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A DIGITAL SIMULATION OF A THREE-PHASE VOLTAGE CONTROLLER FED INDUCTION MOTOR

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S.S. SHOKRALLA, S.A. MAHMOUD, A.A. ELHEFNAWY and S.A. HASSAN,

Team of Control of Electrical Machines (TECEM), Faculty of Eng. & Tech., Menoufia University, Shebin El-Kom, EGYPT.

ABSTRACT

This paper presents a complete digital simulation of a three-phase voltage controller fed induction motor. The modelling of the machine avoids numerical inversion of the inductance matrix even when the stator currents are interrupted. The simulation program is developed using a unified method for the modelling of system including switching devices as well as prediction of the system behaviour. Simulation results for both startingup and steady-state operation are reported and proved to yield good agreement when compared with the relevant experimental results.

LIST OF MAIN SYMBOLS

[i] : current vector

[L] : inductance matrix
[R] : resistance matrix

[V] : voltage vector

f : friction coefficient

J : moment of inertia

L_r: rotor-phase inductance L_s: stator-phase inductance

Mo : maximum mutual inductance between stator -

 $\begin{array}{c} \text{rotor phase} \\ \mathbf{M_{r}} & \text{: mutual inductance between two rotor phases} \end{array}$

M_S: mutual inductance between two stator phases
ρ: differential operator (d/dt)

2P : number of poles

R: stator-phase resistance
R: rotor-phase resistance

1. INTRODUCTION

When an induction machine is operated with static converter, it always operates under transient conditions and generally with unbalanced stator voltages and currents. Whereas many analytical studies have been done on this subject [1,2], the digital simulation is probably the most powerful tool.

The digital simulation of static converter - rotating machine system is a complicated problem concerning many papers [3-9]. For this purpose, the method generally used [3] is to consider the whole system as a network whose topology varies according to the state of conduction of the converters. This method gives a good results for steady-state investigation, but it is not suitable for machine transient.

The transient behaviour is generally investigated by solving a set of nonlinear differential equations, which are convenient to be dealt with in a separate way. Consequently, it seems interesting to find a suitable model allowing study of the behaviour of whole system (source, converter, machine and controllers).

The present paper developes a complete digital simulation of three-phase voltage controller fed induction motor. The method of simulation used allows the user of such simulation to take easily into account all parts of the whole system [6,8]. The model of the three-phase voltage controller has been eastablished using the connection matrices [4], and the model of the machine has been developed such that to reduce large computation time of the digital simulation [7,9].

2. SYSTEM DISCRIPTION AND ASSUMPTIONS

The system under consideration consists of a three-phase induction motor fed by a three-phase voltage controller composed of six thyristors, two of them are connected in antiparallel to the same load phase as shown in Fig. (1). The stator phases as well as the source phases are star-connected, 3-wire or 4-wire.

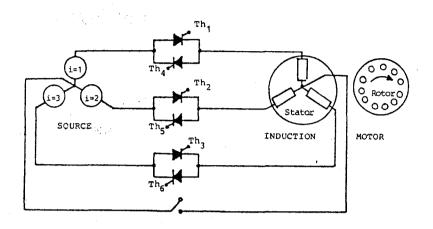


Fig. (1): Induction motor fed by a three-phase voltage controller.

2.1 Firing Devices

Each thyristor is triggered by a so-called "Coincidence pulse generator", i.e. a pulse is generated on $\mathrm{Th}_J(\mathrm{Th}_J)$ is the thyristor of number J connected to the source phase number i) when the control voltage denoted FCR crosses the firing curve TV_{iJ} synchronized on the ith source voltage as shown in Fig. (2).

2.2 Basic Assumptions

The following main assumptions are taken into consideration:

The space harmonic, saturation and iron losses are neglected. Also the m.m.f. is sinusoidal distributed. The thyristor are considered as ideal switches and the commutation phenomenon between the source phases are negligable.

