بسم الله الرحمن الرحيم

Performance Analysis of Air Regenerative-Turbine Compressor

تحليل أداء ضواغط الهواء التربينية الاسترجاعية

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خلاصة في هذا البحث تم تحليل أداء ضواغط الغازات التربينية الاسترجاعية نظريا. بالرغم أن هذا التحليل المستخدم في هذه الدراسة يصلح لكل الغازات إلا أنه ولغرض التركيز فقد اقتصر التحليل هنا على الهواء كوسط عامل. في هذه الدراسة تم وصف السريان خلال الضاغط بواسطة كل من معادلة السريان ومعادلة الحركة ومعادلة الطاقة بالإضافة إلى معادلة الحالة. بفرض أن الهواء غازا مثاليا فقد أمكن تحويل المعادلات الواصفة للسريان إلى صور لابعدية. كذلك أمكن تجميع ظروف التشغيل المختلفة وأبعاد الضاغط إلى عدد محدود من العوامل اللابعدية المؤثرة في السريان. وعلى ذلك وتبعاً لأسلوب المعالجة المتبع في هذه الدراسة يمكن فهم وتحليل أداء الضاغط تحت ظروف التشغيل والأبعاد المختلفة بطريقة أسهل وأعمق مما يمكن من سهولة تصميم أو اختيار الضاغط المناسب للمهمة المطلوبة منه. تم حل النموذج الرياضي عددياً بعمل برنامج للحاسب الآلي. تم في هذه الدراسة استعراض تأثير العوامل اللابعدية المؤثرة وذلك على أداء الضواغط من هذا النوع.

1. Introduction

The regenerative-turbine compressor, as shown schematically in figure (1), consists of impeller and collecting passage. The impeller has pocket, which are formed by flat radial vanes. Gas enters the impeller pockets through a suction port, which are formed in stationary part of compressor. There, gas velocity is increased and hence it is directed to the collecting passage. In collecting passage, gas is collected and then leaves the compressor through delivery port. In collecting passage, some kinetic energy is transferred to enthalpy at higher pressure. As it is clear, the construction of the impeller is similar to the pump runner of hydraulic coupling, while the turbine runner is replaced by the collecting passage [1-2]. In spite of the similarity between this compressor and hydraulic coupling, there are rare knowledge about it in the literature. The performance of regenerative-turbine pump is examined in reference [3].

Because of the simple construction of this type of compressor, it is applied in vapor-compression desalination small units despite higher expected losses. In such applications, a moderate or small pressure ratio is required and moreover the lost power, from turbo machines design point of view, is considered as an added heating source in such units [4]. M. G. Wasel [5] made a simple theoretical model to examine the performance of such compressor when steam is used as working medium.

In present work, seeking for better understanding of the performance of such compressors, a model suitable for perfect gases is proposed. According to this model, the governing equations could, easier, be written in dimensionless form and, in turn, the parameters effecting the flow through the compressor could be eliminated to a definite number of dimensionless parameters. This leads to normalization and generalization of the problem and hence one can, with reasonable effort, determine the proper size of compressor, which can satisfy the design requirements.

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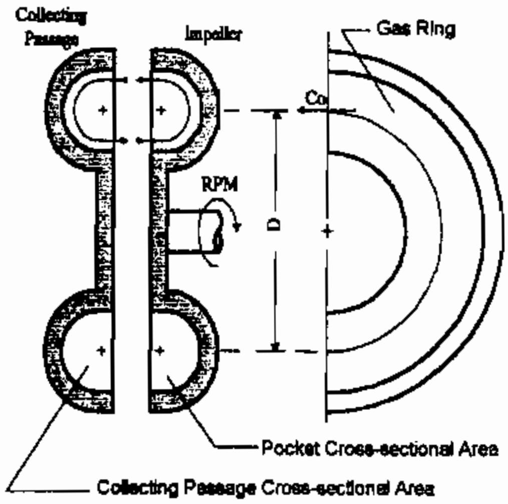


Figure (1) Schematic drawing of compressor showing the gas velocity direction.

