subsystems, and the following parameters are chosen (note that, the elements of the matrix H are chosen so that a largest value of the eigenvalue λ^* is obtained):

$$h^k_{13} = h^k_{24} = h^k_{33} = h^k_{44} = 1.0 \; ; \quad h^k_{66} = h^k_{77} = 300 \qquad , k = 1,2,3$$

$$h^1_{55} = 4.3 \, , \; h^2_{55} = h^3_{55} = 6.7 \quad ; \quad h^1_{88} = 22.5 \, , \; h^2_{88} = 34.0 \, , \; h^3_{88} = 35.0$$

$$h^1_{99} = h^1_{10,10} = 15.5 \, , \; h^1_{11,11} = 1.6 \quad ; \quad h^2_{99} = h^2_{10,10} = 16.5 \, , \; h^2_{11,11} = 2.4$$

$$h^3_{99} = h^3_{10,10} = 11.0 \, , \; h^3_{11,11} = 2.5$$

$$\lambda_1 = \lambda_2 = 4.1 \, , \quad \lambda_3 = \lambda_4 = 4.3 \, , \quad \lambda_5 = \lambda_6 = 4.2 \, , \quad \lambda_7 = 20.0$$

$$T'_{doi} = 3.5 \; ; \; T'_{qoi} = 0.25 \; ; \; K_{Ei} = 10.0 \; ; \; T_{Ei} = 0.40 \qquad , i = 1, 2, 3, \ldots, 7$$

$$\alpha_i = 0.2 \quad ; \quad \mu_i = 30.0 \qquad , i = 1,2,\ldots,7$$

$$\varepsilon_{11} = 0.82 \, , \; \varepsilon_{12} = 0.80 \; ; \; \varepsilon_{21} = 0.83 \, , \; \varepsilon_{22} = 0.81$$

$$\varepsilon_{31} = 0.84 \, , \; \varepsilon_{32} = 0.82$$

Finally, the system aggregation matrix is computed, by using expression 20, as

$$A = \begin{bmatrix} -0.513134 & 0.245680 & 0.292960 \\ 0.400640 & -1.026870 & 0.453568 \\ 0.412400 & 0.382240 & -0.742801 \end{bmatrix}$$

which satisfies the Hick's conditions[21], and is thus a stable matrix. This implies the asymptotic stability of the system equilibrium. Finally, referring to the Appendix of ref. [14], we determine the system asymptotic stability domain estimate E_1:

$$E_1 = \{ X : [V_1(X_1) + V_2(X_2) + V_3(X_3)] \leq 4.8017 \} \tag{22}$$

where V_1, V_2 and V_3 are the free subsystem Lyapunov functions, given by eqn.20.

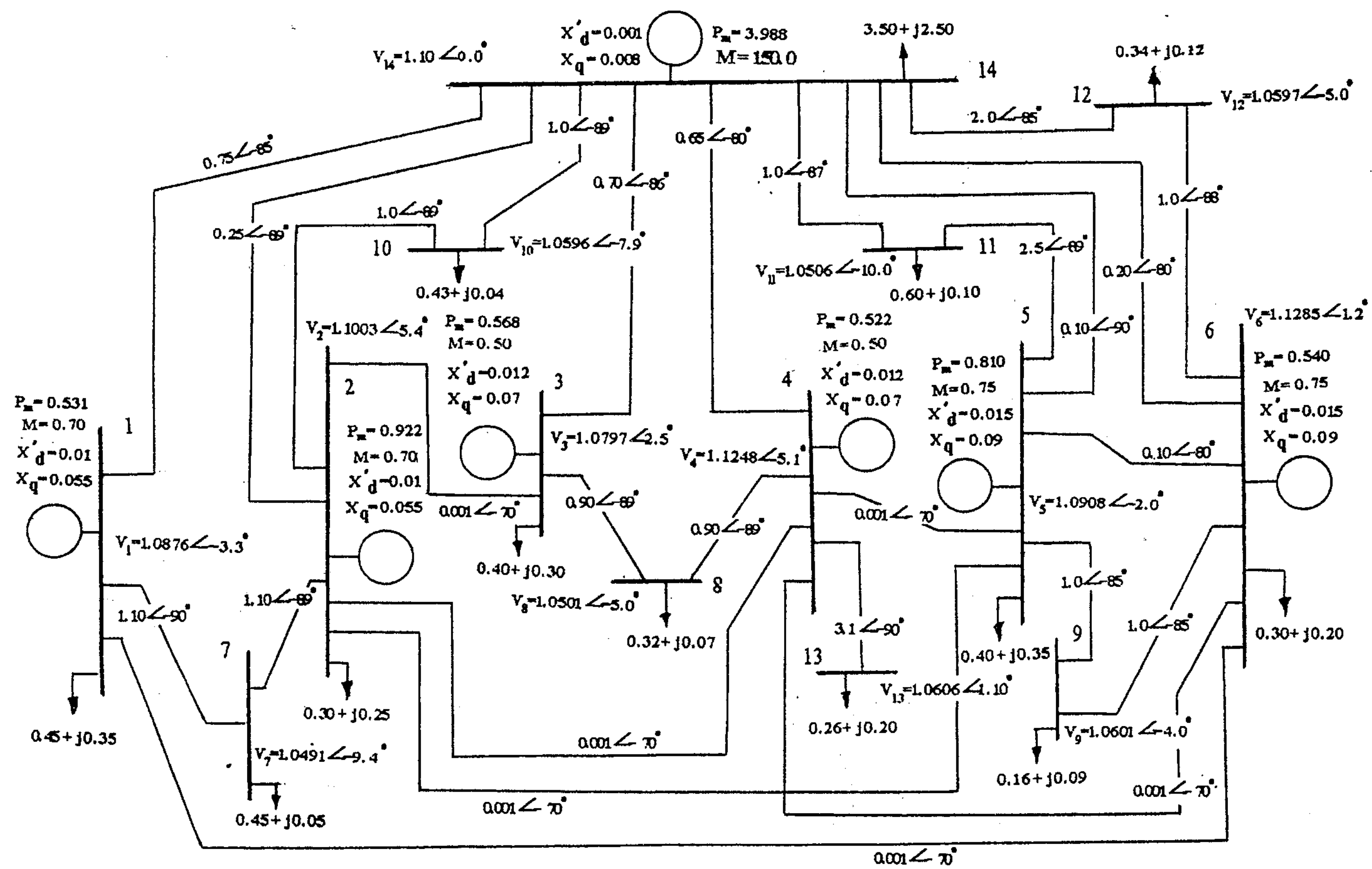

Fig. 1 Seven-machine, 14-bus power system.

Now, the developed stability approach is used to determine directly the critical reclosing time for a 3-phase short circuit fault (with successful reclosure) in three case studies as follows:

1 – The fault is occurred close to generator bus 2 (at 2 % length of the line connecting buses 2 and 7). For isolating the fault, the three lines 2–7, 2–10 and 2–14 are opened simultaneously using 6–cycle C.B. In addition, due to operation of the under-voltage relay, the load connected to bus 2 is also removed using 6-cycle C.B. Referring to eqn.22, it is found that the critical time for reclosing the open lines with reconnecting the removed load is equal to 0.162 sec (measured from the fault instant). Note that, using the standard step-by-step method, the exact critical reclosing time is equal to 0.189 sec. Figs. 2-1, through 2-5, show variations of the first subsystem (including machines 1,2 and 7) states just after reconnecting the removed load and open lines.

2 – It is assumed in this case that the fault is occurred at a point near bus 3 and far a distance 2 % length of the line connecting buses 3 and 8. After elapsing 0.16 sec, the fault is cleared by tripping out the lines 3–8 and 3–14 simultaneously. In addition, the load connected to bus 3 is removed using 10-cycle C.B. It is found, referring to eqn.22, that the open two lines with the removed load should be reconnected after 0.245 sec (the exact time equals 0.295 sec) from the fault instant. Figs. 3-1 through 3-3, show variations of the second subsystem (including machines 3, 4 and 7) states just after reclosing the open lines with reconnecting the removed load.