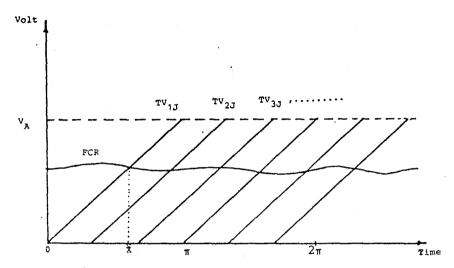


Fig. (2): Principle of pulse generator.

3. MODELLING OF THE SYSTEM

In order to carryout the simulation, the various devices involved in the system are modelled as follows:

3.1 Voltage Controller Model

The three phase voltage controller may be represented by a connection matrix [c]. Denoting one element of the matrix [c] by C_{ik} where, $\text{C}_{ik}\text{=}1$ if the phase "i" of the source is connected to the phase "K" of the Motor, equal zero otherwise. This modelling allows a simple computation for voltages of the motor phases when they are fed by the voltage controller:

where, $[\text{V}_{\text{C}}]$ and $[\text{V}_{\text{S}}]$ are respectively the machine and the source voltages. Furthermore, the source currents may be given by:

$$[I_S] = [C] [I_C] \dots \dots (2)$$

where, $[I_S]$ and $[I_C]$ are the source and motor phase currents respectively.

3.2 Firing Devices Model (Pulse Generators)

It may be considered that, each pulse generator has a model depending on the value of the control voltage (FCR). The generators can be modelled by a "Firing Function" (FAL $_{iJ}$).

 ${\rm FAL}_{iJ} = {\rm FCR}_J - {\rm TV}_{iJ}$... (3) The pulse is generated on the thyristor number J(Th_J) when the control voltage ${\rm FCR}_j \ge$ the timming voltage (TV_{i,J}).

3.3 Machine Model

The general voltage equation of an induction machine may be written in the following matrical form:

$$[V] = [L] \frac{d}{dt} [i] + ([R] + \theta \frac{d}{d\theta} [L]) [i] \dots (4)$$
The this equation by a numerical way, it should

To solve this equation by a numerical way, it should be arranged in a state form as follows:

$$\frac{d}{dt}[i] = [L]^{-1} \{ [V] - ([R] + \theta \frac{d[L]}{d\theta}) [i] \} \dots$$
 (5)

Since [L] is a function of θ , then it is time-varying and thus [L] has to be computed in a repetitive way (three times in each computation step when using a fourth order Runge-Kutta method). This way has two main disadvantages: firstly, it is a largely time consuming and secondly, it may affect the precision of the computation because [L] may found to be ill conditionned.

The usual method used to avoid the matrix inversion is the Park's 0,d,q transformation, which leads to a constant matrix. However, it is not always possible to use this method when the machine is fed by a static converter and especially by a three-phase voltage controller. This is because the three-phases are not always supplied (i.e. one-phase or two-phases are disconnected for a certain time).

In order to solve this problem, a new transformation has been used [8,9] leading to a constant matrices even when one or two stator phases are switched off.

Let us denote 1,2 and 3 to the stator phases, phase 1 being the "dissymetric one" (when one phase is not supplied, it denoted by "1" and when two phases are switched off, "1" in this case denoted the third phase). The rotor phases, are short-circuited, and it is always possible to use the Park's 0,d,q transformation, then the transformed rotor currents are then denoted by \hat{i}_d and \hat{i}_q .

When the three-phase voltage controller fed induction motor with neutral, for such a system, there are four models:

- All the three phases are connected to the source.
- Model (3)
- Model (2) Two-phases are connected to the source.
- One-phase is connected to the source. Model (1)
- All the three phases are disconnected Model (0) from the source.

These models are easily deduced from the connection matrix [c] as given below:

1. Model 3 (three phases are supplied)

This model is corresponding to the presence of three elements equal one in the connection matrix [c]. The conventional d.q. transformation is used with axis linked to the stator. The transformed voltages and currents of stator are respectively, denoted by (V_{α},V_{β}) and (i_{α},i_{β}) . The current equations can be put in the following form:

$$\frac{d}{dt} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_{d} \\ i_{q} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma L_{1}} & 0 & -\frac{M}{\sigma L_{1} L_{2}} & 0 \\ 0 & \frac{1}{\sigma L_{1}} & 0 & -\frac{M}{\sigma L_{1} L_{2}} \\ 0 & -\frac{M}{\sigma L_{1} L_{2}} & 0 & \frac{1}{\sigma L_{2}} \end{bmatrix} \begin{bmatrix} V_{\alpha} \\ V_{\beta} \\ 0 \\ 0 \end{bmatrix}$$