2. Governing Equations

The flow inside the compressor is described by the conservation laws of mass, momentum and energy. Beside these three equations, there is equation of state. Referring to figure (1), the gas enters the collecting passage with tangential component of velocity c_o. The change of velocity in both axial and radial directions is negligible compared with that of tangential component. Figure (2) shows a part of collecting passage. Considering this differential control volume of length δx, one can derive the following governing equations by applying the conservation laws [6-7] on this element as;

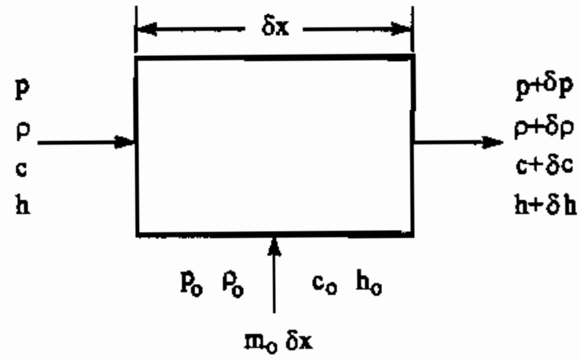


Figure (2) General control volume of length δx isolated from the total length of the collecting passage

$$\delta\, \rho\, c = \frac{m_o}{A}\, \delta\, x \tag{1}$$

$$\delta\, \rho\, c^2 + \delta\, p = \frac{m_o\, c_o}{A}\, \delta\, x \tag{2}$$

$$\delta\, \rho\, c\, H = \frac{m_o\, H_o}{A}\, \delta\, x \tag{3}$$

In present work, the working medium is assumed to be perfect gas. According to this assumption, the equation of state in this case takes the form;

$$p = \rho\, R\, T \qquad \& \qquad h = c_p\, T \tag{4}$$

Where ρ, p, h and c are the density, pressure, enthalpy and velocity of gas at general position; respectively. m_o is the mass flow rate of gas per unit length of collecting passage. The cross-sectional of collecting passage is denoted by A. The suffix o is used to denote the properties of gas entering the collecting passage coming from the compressor impeller. H is the total or stagnation enthalpy of gas, which is defined as the sum of enthalpy and kinetic energy [9]. This definition can, mathematically, expressed as;

$$H = h + \frac{1}{2}\, c^2 \tag{5}$$

In equation (4), the density and temperature of gas are denoted by ρ and T; respectively. R and c_p are the gas constant and constant pressure specific heat; respectively. The velocity of gas at the inlet of the passage is assumed to be nil. The other boundary conditions (p, ρ & h) are taken as the corresponding values of that entering the passage from the compressor impeller.

In this paragraph, attention is made to explain, how the tangential velocity (c_o) acquired due to the flowing of gas in the impeller and the mass flow rate per unit length (m_o) are evaluated. The tangential component of velocity c_o is evaluated according to the following relation as;

$$c_o = \frac{\pi\, N}{60} D \quad , \tag{6}$$

where D is the mean diameter of the impeller as shown in figure (1). N is the impeller speed in *rpm*. Maximum available mass flow rate per unit length of collecting passage $m_{o,max}$ is evaluated according to the following relation;

$$m_{o,max} = \frac{\rho_o\, V_p\, N}{60\, \pi\, D} \tag{7}$$

where ρ_o and V_p are the density of gas at the suction of compressor and the volume of impeller pockets; respectively. The volume of pockets is given according to the following relation as;

$$V_p = V \times \left(1 - \frac{k\ t}{\pi\ D} \right) \quad , \tag{8}$$

where k and t are the total number of pockets and the mean thickness of the vane (mean thickness of pocket wall); respectively. V is the volume of the gas-ring shown in figure (1), its volume is evaluated through the following relation [8];

$$V = \pi D A_p \quad , \tag{9}$$

referring to figure (1), D and A_p are the mean diameter of the gas-ring and the cross-sectional area of a pocket; respectively.

3. Dimensionless Form of Governing Equations

Seeking for better understanding of compressor performance, it is proposed to put the governing equations (1-4) in dimensionless form. Accordingly, the parameters controlling the problem are restricted to a limited number of parameters. To put these equations in dimensionless form, one defines the following dimensionless variables as;

$$p^* = \frac{p - p_o}{\rho_o\, c_o^2} \quad ; \quad c^* = \frac{c}{c_o} \quad ;$$
$$h^* = \frac{h}{c_o^2} \quad ; \quad \rho^* = \frac{\rho}{\rho_o} \quad ; \quad x^* = \frac{x}{L} \quad , \tag{10}$$