3 – In this case the fault is occurred near load bus 12 (at 5 % length of the line connecting buses 12 and 14). Using 10-cycle C.B. the faulted line is isolated, and after elapsing 0.10 sec from the isolation instant a pulsating load, which may simulate a load comprising large motors of a rolling mill, of the power (1.20+j0.50) is added to the load of bus 6. It is found that (eqn.22 is satisfied), the added load can be kept connected with opening the faulted line until a time of 0.345 sec (the exact time equals 0.410 sec) is elapsed from the fault instant. Figs.4-1 through 4-3, show variations of the third subsystem (including machines 5, 6 and 7) states just after reclosing the open line with removing the added load.

It is to be noted that, variations of the states of machine "7" for the considered three fault cases are very small and hence they are not shown in Figs 2, 3 and 4. It is also noted that the computed critical times are nearly equal (about 85 %) the exact times computed by using the standard step-by-step method.

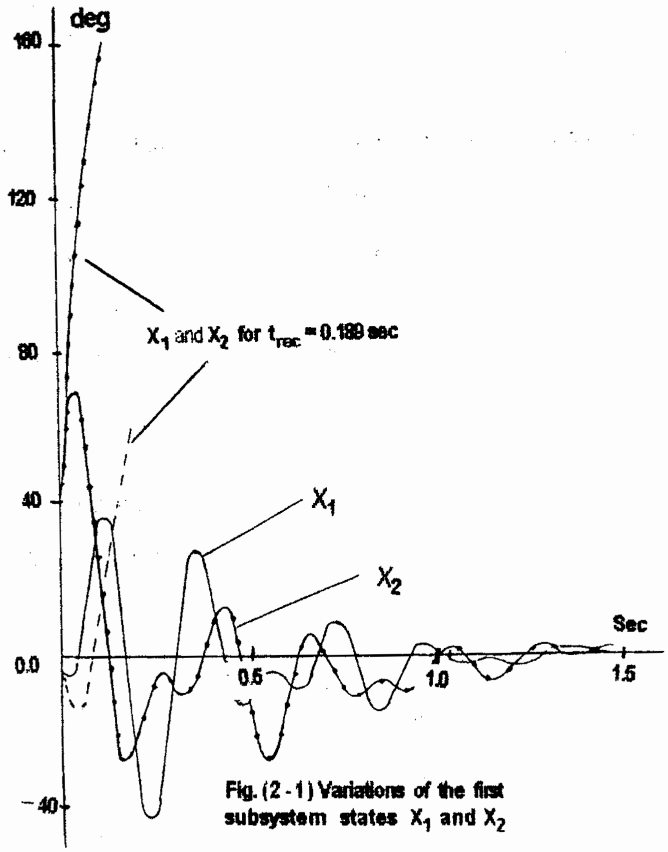

Fig. (2-1) Variations of the first subsystem states X_1 and X_2