$$- \begin{bmatrix} R_{1} & 0 & 0 & 0 \\ 0 & R_{1} & 0 & 0 \\ 0 & M_{\theta} & R_{2} & L_{2} \theta \\ -M_{\theta} & 0 & -L_{2} \theta & R_{2} \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_{d} \\ i_{q} \end{bmatrix}$$
... (6)

The homopolar equation is defined by:

$$v_0 = R_1 i_0 + L_0 \frac{d}{dt} i_0 \dots (7)$$

The motor torque is given by:
$$T = P.M (i_d.i_\beta - i_q.i_\alpha) \dots \dots (8)$$

If the stator is star-connected without neutral, the homopular equation (7) is disapeared and the motor equation is represented by equations(6).

2. Model 2 (one phase is desconnected)

This model is corresponding to the presence of two elements equal one in the connection matrix[c]. When one motor phase is disconnected the following transformation on the stator variables are made[4,9]:

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = [D] \begin{bmatrix} g_1 \\ g_{\gamma} \\ g_{\beta} \end{bmatrix} \text{ where } [D] = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = [D]$$
 (9)

This transformation leads to the following equations, which in trun gives the currents in the other phases:

$$\frac{d}{dt} \begin{bmatrix} i_{\gamma} \\ i_{\beta} \\ \tilde{i}_{d} \\ \tilde{i}_{d} \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_{1}L_{3}} & 0 & \frac{M_{2}}{\sigma_{1}L_{2}L_{3}} & 0 \\ 0 & \frac{1}{\sigma_{L_{1}}} & 0 & \frac{-M}{\sigma_{L_{1}L_{2}}} \\ \frac{M_{2}}{\sigma_{3}L_{2}L_{3}} & 0 & \frac{1}{\sigma_{1}L_{2}} & 0 \\ 0 & \frac{-M}{\sigma_{L_{1}L_{2}}} & 0 & \frac{1}{L_{2}} \end{bmatrix} \begin{bmatrix} v_{\gamma} \\ v_{\beta} \\ 0 \\ 0 \end{bmatrix} \\
- \begin{bmatrix} R_{1} & 0 & 0 & 0 \\ 0 & R_{1} & 0 & 0 \\ 0 & M\hat{e} & R_{2} & L_{2}\hat{e} \\ M_{2}\hat{e} & 0 & -L_{2}\hat{e} & R_{2} \end{bmatrix} \begin{bmatrix} i_{\gamma} \\ i_{\beta} \\ i_{d} \\ i_{q} \end{bmatrix} \end{bmatrix} \cdots (10)$$

The voltage of the disconnected phases may be given by:

$$V_1 = (M_3, M_1) \frac{d}{dt} \begin{bmatrix} i_{\gamma} \\ i_{d} \end{bmatrix} \dots \dots (11)$$

The motor troque is given by:

$$T = P(M.i_d.i_{\beta} + M_2.i_{q}.i_{\gamma}) \dots$$
 (12)

If the motor has a star-connected stator with its neutral is disconnected and only one-phase is opened, the machine currents can be computed (using the transformation of Equations 9) from the following form,

$$\frac{d}{dt} \begin{bmatrix} i_{\beta} \\ \tilde{i}_{d} \\ \tilde{i}_{q} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma L_{1}} & 0 & -\frac{M}{\sigma L_{1} L_{2}} \\ 0 & \frac{1}{L_{2}} & 0 \\ -\frac{M}{\sigma L_{1} L_{2}} & 0 & \frac{1}{\sigma L_{2}} \end{bmatrix} \begin{bmatrix} v_{\beta} \\ 0 \\ 0 \end{bmatrix} \\
-\begin{bmatrix} R_{1} & 0 & 0 \\ M\hat{\theta} & R_{2} & L_{2}\hat{\theta} \\ 0 & -L_{2}\hat{\theta} & R_{2} \end{bmatrix} \begin{bmatrix} i_{\beta} \\ \tilde{i}_{d} \\ \tilde{i}_{q} \end{bmatrix} \right\} \dots (13)$$

