

**Post graduate Exam (Basic Engineering Sciences)**  
**Branch: Engineering Mathematics (Master 600)**

Menofia University  
 Faculty of Engineering  
 Academic Year: 2016-2017  
 Department: Basic Eng. Sci.



**Subject: Integral Equations**  
**Code: BES 625**  
**Time Allowed: 3 hours**  
**Date: 14 / 1 / 2017**  
**Max Marks: 100**

**Answer all the following questions:**

**Q.1** (A) State whether of the following integral equations are (Voletra IE or Fredholm IE), (First kind or Second kind), (homogenous or non-homogenous):

$$\bullet f(x) = \int_a^b k(x,s)\phi(s)ds \quad \bullet \phi(x) = \sin(x) + \lambda \int_0^{\pi/2} \cos(x^2s)\phi(s)ds$$

(B) Using the recursion series method solve the following IE.

$$\phi(x) = x + \lambda \int_0^x \phi(s)ds$$

(C) Find the first two terms of the Neumann series for the equation:

$$\phi(x) = \sin(x) + \lambda \int_0^{\pi/2} \cos(x^2s)\phi(s)ds$$

(D) Solve the following equations using degenerate kernels method:

$$\phi(x) = x^2 + \lambda \int_0^1 x^3s^2\phi(s)ds$$

(E) Find the I.V.P of  $u''(y) + yu'(y) + 2u(y) = 0$  Subject to  $u(0) = \alpha$ ,  $u'(0) = \beta$ , as a voletra IE?

[Q.1 (50 mark)]

**Q.2** (A) Consider the IE  $f(x) = g(x) + \lambda \int_0^{\pi} \sin(x-y)f(y)dy$  Find:

- 1) The values of ( $\lambda$ ) for which it has a unique solution.
- 2) The solution in this case
- 3) The resolvent kernel
- 4) The values of ( $\lambda$ ) for which the solution is not unique.

(B) Write the Voletra IE for the following O.D.E.

$$y'' + \omega^2y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

(C) Using the resolvent kernel method to solve the following IE.

$$g(x) = \cos(2x) + \int_0^{2\pi} \sin(x)\cos(s)g(s)ds$$

(D) Convert the following IE into (O.D.E) and solve it:

$$g(x) = \sin(x) + \int_0^x \sin(x-s)g(s)ds$$

[Q.2 (50 mark)]

Good luck

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