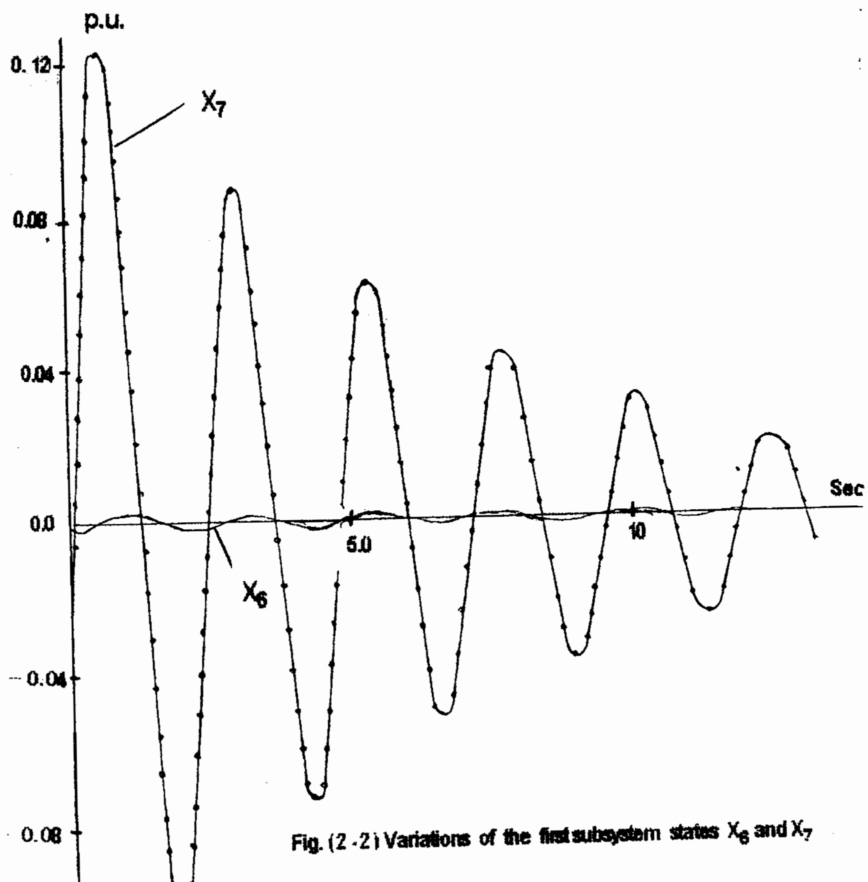

Fig. (2-2) Variations of the first subsystem states X_6 and X_7

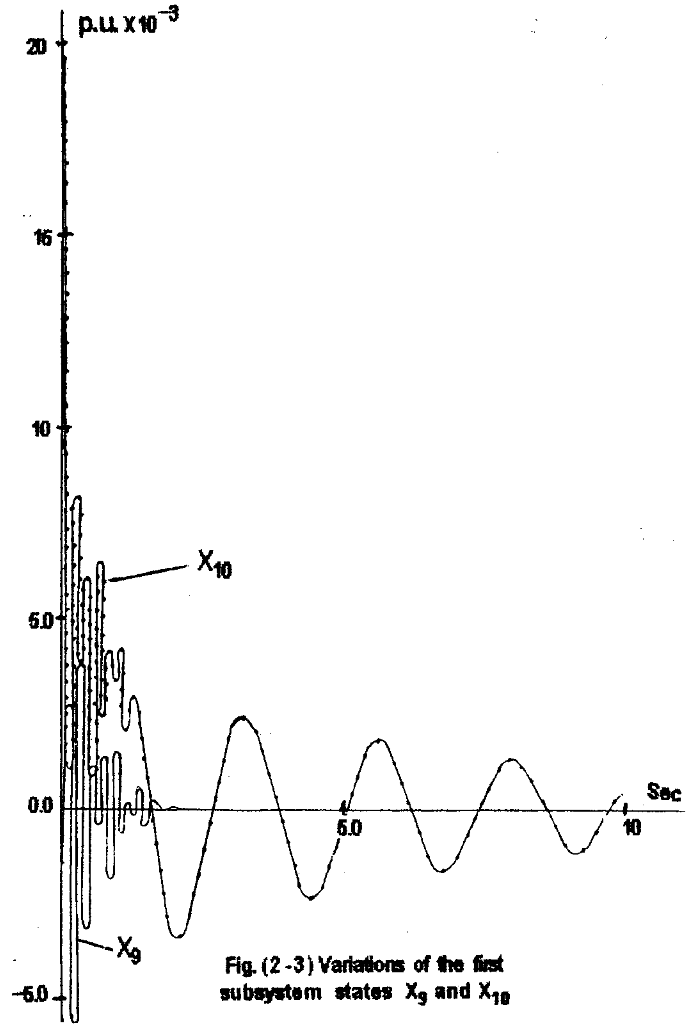

Fig. (2-3) Variations of the first subsystem states X_9 and X_{10}

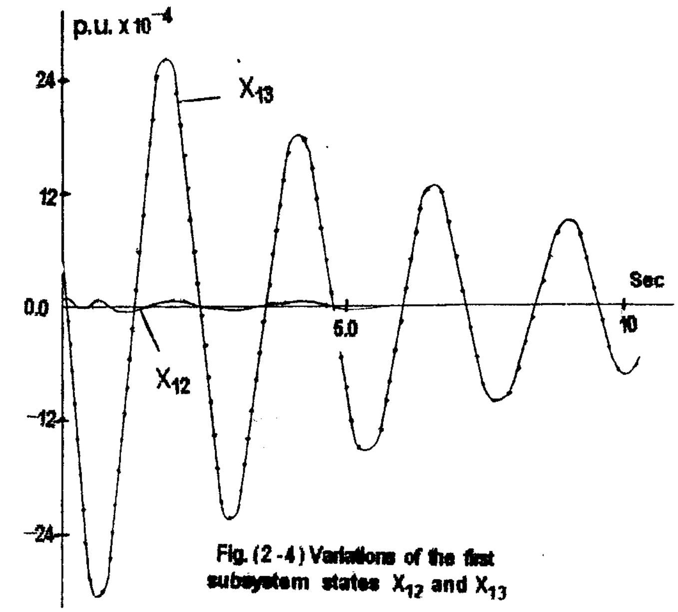

Fig. (2-4) Variations of the first subsystem states X_{12} and X_{13}

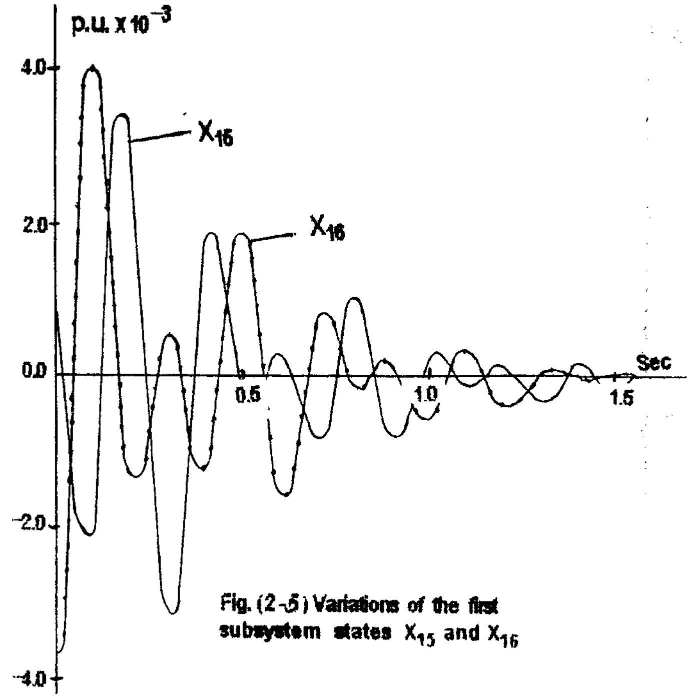

Fig. (2-5) Variations of the first subsystem states X_{15} and X_{16}

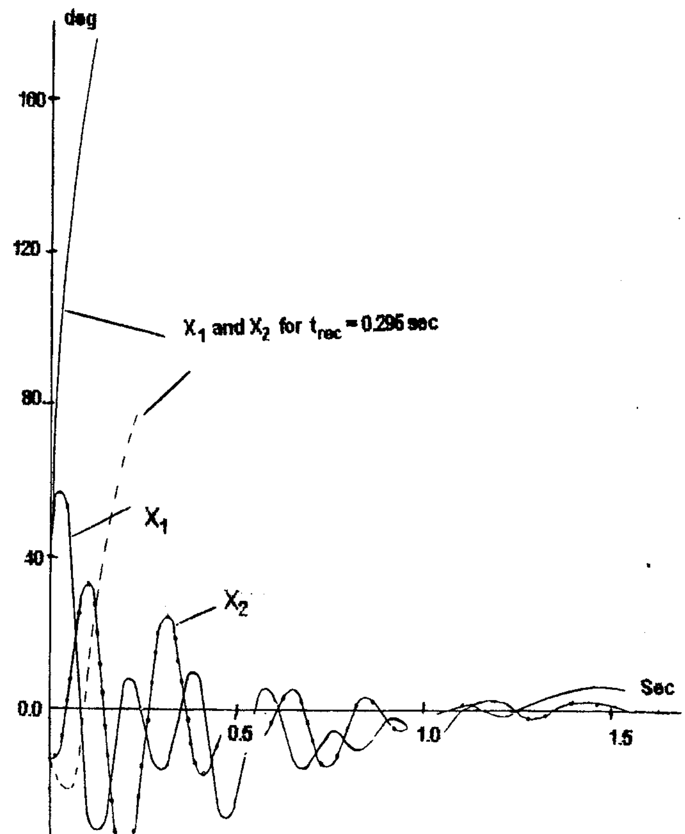

Fig. (3-1) Variations of the second subsystem states X_1 and X_2

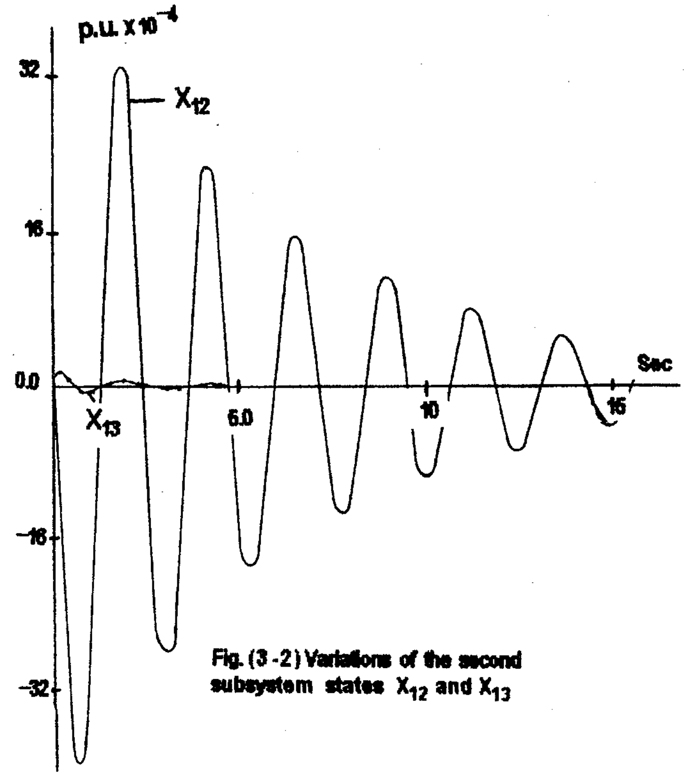

Fig. (3 -2) Variations of the second subsystem states X_{12} and X_{13}

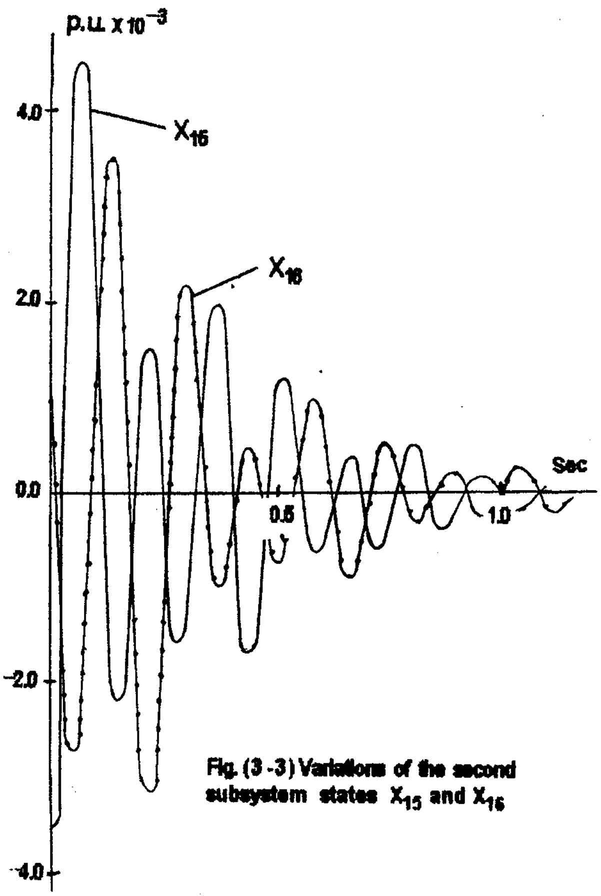

Fig. (3 -3) Variations of the second subsystem states X_{15} and X_{16}

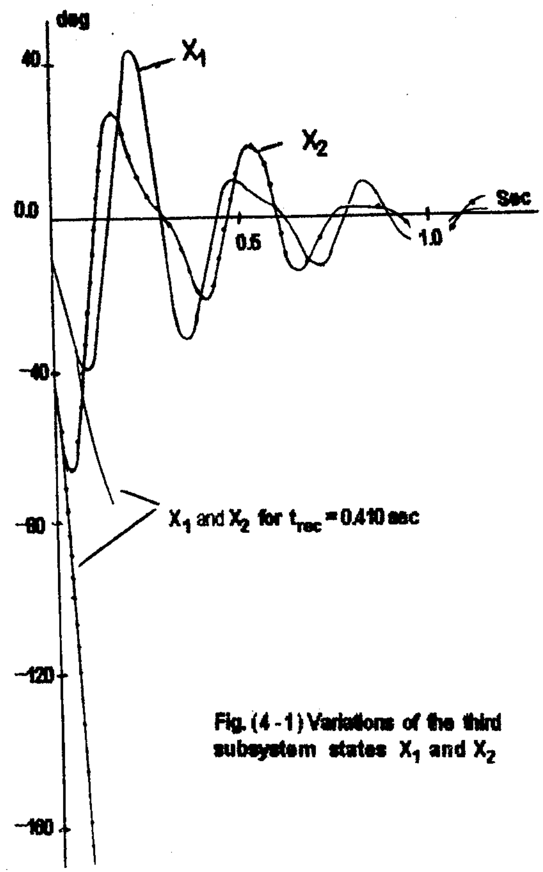

Fig. (4 - 1) Variations of the third subsystem states X_1 and X_2

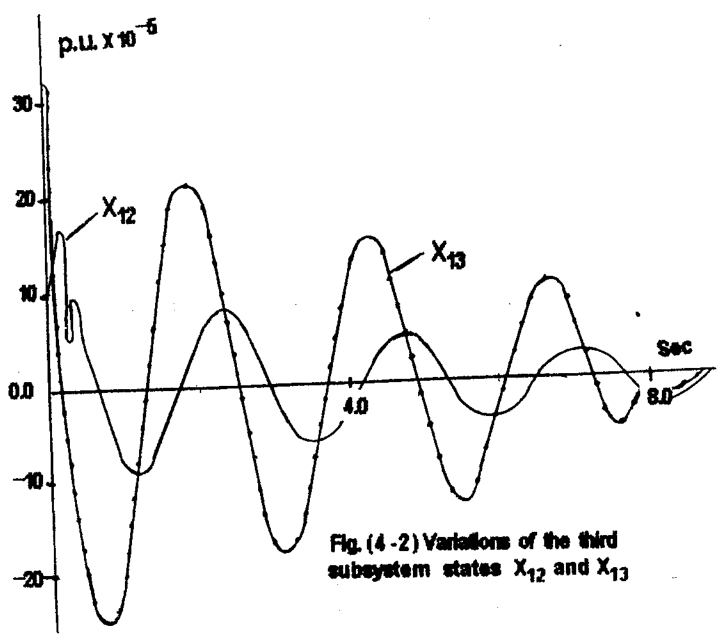

Fig. (4 -2) Variations of the third subsystem states X_{12} and X_{13}

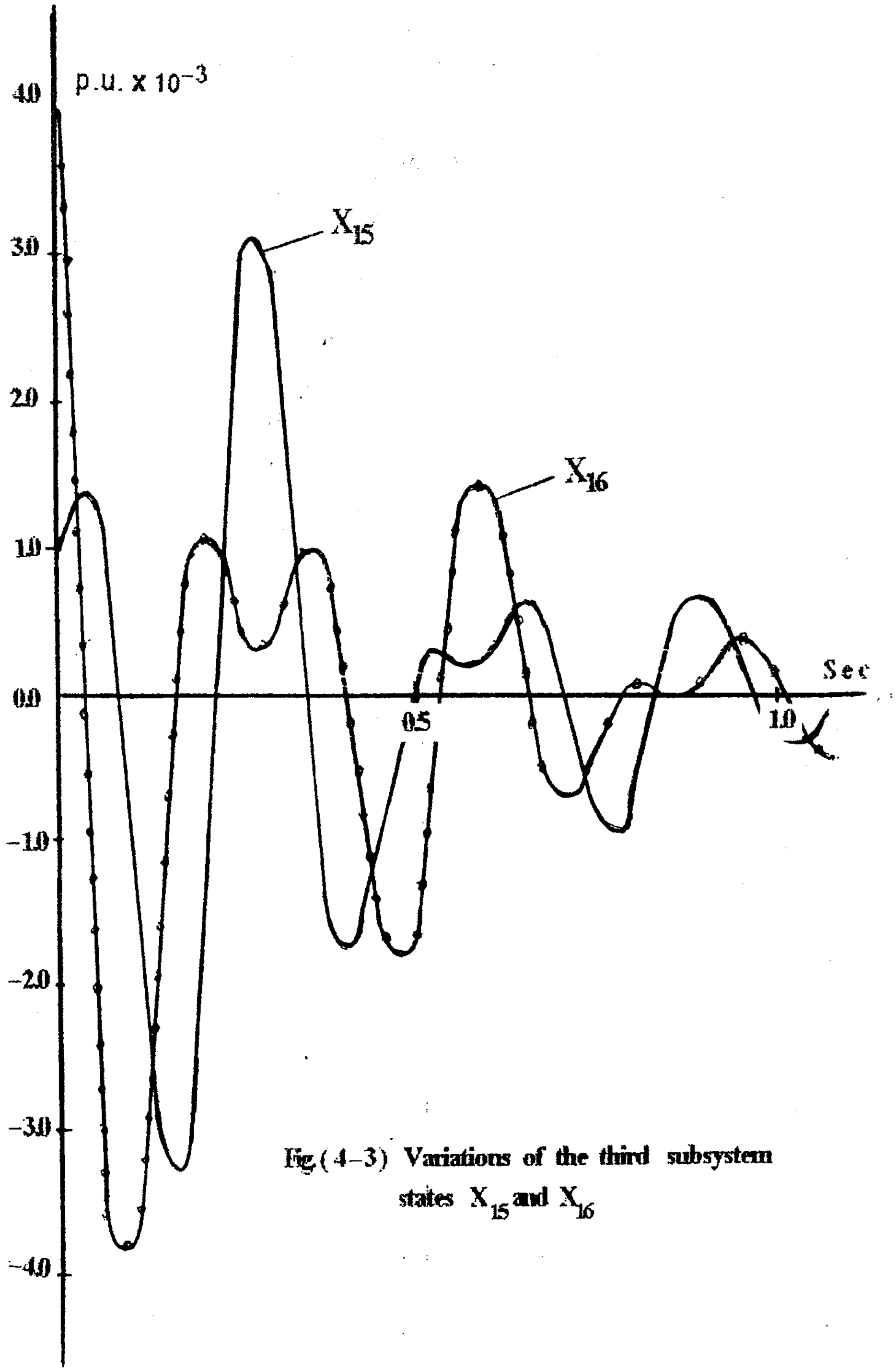

Fig.(4-3) Variations of the third subsystem states X_{15} and X_{16}

6 Conclusions :

A new transient stability approach is developed, in the paper, and the following conclusions are drawn:

1–For transient stability studies of real power systems, the developed stability approach is more suitable than the stability approaches developed so far [9–18]. Note that, in the developed approach changes of the generator internal voltage components E'_q and E'_d are considered. In addition, the automatic voltage regulator (AVR) effect and the speed governor action are taken into consideration.

2– Using the developed approach (the transfer conductances are taken into consideration), resistance of the system lines can be considered and the system network can be simplified by eliminating non-generator nodes.

3– For a real power system (the number of generators is, in general, much less than the number of buses), computations of the developed aggregation matrix and its stability conditions are simple. Note that, order of the developed aggregation matrix depends only on number of the system machines.

4– The developed approach is powerful and it may be simply used to carry out practical stability studies of multimachine power systems.