The voltage of the disconnected phases may be given by:

$$\begin{bmatrix} v_1 \\ v_{\gamma} \end{bmatrix} = \begin{bmatrix} M_1 \\ -M_2 \end{bmatrix} \frac{d}{dt} \tilde{i}_d \qquad \dots \qquad \dots \qquad (14)$$

In this case, the motor torque is given by:

3. Model 1 (Two-phases are disconnected):

For this model, only one element equal one is presence in the connection matrix [c]. Using the previous transformation (Eqs. 9), the currents are computed by the following equations:

$$\frac{d}{dt} \begin{bmatrix} i_{1} \\ \tilde{i}_{d} \\ \tilde{i}_{q} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma_{3}L_{s}} & -\frac{M_{1}}{\sigma_{3}L_{2}L_{s}} & 0 \\ \frac{1}{\sigma_{3}L_{2}L_{s}} & \frac{1}{\sigma_{3}L_{2}} & 0 \\ 0 & 0 & \frac{1}{L_{2}} \end{bmatrix} \begin{bmatrix} v_{1} \\ 0 \\ 0 \end{bmatrix} \\
- \begin{bmatrix} R_{1} & 0 & 0 \\ 0 & R_{2} & L_{2}\dot{\theta} \\ -M_{1}\dot{\theta} & -L_{2}\dot{\theta} & R_{2} \end{bmatrix} \begin{bmatrix} i_{1} \\ \tilde{i}_{d} \\ \tilde{i}_{q} \end{bmatrix} \cdots (16)$$

and the voltage of the disconnected phases are given by:

$$\begin{bmatrix} \mathbf{v}_{\mathbf{Y}} \\ \mathbf{v}_{\mathbf{\beta}} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{3} & -\mathbf{M}_{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M} \end{bmatrix} \frac{\mathbf{d}}{\mathbf{d}\mathbf{t}} \begin{bmatrix} \mathbf{i}_{1} \\ \mathbf{i}_{\mathbf{d}} \\ \mathbf{i}_{\mathbf{q}} \end{bmatrix} \qquad \dots \tag{17}$$

The motor torque is given by:

$$T = -P \sqrt{2/3} M i_1 \tilde{i}_{\alpha} \dots \dots (18)$$

4. Model 0 (Three-phases are disconnected):

This model corresponds to a connection matrix having its elements equal to zero, the electrical equations are reduced to:

$$\frac{d}{dt} \begin{bmatrix} \tilde{i}_d \\ \tilde{i}_q \end{bmatrix} = - \begin{bmatrix} \frac{R_2}{L_2} & \hat{\theta} \\ -\hat{\theta} & \frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} \tilde{i}_d \\ \tilde{i}_q \end{bmatrix} \dots \dots (19)$$

and the voltage of the stator phases are computed by:

$$\begin{bmatrix} V_{1} \\ V_{2} \\ V_{3} \end{bmatrix} = -\frac{M}{L_{2}} \sqrt{\frac{2}{3}} \begin{bmatrix} R_{2} & L_{2}\dot{\theta} \\ -R_{2} & \frac{\sqrt{3}}{2} L_{2}\dot{\theta} & \frac{-L_{2}}{2}\dot{\theta} + \frac{\sqrt{3}}{2} R_{2} \\ \frac{-R_{2}}{2} & -\frac{\sqrt{3}}{2} L_{2}\dot{\theta} & \frac{-L_{2}}{2}\dot{\theta} - \frac{\sqrt{3}}{2} R_{2} \end{bmatrix} \begin{bmatrix} \tilde{i}_{d} \\ \tilde{i}_{q} \end{bmatrix} (20)$$

Obviously, the torque of the motor becomes zero (i.e. τ = 0).

4. METHOD OF SIMULATION

To simulate the above-mentioned system, the whole system is divided into two subsystems[6,8]:
"a completed analogical subsystem" which is the whole set of models of each different parts of the system, and "a finite state automata", whose function is for choosing the right models at any moment. The analogical subsystem and the automate are linked by an "interface", which associates the logical inputs of the automata to analogical variable.