where p^*, c^*, h^*, ρ^* and x^* are dimensionless pressure, velocity, enthalpy, density and dimensionless coordinate along the collecting passage. L is a characteristic length and is equal πD. According to equations (5,10), one can define the dimensionless total enthalpy H^* as;

$$H^* = h^* + \frac{c^{*2}}{2} \quad . \tag{11}$$

With the aid of equations (1-4) and equations (10-11), one can derive the dimensionless form of the flow describing equations as;

$$\delta\, \rho^*\, c^* = \frac{m_o\, L}{\rho_o\, c_o\, A}\ \delta\, x^* \quad , \tag{12}$$

$$\delta p^* = \frac{m_o L}{\rho_o c_o A} \delta x^* - \delta \rho^* c^{*2} \quad , \tag{13}$$

$$\delta \rho^* c^* H^* = \left(\frac{m_o L}{\rho_o c_o A} \right) \cdot \left(\frac{H_o}{c_o^2} \right) \delta x^* \quad , \tag{14}$$

$$p^* = \frac{R}{c_p} \rho^* h^* - \frac{p_o}{\rho_o c_o^2} \quad ;$$

$$\frac{p_o}{\rho_o c_o^2} = \frac{R}{c_p} \frac{h_o}{c_o^2} = \frac{R}{c_p} h_o^* \tag{15}$$

The term $(p_o / \rho_o c_o^2)$ is expressed as a function of the enthalpy ratio (h_o / c_o^2), by applying equation of state for the condition of gas leaving the compressor impeller. Referring to equations (8-9), one can define the volume ratio β as the volume occupied by the gas to the total volume of gas ring ($\beta = V_p / V = 1 - kt / \pi D$). As it is clear, the volume ratio is dependant on the design considerations. The degree of filling ϕ is defined as the ratio between the actual mass flow rate of the compressor and the maximum possible flow rate ($\phi = m_o / m_{o,max}$, $1 \geq \phi \leq 0$). Moreover, the area ratio ψ is defined as the ratio between the cross-sectional area of gas ring A_p and that of the collecting passage A. Accordingly and with the aid of equations (6-9), the term ($\frac{m_o L}{\rho_o c_o A}$) appears in equations (12-14) can be written as;

$$\frac{m_o L}{\rho_o c_o A} = \beta \phi \psi \tag{16}$$

According to the definition of the dimentionless total enthalpy [eqn.(11)], one can express the term (H_o / c_o^2) by;

$$\frac{H_o}{c_o^2} = h_o^* + \frac{1}{2} = H_o^* \quad , \tag{17}$$

where H_o^* is the dimensionless total enthalpy of gas entering the collecting passage from the gas ring.

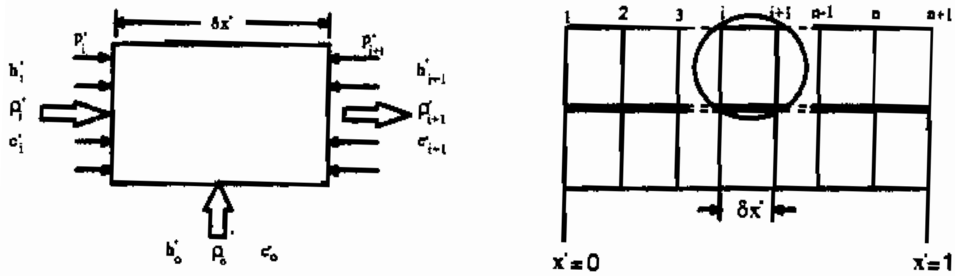
4. The Numerical Procedure

The dimensionless form of governing equations (12-14) is, numerically, integrated by dividing the collecting passage to n cells, as shown in figure (3). Referring to figure (3-a), the application of conservation laws (12-14) on the shown general cell leads to the following recursive relations as;

$$\rho^*_{i+1}\, c^*_{i+1} = \rho^*_i\, c^*_i + (\phi\, \beta\, \psi)\cdot \delta\, x^* \quad , \tag{18}$$

$$p^*_{i+1} = p^*_i + \rho^*_i\, c^{*2}_i + (\phi\, \beta\, \psi)\cdot \delta\, x^* - \rho^*_{i+1}\, c^{*\,2}_{i+1} \quad , \tag{19}$$

$$\rho^*_{i+1}\, c^*_{i+1}\, H^*_{i+1} = \rho^*_i\, c^*_i\, H^*_i + (\phi\, \beta\, \psi)\cdot (H^*_o)\, \delta\, x^* \quad . \tag{20}$$