5- The developed approach can open new horizons to sensitivity analysis problem of power systems. Note that the generator parameters as well as the voltage regulator and speed governor parameters are considered.

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APPENDIX

Table I : The system reduced admittance matrix (moduli in p.u. and arguments in deg.)

1	1.68799 $\angle -71.68$	0.51323 $\angle 79.44$	0.00051 $\angle 98.07$	0.00048 $\angle 104.0$
	0.00058 $\angle 88.92$	0.00155 $\angle 103.64$	0.73660 $\angle 94.39$	

2	1.60395 ∠−74.06 0.00060 ∠84.22	0.00144 ∠101.94 0.71713 ∠83.55	0.00142 ∠104.24	0.00146 ∠97.72
3	1.47795 ∠−72.23 0.68687 ∠93.48	0.41549 ∠81.64	0.00054 ∠87.73	0.00057 ∠93.72
4	1.33515 ∠−73.44	0.00145 ∠103.91	0.00149 ∠105.97	0.63920 ∠99.46
5	1.89280 ∠−69.0	0.54867 ∠91.89	0.76704 ∠83.94	
6	1.66321 ∠−75.80	0.81418 ∠ 89.98		
7	7.43937 ∠−62.43			

Definition of the elements of 18-th order matrix R :

$$r^{I}_{11} = 2\, a_I \{D_{iI}\, \varepsilon_{I1} - \tilde{D}_{iI} - m_{iI,\, iI+1} - \Sigma\, U_{iI,\, j}\}$$

$$r^{I}_{22} = 2\, \bar{a}_I \{D_{iI+1}\, \varepsilon_{I2} - \tilde{D}_{iI+1} - m_{iI+1,\, iI} - \Sigma\, U_{iI+1,\, j}\}$$

$$r^{I}_{33} = 2\,(\lambda_{iI}\, h^{I}_{33} - h^{I}_{13}) \quad , \quad r^{I}_{44} = 2\,(\lambda_{iI+1}\, h^{I}_{44} - h^{I}_{24})$$

$$r^{I}_{55} = 2\ \lambda_N\, h^{I}_{55} \quad , \quad r^{I}_{66} = 2\, \Gamma_{iI}\, h^{I}_{66}$$

$$r^{I}_{77} = 2\, \Gamma_{iI+1}\, h^{I}_{77} \quad , \quad r^{I}_{88} = 2\, \Gamma_N\, h^{I}_{88}$$

$$r^{I}_{99} = 2\, \beta_{iI}\, h^{I}_{99} \quad , \quad r^{I}_{10,10} = 2\, \beta_{iI+1}\, h^{I}_{10,10}$$

$$r^{I}_{11,11} = 2\, \beta_N\, h^{I}_{11,11} \quad , \quad r^{I}_{12,12} = 2\, \rho_{iI}\, h^{I}_{66}$$

$$r^{I}_{13,13} = 2\, \rho_{iI+1}\, h^{I}_{77} \quad , \quad r^{I}_{14,14} = 2\, h^{I}_{88}\, \rho_N$$

$$r^{I}_{15,15} = 2\, \rho^{*}_{iI}\, [b_I + (a_I / \mu_{iI})]$$

$$r^{I}_{16,16} = 2\, \rho^{*}_{iI+1}\, [\bar{b}_I + (\bar{a}_I / \mu_{iI+1})]$$

$$r^{I}_{17,17} = 2\ \rho^{*}_{N}\, b_N$$

$$r^{I}_{12} = -\left|\, (a_I - \bar{a}_I)\, g_{iI,\, iI+1} - (a_I + \bar{a}_I)\, \hat{g}_{iI,\, iI+1} \right|\, \zeta_{I3}$$

$$r^{I}_{13} = -\, b_I\, [\tilde{D}_{iI} + m_{iI,\, iI+1} + \Sigma\, \overline{U}_{iI,\, j}] \quad , \quad r^{I}_{14} = -\, \bar{b}_I\, m_{iI+1,\, iI}$$

$$r^{I}_{15} = -\, b_I\, D_{iI}\, \xi_{iI} - b_N\, \bar{g}_{iI} - \mathrm{Max}\, [\, \tilde{T}_{iI}\, h^{I}_{13}\ ;\ b_N\, Đ_{iI}]$$

$$r^{I}_{16} = -a_I\, [n_{iI} + H_{N,\, iI} + H_{iI+1,\, iI}] - c_I\, [\breve{G}_{iI} + \tilde{G}_{iI} + \overline{G}_{iI,\, iI+1}] - (a_I + c_I)\ \Sigma\ \tilde{U}_{iI,\, j}$$

$$r^{I}_{17} = -\, a_I\, H_{iI,\, iI+1} - \bar{c}_I\ \overline{G}_{iI+1,\, iI}$$

$$r^{I}_{18} = -\, a_I H_{iI,\, N} - c_N\, [\, \breve{g}_{iI} + \tilde{N}_{iI}]$$

$$r^{I}_{1,9} = -\, a_I\, [\bar{n}_{iI} + H_{N,\, iI} + H_{iI+1,\, iI}] - d_I\ [(\breve{G}_{iI} / \tau_{iI}) + \tilde{G}_{iI}\, \tau_{iI} + \mathcal{G}_{iI,\, iI+1}\,] - (a_I + d_I)\, \Sigma\, \hat{U}_{iI,\, j}$$

$$r^{I}_{1,10} = -\, a_I\, H_{iI,\, iI+1} - \bar{d}_I\, \mathcal{G}_{iI+1,\, iI}$$

$$r^{I}_{1,11} = -a_I H_{iI,N} - d_N[(\breve{g}_{iI}/\tau_{iI}) + \tilde{N}_{iI}\tau_{iI}]$$

$$r^{I}_{1,12} = -\breve{n}_{I6}[\breve{G}_{iI} + \overline{G}_{iI,iI+1} + \tilde{G}_{iI} + \Sigma\,\tilde{\breve{U}}_{iI,j}]$$

$$r^{I}_{1,13} = -\breve{n}_{I7}\overline{G}_{iI+1,iI} \quad , \quad r^{I}_{1,14} = -\breve{n}_{I8}[\breve{g}_{iI} + \tilde{N}_{iI}]$$

$$r^{I}_{23} = -b_I m_{iI,iI+1}$$

$$r^{I}_{24} = -\bar{b}_I[\tilde{D}_{iI+1} + m_{iI+1,iI} + \Sigma\,\overline{U}_{iI+1,j}]$$

$$r^{I}_{25} = -\bar{b}_I D_{iI+1}\xi_{iI+1} - b_N \bar{g}_{iI+1} - \mathrm{Max}[\tilde{T}_{iI+1} h^{I}_{24} \; ; \; b_N Đ_{iI+1}]$$

$$r^{I}_{26} = -\bar{a}_I H_{iI+1,iI} - c_I \overline{G}_{iI,iI+1}$$

$$r^{I}_{27} = -\bar{a}_I[n_{iI+1} + H_{N,iI+1} + H_{iI,iI+1}] - \bar{c}_I[\breve{G}_{iI+1} + \tilde{G}_{iI+1} + \overline{G}_{iI+1,iI}] -$$
$$-(\bar{a}_I + \bar{c}_I)\,\Sigma\,\tilde{\breve{U}}_{iI+1,j}$$

$$r^{I}_{28} = -\bar{a}_I H_{iI+1,N} - c_N[\breve{g}_{iI+1} + \tilde{N}_{iI+1}]$$

$$r^{I}_{2,9} = -\bar{a}_I H_{iI+1,iI} - d_I\, Ģ_{iI,iI+1}$$

$$r^{I}_{2,10} = -\bar{a}_I[\bar{n}_{iI+1} + H_{iI,iI+1} + H_{N,iI+1}] - \bar{d}_I[(\breve{G}_{iI+1}/\tau_{iI+1}) +$$
$$+\tilde{G}_{iI+1}\tau_{iI+1} + Ģ_{iI+1,iI}] - (\bar{a}_I + \bar{d}_I)\,\Sigma\,\hat{U}_{iI+1,j}$$