The general flow-chart of the simulation is shown in Fig.(3).

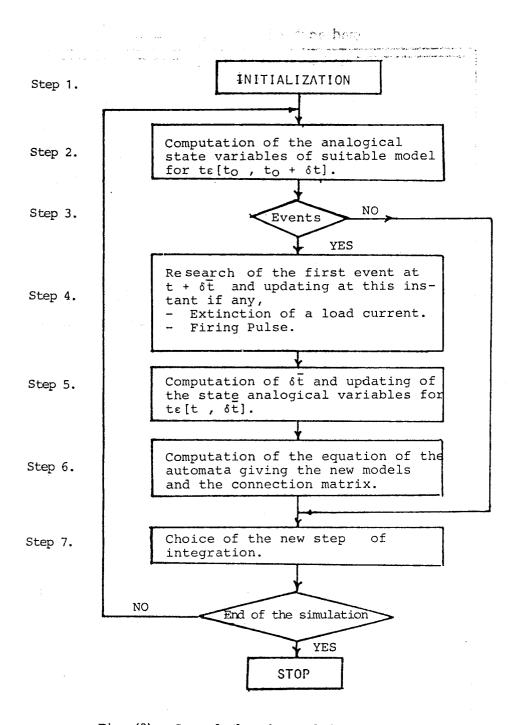


Fig. (3): General Flow Chart of the Simulation.

- Step 1, in which all the constants (the analogical and logical variables) are initialized in the main program.
- Step 2, is achieved with independent subroutines, so that a change in the load or in the controller should lead to modify only one subroutine. A fourth order Runge-Kutta method is used to integrate the differential equation.
- Steps 3 to 7, are achieved by a single subroutine named "GRAD". This subroutine represents the characteristic of the three-phase voltage controller, and helps to choose the suitable model for the simulation.

5. COMPUTED AND EXPERIMENTAL RESULTS:

The following results of the simulation have been obtained using an XTIBM computer. The startingup and steady-state behaviour of the three-phase voltage controller fed induction motor are selected in order to show the advantages, flexibility and accuracy of the developed simulation. The parameters of the motor as well as the system conditions are given in the Appendix. The results are based on starting-up period with a load torque of $T_{\tau} = 5.5(1-S_1)^2$.

Figs. (4,12) and (5,13) show the computed voltage and current for one phase of the motor during starting-up period. Also during this period the electromagnetic torque of the motor is indicated by Figs. (6,14). It should be noted that the maximum value of current and torque for the motor without neutral are smaller than that of the motor with neutral. This is due to the presence of the third harmonic in the later case. Figs. (7,15) illustrate the speed variation when the motor started from rest. It can be detected from the Figures that for same runup time the motor speed with neutral connected is higher than that without neutral. This is due to the high rate of change of speed and the increase of the developed electromagnetic torque.

Figs. [(21-A), (22-A), 8, 16] show the computed and experimental steady-state results of stator phase voltages. Figs. [(21-B),(22-B), 9, 17] show the same results for stator phase currents. It can be noted that the maximum and effective value of currents of the machine with neutral are higher The state of the s

than that of the machine without neutral. Also it can be concluded that the computed results are in good correlation with the results obtained experimentaly.

Figs. (10,18) show the steady-state developed torque. Theaverage developed torque for machine without neutral is less than that of machine with neutral. This is due to the decreased stator currents. Figs. (11,19) show the steady-state speed. The machine speed without neutral is less than that with neutral. This is because the voltage is decreased in the case of machine without neutral for the same value of firing angle. In order to show the accuracy of the simulation program, the experimental set-up is used for recording the volt-speed characteristics of the motor. These results are compared with the simulation results for two firing angles $[\alpha=76.2\,^{\circ}$ and $\alpha=83.4\,^{\circ}]$ as shown in Fig. (20).