(a) Mass and energy Transfer through a general cell of collecting passage

(b) Cells identifications of both collecting passage (above) and impeller

Figure (3) Devisions of both collecting passage and impeller of compressor used in numerical technique

Where i varies from 1 to $n+1$ (n is the total number of cells). According to the definition of dimensionless variables equations (10), the total length of the collecting passage is unity and hence the step size (δx^*) is equal to $1/n$. Though out the calculations, the thermodynamic properties (ρ, p, h,.....etc.) remain constant inside the cells and the change of them takes place, only, at the boundary of each cell. The boundary conditions in dimensionless form (at $i=1$) can be listed as the following;

$$x_1^* = 0 \;;\; p_1^* = 0 \;;\; c_1^* = 0 \;;\; \rho_1^* = 0 \;\&\; h_1^* = h_o^* / c_o^2 \quad . \tag{21}$$

The final form of conservation laws (18-20) and their boundary conditions (21) accompanied by the final form of equation of state (15) , now, can be solved for the unknowns c^* , p^* , h^* and ρ^* along the collecting passage for different values of problem parameters.

A computer program is designed to solve the present proposed model. Through out all carried out runs of the computer program; the total number of cells n is, reasonably, found to be 100. The ratio between gas constant R and the specific heat c_p (R/c_p) is taken 0.285, which is associated with the condition of air as a working medium. The volume ratio β is taken , through out all runs, as 0.95. Practically, from design of impeller point of view, it is found to be O(0.9). Three compressors, associated with $\psi = 0.8$, 1.0 & 1.2, are tested. For all runs, the behavior of air is studied at different values of mass flow rate (according to $\phi = 0.0$ to 1.0, with $\Delta\phi = 0.05$).

5. Results and Discussions

In the following sections, the effect of operating conditions and the effect of compressor dimensions are examined by studying the effect of the enthalpy ratio (h/c_o^2) and area ratio (ψ); respectively. First of all, the thermodynamic properties of air through out the collecting passage are depicted by figures (4). These figures show the air properties in case of enthalpy ratio (h_o/c_o^2) of 5.0 and area ratio (ψ) of 0.8. The air properties are shown at three different values of degree of filling 1.0, 0.5 and 0.1. Figure (4-a) shows the dimensionless density distribution along the collecting passage. In general, the dimensionless density increases in down stream direction till it reaches its maximum value at the exit of the collecting