$$r^{I}_{2,11} = -\bar{a}_I H_{iI+1,N} - d_N[(\breve{g}_{iI+1}/\tau_{iI+1}) + \tilde{N}_{iI+1}\tau_{iI+1}]$$

$$r^{I}_{2,12} = -\breve{n}_{I6}\overline{G}_{iI,iI+1}$$

$$r^{I}_{2,13} = -\tilde{n}_{iI+1} h^{I}_{77}[\breve{G}_{iI+1} + \overline{G}_{iI+1,iI} + \tilde{G}_{iI+1} + \Sigma\,\tilde{\breve{U}}_{iI+1,j}]$$

$$r^{I}_{2,14} = -\breve{n}_{I8}[\breve{g}_{iI+1} + \tilde{N}_{iI+1}]$$

$$r^{I}_{2,18} = -\left|(\bar{a}_I - a_I)\dot{g}_{iI,iI+1} - (\bar{a}_I + a_I)\bar{e}_{iI,iI+1}\right|$$

$$r^{I}_{35} = -h^{I}_{13} \quad , \quad r^{I}_{36} = -b_I[n_{iI} + H_{N,iI} + H_{iI+1,iI} + \Sigma\,\tilde{\breve{U}}_{iI,j}]$$

$$r^{I}_{37} = -b_I H_{iI,iI+1} \quad , \quad r^{I}_{38} = -b_I H_{iI,N}$$

$$r^{I}_{3,9} = -b_I[\bar{n}_{iI} + H_{iI+1,iI} + H_{N,iI} + \Sigma\,\hat{U}_{iI,j}]$$

$$r^{I}_{3,10} = r^{I}_{37} \quad , \quad r^{I}_{3,11} = r^{I}_{38}$$

$$r^{I}_{45} = -h^{I}_{24} \quad , \quad r^{I}_{46} = -\bar{b}_I H_{iI+1,iI}$$

$$r^{I}_{47} = -\bar{b}_I[n_{iI+1} + H_{N,iI+1} + H_{iI,iI+1} + \Sigma\,\tilde{\breve{U}}_{iI+1,j}]$$

$$r^{I}_{48} = -\bar{b}_I H_{iI+1,N} \quad , \quad r^{I}_{49} = r^{I}_{46}$$

$$r^{I}_{4,10} = -\bar{b}_I[\bar{n}_{iI+1} + H_{N,iI+1} + H_{iI,iI+1} + \Sigma\,\hat{U}_{iI+1,j}]$$

$$r^{I}_{4,11} = r^{I}_{4,8} \quad , \quad r^{I}_{4,18} = -\left|(\bar{b}_I - b_I)\dot{g}_{iI,iI+1} - (\bar{b}_I + b_I)\bar{e}_{iI,iI+1}\right|$$

$$r^{I}_{56} = -b_N H_{N,iI} \quad , \quad r^{I}_{57} = -b_N H_{N,iI+1}$$

$$r^{I}_{58} = -b_N[n_N + H_{iI,N} + H_{iI+1,N} + \Sigma\,\tilde{\breve{U}}_{N,j}]$$

$$r^{I}_{59} = r^{I}_{56} \quad , \quad r^{I}_{5,10} = r^{I}_{57}$$

$$r^{I}_{5,11} = -b_N[\bar{n}_N + H_{iI,N} + H_{iI+1,N} + \Sigma\,\hat{U}_{N,j}]$$

$r^{I}_{5,15} = -(a_I/\mu_{iI})$, $r^{I}_{5,16} = -(\bar{a}_I/\mu_{iI+1})$

$r^{I}_{67} = -Y_{iI,\,iI+1}\sqrt{c_I^2+\bar{c}_I^2-2\bar{c}_I c_I\cos(2\theta_{iI,\,iI+1})}$

$r^{I}_{68} = -Y_{iI,\,N}\sqrt{c_I^2+c_N^2-2c_I\,c_N\cos(2\theta_{iI,\,N})}$

$r^{I}_{69} = -|c_I - d_I|\,G_{iI,\,iI}$

$r^{I}_{6,10} = -Y_{iI,\,iI+1}\sqrt{c_I^2+\bar{d}_I^2-2c_I\,\bar{d}_I\cos(2\theta_{iI,\,iI+1})}$

$r^{I}_{6,11} = -Y_{iI,\,N}\sqrt{c_I^2+d_N^2-2\,c_I\,d_N\cos(2\theta_{iI,\,N})}$

$r^{I}_{6,13} = -\check{n}_{I7}Y_{iI,\,iI+1}$, $r^{I}_{6,14} = -\check{n}_{I8}Y_{iI,\,N}$

$r^{I}_{6,18} = -c_I\,\breve{e}_{iI+1,\,iI}$

$r^{I}_{78} = -Y_{iI+1,\,N}\sqrt{\{\bar{c}_I^2+c_N^2-2\bar{c}_I\,c_N\cos(2\theta_{iI+1,\,N})\}}$

$r^{I}_{79} = -Y_{iI,\,iI+1}\sqrt{\{\bar{c}_I^2+d_I^2-2\bar{c}_I\,d_I\cos(2\theta_{iI,\,iI+1})\}}$

$r^{I}_{7,10} = -|\bar{c}_I - \bar{d}_I|\,G_{iI+1,\,iI+1}$

$r^{I}_{7,11} = -Y_{iI+1,\,N}\sqrt{\{\bar{c}_I^2+d_N^2-2\bar{c}_I\,d_N\cos(2\theta_{iI+1,\,N})\}}$

$r^{I}_{7,12} = -\check{n}_{I6}Y_{iI,\,iI+1}$, $r^{I}_{7,14} = -\check{n}_{I8}Y_{iI+1,\,N}$

$r^{I}_{7,18} = -\bar{c}_I\,\breve{e}_{iI,\,iI+1}$

$r^{I}_{89} = -Y_{iI,\,N}\sqrt{c_N^2+d_I^2-2\,c_N\,d_I\cos(2\theta_{iI,\,N})}$

$r^{I}_{8,10} = -Y_{iI+1,\,N}\sqrt{c_N^2+\bar{d}_I^2-2\,c_N\bar{d}_I\cos(2\theta_{iI+1,\,N})}$

$r^{I}_{8,11} = -|c_N - d_N|\,G_{N,N}$, $r^{I}_{8,12} = -\check{n}_{I6}Y_{iI,\,N}$

$r^{I}_{8,13} = -\check{n}_{I7}Y_{iI+1,\,N}$

$r^{I}_{9,10} = -Y_{iI,\,iI+1}\sqrt{d_I^2+\bar{d}_I^2-2\,d_I\bar{d}_I\cos(2\theta_{iI,\,iI+1})}$

$r^{I}_{9,11} = -Y_{iI,\,N}\sqrt{d_I^2+d_N^2-2\,d_I\,d_N\cos(2\theta_{iI,\,N})}$

$r^{I}_{9,12} = -\hat{\Lambda}_{iI}\,h^{I}_{66}$, $r^{I}_{9,13} = -\check{n}_{I7}Y_{iI,\,iI+1}$

$r^{I}_{9,14} = -\check{n}_{I8}Y_{iI,\,N}$, $r^{I}_{9,18} = -d_I\,\hat{G}_{iI,\,iI+1}$

$r^{I}_{10,11} = -Y_{iI+1,\,N}\sqrt{d_N^2+\bar{d}_I^2-2\,d_N\bar{d}_I\cos(2\theta_{iI+1,\,N})}$

$r^{I}_{10,12} = -\check{n}_{I6}\,Y_{iI,\,iI+1}$, $r^{I}_{10,13} = -\hat{\Lambda}_{iI+1}\,h^{I}_{77}$

$r^{I}_{10,14} = -\check{n}_{I8}Y_{iI+1,\,N}$, $r^{I}_{10,18} = -\bar{d}_I\hat{G}_{iI+1,\,iI}$

$r^{I}_{11,12} = -\check{n}_{I6}Y_{iI,\,N}$, $r^{I}_{11,13} = -\check{n}_{I7}\,Y_{iI+1,\,N}$

$r^{I}_{11,14} = -\hat{\Lambda}_N\,h^{I}_{88}$. $r^{I}_{12,18} = -\check{n}_{I6}\,\breve{e}_{iI+1,\,iI}$

$r^{I}_{13,18} = -\check{n}_{I7}\,\breve{e}_{iI,\,iI+1}$

$$r^{I}_{18,18} = 2\,a_I\,(\dot{g}_{iI,\,iI+1} + \bar{e}_{iI,\,iI+1})\,/\,\xi_{i3} \qquad \text{(A -1)}$$

where the other elements of this matrix are zero.

