EL-MANSOURA UNIVERSITY
FACULTY OF ENGINEERING
DEPT. OF PROD. & MECH. DESIGN ENG.
TIME ALLOWED: 3 HOURS

FINAL EXAM., JAN. 2011
SYSTEMS OF STRESS ANALYSIS
2nd YEAR STUDENTS
EQUATION SHEET IS PROVIDED

SOLVE THE FOLLOWING PROBLEMS; NEAT SKETCHES ARE REQUIRED. ALL PROBLEMS HAVE SAME MARKS.

PROBLEM # 1:

a) Outline the importance of stress analysis to the design engineer.
b) Stress analysis may be performed: analytically, experimentally & numerically.
   1. Explain basic aspects of each technique.
   2. Which of these techniques will be suitable for :

      i) Simple geometries?
      ii) Complex geometries?
      Explain procedure of applying the selected method.

PROBLEM # 2:

Given the special function:

$$\emptyset_1 = 80\,x^2 - 20\,y^2 - 30\,xy + 15$$

It is required to:

(a) Show that $\emptyset_1$ is a stress function.
(b) Find the stresses σ_{xx}, σ_{yy} and τ_{xy}.
(c) Assuming that $\sigma_{zz} = \tau_{xz} = \tau_{zy} = 0$, draw the 3-dimensional Mohr's stress circle
(d) Find the maximum shear stress τ_{max}
(e) Determine the six stress components : $\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{z'z'}$, $\tau_{x'y'}$, $\tau_{y'z'}$, $\tau_{z'x'}$, in the new system of axes, **O x′ y′ z′**, which is defined by the direction cosines shown :

	x	y	z
x′	π/3	π/3	π/4
y′	3π/4	π/4	π/2
z′	π/3	π/3	3π/4

PROBLEM # 3:

(a) Explain what is meant by "Strain rosette". What is it used for? What are its types?

(b) The following results were obtained from a three-element delta (equiangular -120°) strain rosette mounted on a steel (**E = 200 GPa** and υ **= 0.3**) specimen. Determine the principal strains ϵ_1 & ϵ_2 and the corresponding principal stresses σ_1 & σ_2. Calculate the maximum shear stress and strain, τ_{max} & γ_{max}.

$\epsilon_A = \epsilon_o = \mathbf{1600 \times 10^{-6}}$, $\epsilon_B = \epsilon_{120} = \mathbf{800 \times 10^{-6}}$, $\epsilon_C = \epsilon_{-120} = \mathbf{0}$

- If a 3-element rectangular strain rosette is used for the same specimen, what would be the readings of each strain gage?

PROBLEM # 4:

(a)"**Photoelasticity is a whole-field stress analysis technique.**"

Explain the above statement considering the following:

i) Theoretical background.
ii) Experimental set-up.
iii) Method of analysis.

(b) Fig.1 shows isochromatic fringe pattern in a photoelastic gear tooth model subjected to a contact force. If the specimen thickness is 4 mm and the material constant f_σ = **60 KN/m**, determine the maximum shear stress at points A, B, C.

At which point is the contact force applied?

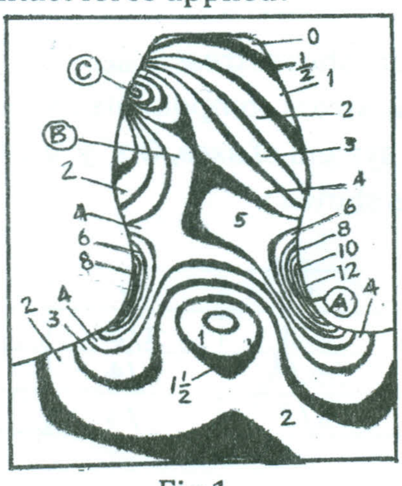

Fig.1

PROBLEM # 5 :

Rewrite the following statements after selecting the correct expression:

1. The principal stresses are (dependent/independent) on the cartesian coordinate system.
2. Addition of a hydrostatic pressure to a given stress state (affects/does not affect) the magnitude of the maximum shear stress.
3. Rigid-body motion (produces/does not produce) linear strain.
4. Mechanical strain gages have (high/low) sensitivity.
5. The Whitstone bridge electrical circuit is (suitable/not suitable) for dynamic strain measurements.
6. Moiré fringe technique is a (whole field/point-by-point) stress analysis technique.
7. Brittle coating on a circular rod subjected to a twisting moment (torque) will develop (axial/circular/helical) cracks.
8. Finite-element technique (is/is not) used in numerical stress analysis.
9. Acoustical strain gages (are/are not) based on vibration of strings.
10. Coating methods of stress analysis (requires/does not require) making of special specimens.