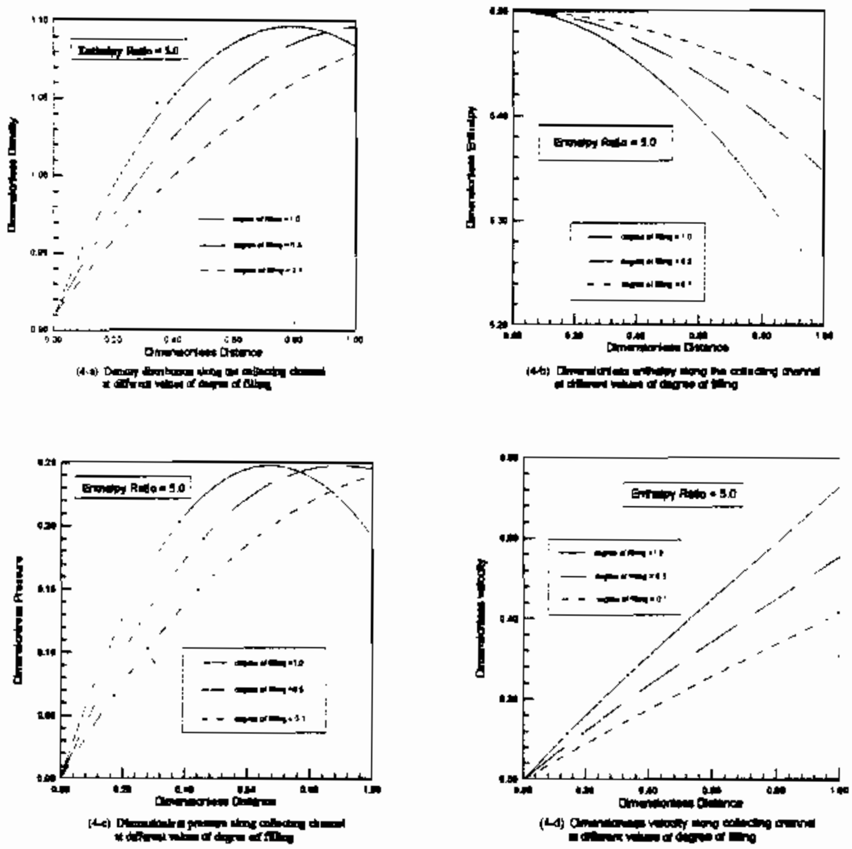


Figure (4) The thermodynamic properties of air along the collecting passage.

passage. It is true for flow rates corresponding to small and moderate values of degree of filling. For higher values of degree of filling(e.g. $\phi=1$), dimensionless density has a peak value near the exit of passage. Dimensionless total enthalpy ($h/c_o^2 + ½$) along the collecting channel is shown in figure (4-b). Dimensionless total enthalpy, in general, decreases in down stream direction for all values of degree of filling. It decreases in higher rate for higher values of degree of filling. Dimensionless pressure distribution is shown in figure (4-c). The pressure increases at a higher rate for higher values of degree of filling. It has a maximum value, which is nearer to the exit of collecting passage for lower values of degree of filling. For smaller values of degree of filling, no maximal is reached. In figure (4-d), the dimensionless velocity increases almost linearly up to the exit cross section. The rate of increase is higher as the degree of filling is greater.

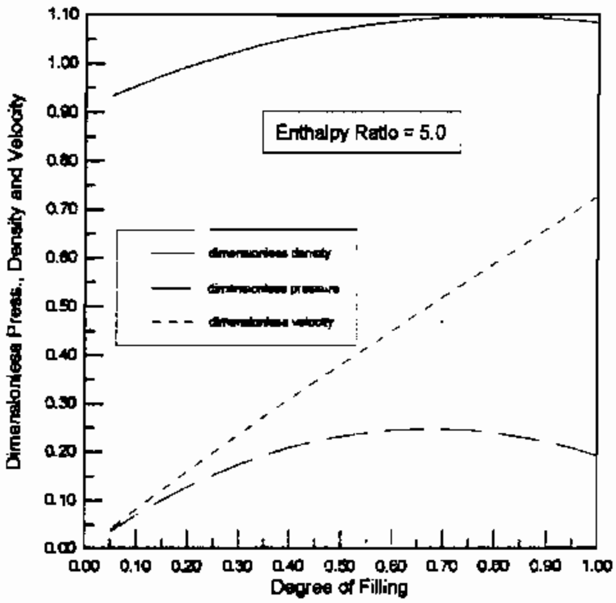


Figure (5) Dimensionless properties at delivery of compressor

Figure (5) shows the air properties versus mass flow rate represented by degree of filling. The behavior of both density and pressure is, in general, similar. Both of them increase till it reaches a maximum value. Hence they start to decrease continuously up to the maximum mass flow rate corresponding to degree of filling of unity. The dimensionless velocity increases, almost, linearly.

Figures (6-7) show the effect of area ratio (ψ) on the compressor performance. In figure(6), the relation between dimensionless pressure and degree of filling at area ratio (ψ) of 0.8, 1.0 &1.2 is shown. At all values of area ratio, the dimensionless pressure has a maximum value. This maximum value lies at higher value of ϕ as the area ratio is lower. In case of $\psi = 1.2$, the compressor fails to do its job at a value of $\phi \sim 0.9$ and greater (where $p^* \leq 0$). As a result of curve fitting-process, it is found that the second degree polynomial fits properly the obtained results of pressure for all values of ψ. As a result of this fitting process, the value of maximum pressure is nearly the same for all values of ψ and is equal to 0.248. This maximal lie at ϕ equals to 0.6727, 0.5400 and 0.4496 in case of ψ= 0.8, 1.0 and 1.2; respectively. Figure (7) depicts the dimensionless air velocity versus the degree of filling in case of area ratio ψ = 0.8, 1.0& 1.2. The velocity increases, almost, linearly for all values of ψ

It has higher values for higher values of area ratio. Figure (8) shows dimensionless pressure against the product of degree of filling and area ratio in the case of area ratio ψ=0.8, 1.0& 1.2.