Definition of the aggregation matrix off-diagonal elements:

In eqn.21, Z_{IK} is given as (see Notation),

$$Z_{IK} = Z_3\,[Z_2(\,\overline{Z}_{I,K}\,;\,\overline{Z}_{I,K+1})\,;\,Z_2(\,\hat{Z}_{I,K}\,;\,\hat{Z}_{I,K+1})\,;\;;\,Z_2(\,\breve{Z}_{I,K}\,;\,\breve{Z}_{I,K+1})] \qquad (A-2)$$

where

$$\overline{Z}_{I,K} = Z_3[\tilde{Z}_{I,K}\,;\,b_N\,\overline{U}_{N,k}\,;\,d_N\,\hat{U}_{N,k}]$$

$$\overline{Z}_{I,K+1} = Z_3[\tilde{Z}_{I,K+1}\,;\,b_N\,\overline{U}_{N,k1}\,;\,d_N\hat{U}_{N,k1}]$$

$$\hat{Z}_{I,K} = Z_3[\dot{Z}_{I,K}\,;\,b_N\,H_{N,k}\,;\,c_N\,\tilde{a}_{N,k}]$$

$$\hat{Z}_{I,K+1} = Z_3[\dot{Z}_{I,K+1}\,;\,b_N H_{N,k1}\,;\,c_N\,\tilde{a}_{N,k1}]$$

$$\breve{Z}_{I,K} = Z_3[Z^{\circ}_{I,K}\,;\,b_N\,H_{N,k}\,;\,d_N\,\tilde{a}_{N,k}]$$

$$\breve{Z}_{I,K+1} = Z_3[Z^{\circ}_{I,K+1}\,;\,b_N\,H_{N,k1}\,;\,c_N\,\tilde{a}_{N,k1}]$$

and where,

$$\tilde{Z}_{I,K} = Z_4[\,Z_2(\hat{R}_I\,\overline{U}_{i,k}\,;\,\tilde{R}_I\overline{U}_{i1,k})\,;\,Z_3(\,c_I\,\tilde{U}_{i,k}\,;\,\bar{c}_I\,\tilde{U}_{i1,k}\,;\;;\,c_N\,\tilde{U}_{N,k})\,;\,Z_2(\,d_I\check{U}_{i,k}\,;\,\bar{d}_I\,\hat{U}_{i1,k})\,;\,Z_3(\,\check{n}_{I6}\,\tilde{U}_{i,k}\,;\;;\,\check{n}_{I7}\,\tilde{U}_{i1,k}\,;\,\check{n}_{I8}\,\tilde{U}_{N,k})]$$

$$\tilde{Z}_{I,K+1} = Z_4[\,Z_2(\hat{R}_I\,\overline{U}_{i,k1}\,;\,\tilde{R}_I\overline{U}_{i1,k1})\,;\,Z_3(\,c_I\,\tilde{U}_{i,k1}\,;\;;\bar{c}_I\,\tilde{U}_{i1,k1}\,;\,c_N\,\tilde{U}_{N,k1})\,;\,Z_2(\,d_I\check{U}_{i,k1}\,;\,\bar{d}_I\,\hat{U}_{i1,k1})\,;\;;\,Z_3(\check{n}_{I6}\,\tilde{U}_{i,k1}\,;\,\check{n}_{I7}\,\tilde{U}_{i1,k1}\,;\,\check{n}_{I8}\,\tilde{U}_{N,k1})]$$

$$\dot{Z}_{I,K} = Z_4[\,Z_2(\overline{R}_I\,H_{i,k}\,;\,\tilde{R}_I H_{i1,k})\,;\,Z_2(c_I\,\tilde{a}_{i,k};\bar{c}_I\,\tilde{a}_{i1,k})\,;\;;Z_3(d_I\,\tilde{b}_{i,k}\,;\,\bar{d}_I\tilde{b}_{i1,k}\,;\,d_N\,\tilde{b}_{N,k})\,;\,Z_3\,(\check{n}_{I6}\,\tilde{a}_{i,k}\,;\;;\,\check{n}_{I7}\,\tilde{a}_{i1,k}\,;\,\check{n}_{I8}\,\tilde{a}_{N,k})]$$

$$\dot{Z}_{I,K+1} = Z_4[\,Z_2(\overline{R}_I\,H_{i,k1}\,;\,\tilde{R}_I H_{i1,k1})\,;\,Z_2(c_I\,\tilde{a}_{i,k1}\,;\;;\bar{c}_I\,\tilde{a}_{i1,k1})\,;\,Z_3(d_I\,\tilde{b}_{i,k1}\,;\bar{d}_I\,\tilde{b}_{i1,k1}\,;\,d_N\,\tilde{b}_{N,k1})\,;\;;\,Z_3(\,\check{n}_{I6}\,\tilde{a}_{i,k1}\,;\,\check{n}_{I7}\,\tilde{a}_{i1,k1}\,;\,\check{n}_{I8}\,\tilde{a}_{N,k1})]$$

$$Z^{\circ}_{I,K} = Z_4[\,Z_2(\overline{R}_I\,H_{i,k}\,;\,\tilde{R}_I H_{i1,k})\,;\,Z_3\,(c_I\,\tilde{b}_{i,k}\,;\,\bar{c}_I\,\tilde{b}_{i1,k}\,;\;;\,c_N\,\tilde{b}_{N,k})\,;\,Z_2(d_I\,\tilde{a}_{i,k};\bar{d}_I\,\tilde{a}_{i1,k})\,;\,Z_3\,(\check{n}_{I6}\tilde{b}_{i,k}\,;\;;\,\check{n}_{I7}\tilde{b}_{i1,k}\,;\,\check{n}_{I8}\,\tilde{b}_{N,k})]$$

$$Z^{\circ}_{I,K+1} = Z_4[\,Z_2(\overline{R}_I\,H_{i,k1}\,;\,\tilde{R}_I H_{i1,k1})\,;\,Z_3\,(c_I\,\tilde{b}_{i,k1}\,;\;;\bar{c}_I\,\tilde{b}_{i1,k1}\,;\,c_N\,\tilde{b}_{N,k1})\,;\,Z_2(d_I\,\tilde{a}_{i,k1}\,;\bar{d}_I\,\tilde{a}_{i1,k1})\,;\;;\,Z_3(\check{n}_{I6}\tilde{b}_{i,k1}\,;\,\check{n}_{I7}\tilde{b}_{i1,k1}\,;\,\check{n}_{I8}\,\tilde{b}_{N,k1})]$$

In eqns.(A-1) and (A - 2), the following constants are defined,

$a_I = h^I_{13} / M_{iI}$, $\bar{a}_I = h^I_{24} / M_{iI+1}$

$b_I = h^I_{33} / M_{iI}$, $\bar{b}_I = h^I_{44} / M_{iI+1}$, $b_N = h^I_{55} / M_N$

$c_I = K_{iI}\, h^I_{66}$, $\bar{c}_I = K_{iI+1}\, h^I_{77}$, $c_N = K_N\, h^I_{88}$

$d_I = L_{iI}\, h^I_{99}$, $\bar{d}_I = L_{iI+1}\, h^I_{10,10}$, $d_N = L_N\, h^I_{11,11}$

$\xi_{I3} = \cos\delta^{o}_{iI,\, iI+1}$, $\zeta_{I3} = |\sin\delta^{o}_{iI,\, iI+1}|$

$\hat{R}_I = Z_2(h^I_{13} ; h^I_{33}) / M_{iI}$

$\tilde{R}_I = Z_2(h^I_{24} ; h^I_{44}) / M_{iI+1}$

$D_j = (A_{jN}\, B_{jN} + \breve{A}_{jN}\ G_{jN})$, $\zeta_j = |\sin\delta^{o}_{j,N}|$