5.1 Computed Results:

5.1.1 Machine is Star-Connected With Neutral:

(a) During Starting-up:

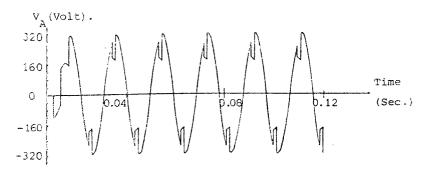


Fig. (4): Voltage of Stator Phase A.

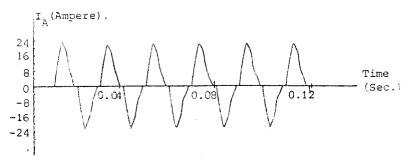


Fig. (5): Current of Stator Phase A.

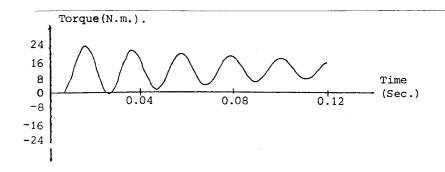


Fig. (6): Motor Developed Torque.

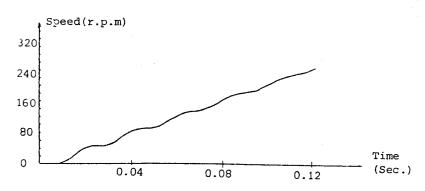


Fig. (7): Motor Speed.

(b) <u>During Steady-State</u>:

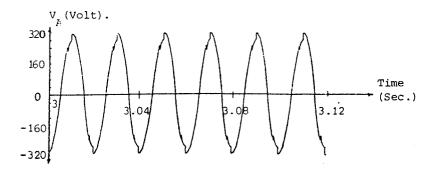
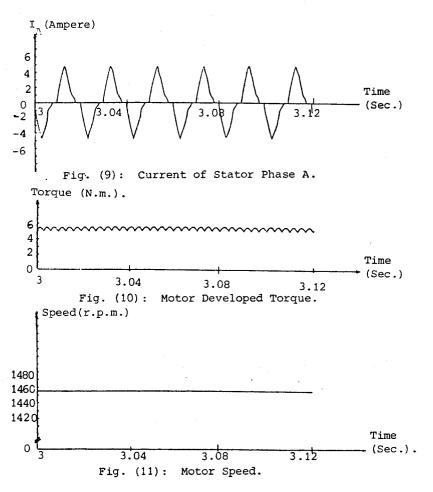
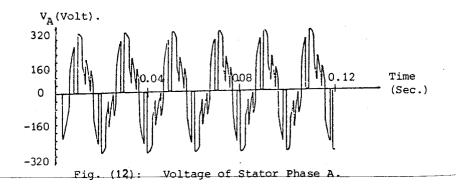


Fig. (8): Voltage of Stator Phase A. -67-



5.1.2 Machine Is Star-Connected Without Neutral: (a) During Starting-up:



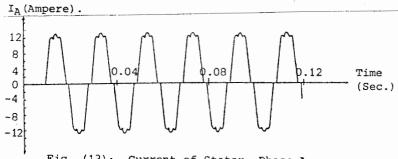


Fig. (13): Current of Stator Phase A.

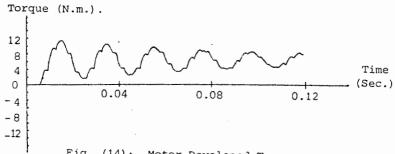
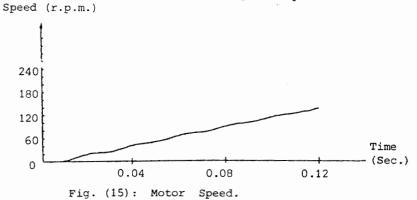


Fig. (14): Motor Developed Torque.



(b) During Steady-State:

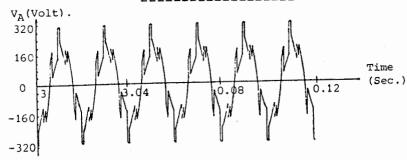


Fig. (16): Voltage of Stator Phase A.

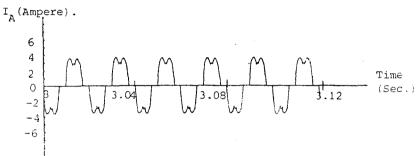


Fig. (17): Current of Stator Phase A.