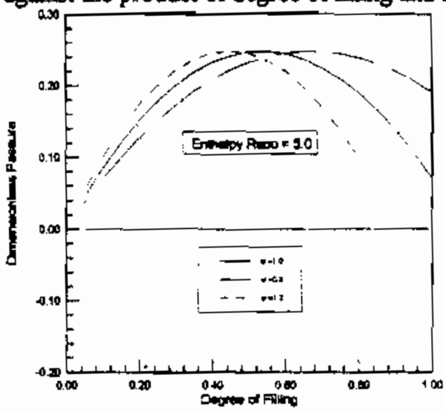


Figure (6) Dimensionless pressure against degree of filling at different values of area ratio

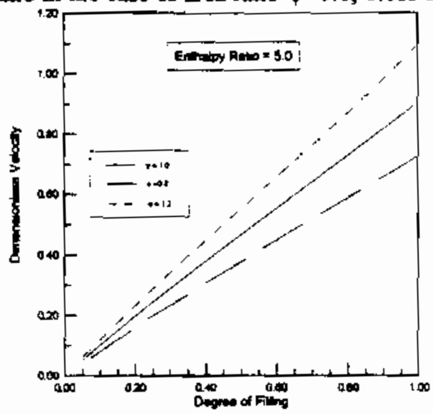


Figure (7) Dimensionless velocity against degree of filling at different values of area ratio

As it is clear, the dimensionless delivery pressure can be represented by a single curve, whatever the value of area ratio is. This curve is found to be properly fitted by a second degree polynomial, which has the following form;

$$p* = 0.00316014 + 0.908175\,(\phi\psi) - 0.841675\,(\phi\psi)^2$$

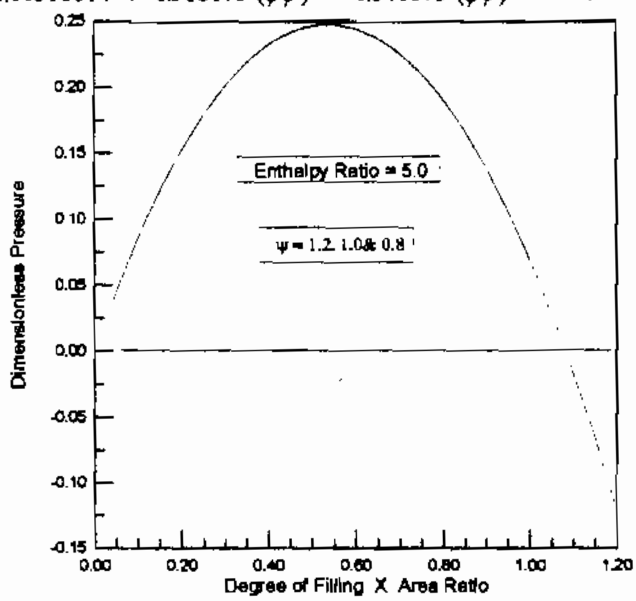


Figure (8) Dimensionless pressure at delivery of compressor at different values of area ratio

With the aid of this relation one can deduce the following important values;

$$p^* = p^*_{max} = 0.248 \qquad \text{at} \qquad (\phi\,\psi) = 0.540 \quad ,$$

$p^{*} = 0.0$ at $(\phi\ \psi) = 1.083$.

One can see from the foregoing relations, that the range of stable operation of the compressor is from $(\phi\ \psi)$ = 0.540 to 1.083 and the maximum available pressure gain is that corresponding to p^{*}_{max} =0.248. These mentioned values are valid for the value of enthalpy ratio (h / c_o^2) of 5.0 .