With my best wishes

Prof. Dr. M. Shabara

التغييرات الضرورية :

في المعادلة ②

$n \to x'$ — (a)

$n \to y'$ — (b)

$n \to z'$ (c)

في المعادلة ③

$n \to x'$, $n' \to y'$ } (d)

$n \to y'$, $n' \to z'$, $x \to y$, $y \to z$ } (e)

$n \to z'$, $n' \to x'$, $x \to z$, $y \to x$ } (f)

EQUATION SHEET

$$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) ; \qquad (\sigma_1 - \sigma_2) = \frac{N f_\sigma}{h} \quad (1)$$

$$\sigma_{nn} = \sigma_{xx}\cos^2(n,x) + \sigma_{yy}\cos^2(n,y) + \sigma_{zz}\cos^2(n,z) + 2\tau_{xy}\cos(n,x)\cos(n,y) + 2\tau_{yz}\cos(n,y)\cos(n,z) + 2\tau_{zx}\cos(n,z)\cos(n,x) \quad (2)$$

$$\tau_{nn'} = \sigma_{xx}\cos(n,x)\cos(n',x) + \sigma_{yy}\cos(n,y)\cos(n',y) + \sigma_{zz}\cos(n,z)\cos(n',z) + \tau_{xy}[\cos(n,x)\cos(n',y) + \cos(n,y)\cos(n',x)] + \tau_{yz}[\cos(n,y)\cos(n',z) + \cos(n,z)\cos(n',y)] + \tau_{zx}[\cos(n,z)\cos(n',x) + \cos(n,x)\cos(n',z)] \quad (3)$$

$$\epsilon_{\frac{1}{2}} = \tfrac{1}{2}(\epsilon_A + \epsilon_C) \pm \tfrac{1}{2}\sqrt{(\epsilon_A - \epsilon_C)^2 + (2\epsilon_B - \epsilon_A - \epsilon_C)^2} \quad (4)$$

$$\sigma_{\frac{1}{2}} = \frac{E}{2}\frac{(\epsilon_A + \epsilon_C)}{1-\nu} \pm \frac{E}{2}\sqrt{(\epsilon_A - \epsilon_C)^2 + (2\epsilon_B - \epsilon_A - \epsilon_C)^2}/(1+\nu) \quad (5)$$

$$\epsilon_{\frac{1}{2}} = \frac{\epsilon_A + \epsilon_B + \epsilon_C}{3} \pm \frac{\sqrt{2}}{3}\sqrt{(\epsilon_A - \epsilon_B)^2 + (\epsilon_B - \epsilon_C)^2 + (\epsilon_C - \epsilon_A)^2} \quad (6)$$

$$\sigma_{\frac{1}{2}} = \frac{E(\epsilon_A + \epsilon_B + \epsilon_C)}{3(1-\nu)} \pm \frac{E\sqrt{2}}{3}\sqrt{(\epsilon_A - \epsilon_B)^2 + (\epsilon_B - \epsilon_C)^2 + (\epsilon_C - \epsilon_A)^2}/(1+\nu) \quad (7)$$

$$\nabla^4\phi = \frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} ; \quad \sigma_{xx} = \frac{\partial^2\phi}{\partial y^2} ; \quad \tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} \quad (8)$$

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_{xx} + \nu(\epsilon_{yy} + \epsilon_{zz})] ; \qquad \tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \quad (9)$$

$$\epsilon_{\frac{1}{2}} = \tfrac{1}{2}(\epsilon_{xx} + \epsilon_{yy}) \pm \tfrac{1}{2}\sqrt{(\epsilon_{xx} - \epsilon_{yy})^2 + \gamma_{xy}^2} ; \qquad \epsilon_{zz} = -\frac{\nu}{1-\nu}(\epsilon_{xx} + \epsilon_{yy}) \quad (10)$$