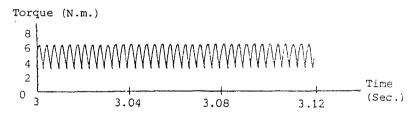
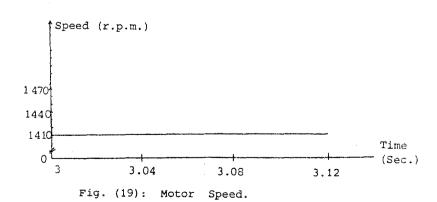
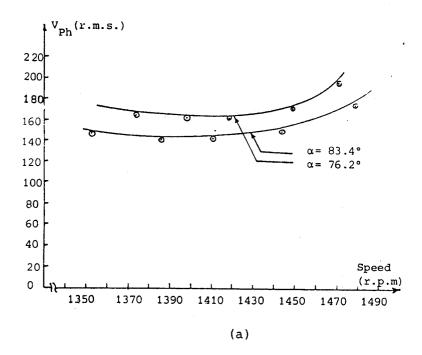


Fig. (18): Motor Developed Torque.





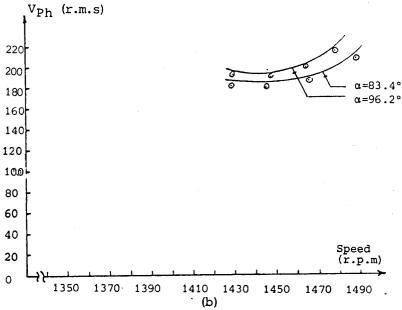


Fig. (20): Volt-Speed Characteristics for Induction Motor.

a. Without neutral.

Computed results.

Computed results.

5.2 Experimental Results:

5.2.1 Machine is Star-Connected With Neutral:

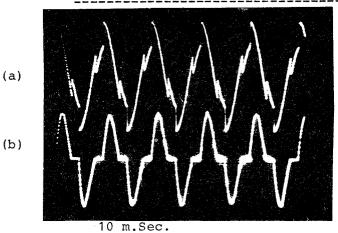


Fig.(21): Voltage and Current of Stator Phase A.

- (a) V_A (the maximum peak value is 311 V.). (b) I_A (the maximum peak value is 4.7 A.).

5.2.2 Machine is Star-Connected Without Neutral:

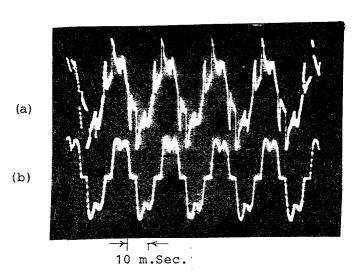


Fig. (22): Voltage and Current of Stator Phase A.

- (a) V_A (the maximum peak value is 311 V.).
- (b) $I_{\rm A}$ (the maximum peak value is 3.6 A.).

The present paper developes a complete digital simulation of a three-phase voltage controller fed induction motor. The new modelling approach of the motor, yields to avoid the numerical inversion of the inductance and exhibit a large saving in computation time. Also the method of simulation leads to modular structure of the program, which make it well suited to study dynamic behaviour, starting and steady-state of an induction motor.

The computed results which are obtained from the simulation for steady-state condition are proved to yield good agreement when compared with the relevant experimental results.

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8. APPENDIX:

The test machine having the following particulars: squirrel - cage, three - phase, 2-horse - power, 4-poles, 380/220 volt, 3.6/6.2 ampere, star - delta connected induction machine.

The measured parameters are: $R_1 = 4.7 \text{ ohm}$, $L_s = 0.228 \text{ H}$, $M_s = -0.112 \text{ H}$ $M_o = 0.212 \text{ H}$, $R_2 = 4.1 \text{ ohm}$, $L_r = 0.228 \text{ H}$ $M_r = -0.114 \text{ H}$, $J = 0.009 \text{kg.m}^2$ and f = 50 Hz.

The control voltage is adjusted to give a firing angle of $[\alpha=83.4^{\circ}]$ for the induction motor with and without neutral.