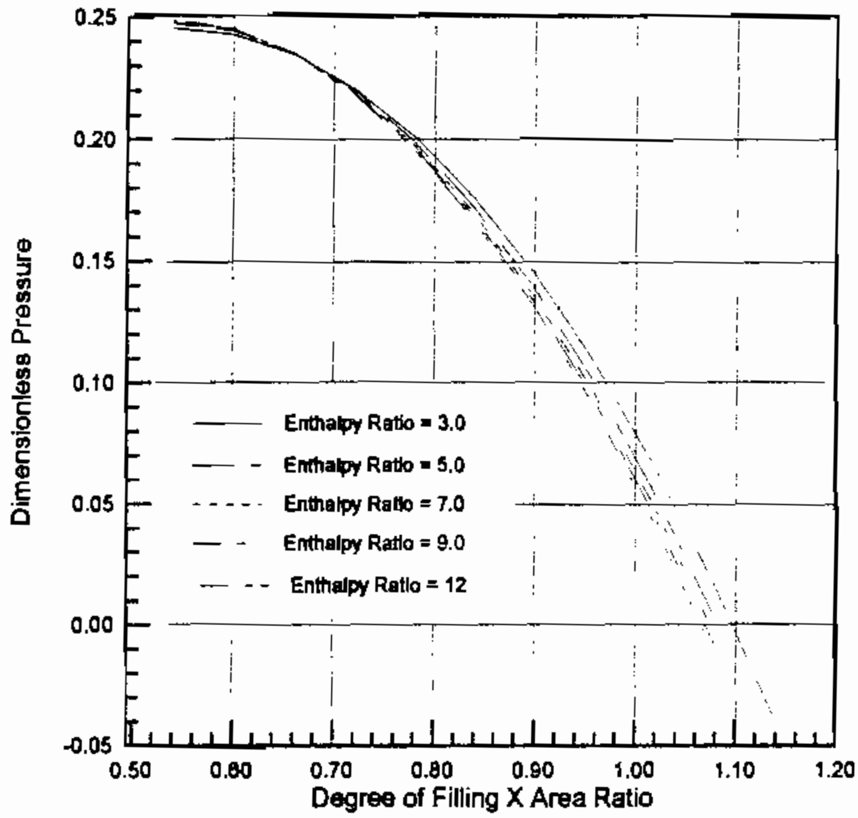


Figure (9) Dimensionless pressure at delivery of compressor for different valuesof enthalpy ratio

Figure (9) shows the performance curves of the studied compressor at enthalpy ratios of 3.0, 5.0, 7.0, 9.0 and 12.0. As it is clear, the stable range of the curves is presented in this figure. All curves can be fitted by second degree polynomials of the form; $a\ (\phi\psi)^2\ + b\ (\phi\psi)\ + d = 0$. The coefficients of the foregoing equation are given in table(1).

h_o/c_o^2	3.0	5.0	7.0	9.0	12.0
a	−0.792113	−0.83049	−0.848942	−0.86002	−0.870139
b	0.861462	0.894405	0.90927	0.918062	0.925982
d	0.0113173	0.00677039	0.00489232	0.00376185	0.00274461

Table (1) The values of the coefficients of performance curves of compressor at different values of enthalpy ratio

6. Conclusions

The proposed theoretical model presented in this work seems to be a satisfactory tool to select or design a compressor of the regenerative-turbine type to do certain job. Although the air is used as a working medium, the present model is valid for all perfect gases. According to this model, operating conditions and required compressor dimensions are collected in a limited number of dimensionless parameters. These parameters are enthalpy ratio, area ratio and the ratio between gas constant and constant pressure specific heat. With the aid of this model, mathematical relationships between effective performance parameter are derived. The maximum available pressure gain represented by the dimensionless pressure is about 0.25, whatever the design parameters are. It is recommended to vertify this model with experimental one to take in consideration the effect of the ignored factors such as friction and leakage.

Nomenclature

A cross-sectional area of collecting passage
c air velocity in collecting passage
c^* dimensionless air velocity
c_o acquired velocity of air due to passing through impeller
c_p constant pressure specific heat
D mean diameter of both impeller and collecting passage
H total or stagnation enthalpy, defined in equation (10)
H_o total or stagnation enthalpy of air entering the collecting passage coming from impeller
h enthalpy of air in collecting passage
h^* dimensionless enthalpy, is defined in equations (10)
h_o enthalpy of air at suction port of compressor
i identifier to denote the position along the passage
k total number of impeller pockets
L characteristic length, $L = \pi D$
m_o mass flow rate per unit length of collecting passage
$m_{o,max}$ maximum possible flow rate per unit length of collecting passage
N impeller speed in rpm
n total number of impeller pockets
p air pressure
p^* dimensionless pressure, is defined in equations (10)
p_o suction pressure
R gas constant