$\tilde{D}_j = |\, A_{jN}\, G_{jN} - \tilde{A}_{jN}\ B_{jN}\, |\, \zeta_j$

$\tilde{G}_j = |\, \hat{E}_{qN}\, B_{jN} + \hat{E}_{dN}\, G_{jN}\, |\, \zeta_j$

$\bar{g}_j = |\, A_{jN}\, G_{jN} + \breve{A}_{jN}\, B_{jN}\, |\, \zeta_j$

$\tilde{N}_j = |\, \hat{E}_{dj}\, G_{jN} + \hat{E}_{qj}\, B_{jN}\, |\, \zeta_j$

$\xi_j = \cos\delta^{o}_{j,N}$, $Ð_j = (A_{jN}\, B_{jN} - \breve{A}_{jN}\ G_{jN})\, \xi_j$

$\breve{G}_j = |\, \hat{E}_{dN}\, B_{jN} - \hat{E}_{qN}\ G_{jN}\, |\, \xi_j$

$\breve{g}_j = |\, \hat{E}_{dj}\, B_{jN} - \hat{E}_{qj}\, G_{jN}\, |\ \xi_j$ $j \in J_I$

$n_j = \hat{E}_{qj}\, G_{jj}$, $\bar{n}_j = |\, \hat{E}_{dj}\, |\, G_{jj}$

$\tilde{n}_j = [\chi_j\, X'_{dj} / \ (1.0 + X'_{dj}\, B_{jj})]$

$\hat{\Lambda}_j = \chi_j\, \phi_j / \eta_j$, $\rho_j = (\chi_j / \eta_j\, T_{Ej})$

$\tau_j = \xi_j / \zeta_j$, $\rho^*_j = \mu_j / \alpha_j$ $j \in J_{IN}$

$m_{k,j} = |\, A_{k,j}\, G_{k,j} - \tilde{A}_{k,j}\, B_{k,j}\, |\, \zeta_{I3}$

$g_{k,j} = A_{k,j}\, |\, G_{kj}\, |$, $\hat{g}_{k,j} = \breve{A}_{k,j}\, B_{kj}$

$\tilde{T}_j = [\lambda_j + (1/ M_j\, \rho^*_j)]$

$$\acute{g}_{k,j} = A_{k,j} B_{k,j} \quad , \quad \bar{e}_{k,j} = \tilde{A}_{k,j} G_{k,j}$$

$$e_{k,j} = \left| (\hat{E}_{qj} G_{k,j} - \hat{E}_{dj} B_{k,j}) \cos \delta^{o}_{k,j} \right|$$

$$\breve{e}_{k,j} = \left| \hat{E}_{dk} B_{k,j} - \hat{E}_{qk} G_{k,j} \right| \quad , \quad \mathcal{G}_{k,j} = \breve{e}_{k,j} \zeta_{I3}$$

$$\overline{G}_{k,j} = \left| \hat{E}_{dj} G_{k,j} + \hat{E}_{qj} B_{k,j} \right| \zeta_{I3}$$

$$\hat{G}_{k,j} = \overline{G}_{k,j} / \zeta_{I3} \qquad , k \neq j \, , k, j \in \mathbf{J}_I$$

$$\tilde{U}_{k,j} = Y_{k,j} (\hat{E}_{qj} \zeta_{k,j} + |\hat{E}_{dj}| \xi_{k,j})$$

$$U_{k,j} = Y_{k,j} \, \mathrm{Max} \, (\hat{E}_{qk} |\hat{E}_{dj}| ; \, |\hat{E}_{dk}| \hat{E}_{qj}) \, \zeta_{k,j}$$

$$\hat{U}_{k,j} = Y_{k,j} \, (\hat{E}_{qj} \, \xi_{k,j} + |\hat{E}_{dj}| \, \zeta_{k,j})$$

$$\overline{U}_{k,j} = [U_{k,j} + Y_{k,j} A_{k,j} \xi_{k,j}]$$

$$\tilde{a}_{k,j} = Y_{k,j} \xi_{k,j} \quad , \quad \tilde{b}_{k,j} = Y_{k,j} \zeta_{k,j}$$

$$H_{k,j} = Y_{k,j} \hat{E}_k \quad , \quad \xi_{k,j} = | \sin (\theta_{k,j} - \delta^{o}_{k,j}) |$$

$$\zeta_{k,j} = | \cos (\theta_{k,j} - \delta^{o}_{k,j}) | \qquad , k \in \mathbf{J}_{IN} \, , j \notin \mathbf{J}_{IN}$$

$$\breve{n}_{I6} = \tilde{n}_{iI} \; h^{I}_{66} \quad , \quad \breve{n}_{I7} = \tilde{n}_{iI+1} \; h^{I}_{77}$$

$$\breve{n}_{I6} = \tilde{n}_{N} \; h^{I}_{88}$$

$$(*)_{iI,\, iK} = (*)_{i,\, k} \quad , \quad (*)_{iI+1,\, iK} = (*)_{i1,\, k}$$

$$(*)_{N,\, iK} = {}^{*}_{N,\, k} \quad , \quad (*)_{iI,\, iK+1} = (*)_{i,\, k1}$$

$$(*)_{iI+1,\, iK+1} = (*)_{i1,\, k1} \quad , \quad (*)_{N,\, iK+1} = (*)_{N,\, k1}$$

أسلوب جديد للاتزان الانتقالي يطبق علي
أنظمة القدرة متعددة الآلات

ملخص البحث

يهدف هذا البحث الي تقديم أسلوب أتزان جديد يستخدم لإنجاز دراسات الاتزان الانتقالي لنظام قدره متعدد الآلات , يحتوى على"ن" آله, وذلك في حالة ما تكون مركبتي الجهد الداخلي (E'_d , E'_q) لكل آله متغيرة مع الزمن.

* مع الأخذ في الاعتبار كل من الآتي:

١ - الإخماد الميكانيكي الغير متماثل

٢ - تأثير منظم الجهد الأوتوماتيكي

٣ - تأثير متحكم السرعة

تم تمثيل كل آله من الآت نظام القدره بنموذج ديناميكي من الدرجة السادسة- ومن ثم أستنتج النموذج الرياضي للنظام.

* بتطبيق طريقة الفك والتراكب, تم فك نظام القدره الي تحت أنظمه كل منها يشتمل علي آلتين إضافة الي الآله المقارنه وذلك باستخدام أسلوب الفك الثلاثي .

* تم لكل تحت نظام حر أختيار دالة ليابونوف مقياسيه في شكل " صوره مربعه من الدرجة السابعة عشر + مجموع تكاملات ثلاثة دوال غير خطيه" – استخدمت هذه الدوال في تكوين دالة ليابونوف متجهه. باستخدام هذه الدالة المتجهة تم أجراء تراكب نظام القدره ومن ثم أمكن الحصول علي مصفوفه تراكب للنظام – أتزان هذه المصفوفه يتضمن الاتزان المقارب لنظام القدره.

* في مثال توضيحي أستخدم أسلوب الاتزان المقدم في أجراء دراسات الاتزان الانتقالي لنظام قدره مكون من سبعة مولدات ويشتمل علي أربعة عشر قضيبا. أفترض حدوث قصر ثلاثي الأوجه (مع أعاده قفل ناجح) في ثلاثة مواضع مختلفة- لكل موضع من هذه المواضع الثلاثة تم بطريقه مباشره أيجاد الزمن الحرج لأعادة القفل. وجد أن الأزمنة الحرجه التي تم الحصول عليها باستخدام الأسلوب المقدم مساويه تقريبا(حوالي ٨٥ ٪) للأزمنة المضبوطة التي تم حسابها باستخدام طريقة الخطوة- خطوه القياسيه.

* وجد أن أسلوب الاتزان المقدم مناسبا ويمكن تطبيقه بسهوله علي أنظمة القدره كبيرة المقياس.كما وجد أيضا أنه لأجراء دراسات الاتزان الانتقالي لأنظمة القدره فأن الأسلوب المقدم يعتبر أنسب من أساليب الفك والتراكب التي تم تقديمها خلال السنوات القليله الماضية.