



**Answer the following questions**

**Question 1 ( 15 marks)**

(A) Show that the set of functions  $\sin\left(\frac{n\pi x}{L}\right)$ ,  $n = 1, 2, 3, \dots$  are orthogonal on the interval  $(0, L)$ . ( 2 Marks)

(B) For the generalized Fourier expansion of the function  $f(x)$  is  $f(x) = \sum_{n=1}^{\infty} a_n \varphi_n(x)$ , Prove that  $a_n = \frac{\int_a^b r(x) f(x) \varphi_n(x) dx}{\int_a^b r(x) \varphi_n^2(x) dx}$ ,  $n = 1, 2, 3, \dots$  ( 3 Marks)

(C) Solve the following PDE

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + x e^{-t}, \quad 0 < x < 1, \quad t > 0$$

Subject to: B.C.s  $u(0, t) = 0, u(1, t) = 1$  and I.C.  $u(x, 0) = f(x)$  ( 10 Marks)

**Question 2 ( 25 MARKS)**

(A) Solve the following PDE

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + x, \quad 0 < x < 1, \quad t > 0$$

Subject to: B.C.s  $u(0, t) = 0, u(1, t) = 1$  and I.C.  $u(x, 0) = f(x), \frac{\partial u(x, 0)}{\partial t} = g(x)$  ( 10 Marks)

(B) Discuss (ناقش) with graphs three drawbacks (عيوب) of Newton-Raphson method for solving non linear algebraic equation. (3 Marks)

(C) Find the real root of the equation  $e^{-x} - x = 0$  using :  
 (i) Newton-Raphson method with  $x_0 = 0$   
 (ii) The Secant method with  $x_0 = 0$  and  $x_1 = 1$  ( 4 Marks)

(D) Given the following system of algebraic equations:

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &= 15 \\ 2x_1 + x_2 + 5x_3 &= 19 \\ 3x_1 + x_2 + x_3 &= 8 \end{aligned}$$

(i) If you solve this system without ordering the equations. What do you expect? Discuss the convergence of this system through Scarborough criteria.  
 (ii) Order your equations in an appropriate way. Use Gauss-Siedel iterative method to make two iterations. Use  $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 1$  ( 8 Marks)

**Question 3 ( 15 MARKS)**

(A) Determine the value of the function  $f(x)$  at  $x=3$  using the direct method interpolation using first and second order polynomial.

$x$	0	1	2	3
$f(x)$	1	4	15	85

(3 Marks)

(B) (i) Prove that the normal equations to the curve  $y = ax + b$  using least squares method are  $\sum y = a\sum x + nb$  and  $\sum xy = a\sum x^2 + b\sum x$ .

(ii) Fit the straight line  $y = ax + b$  and also a parabola

$y = a + bx + cx^2$  to the following set of observations using least squares method:

$x$	0	1	2	3	4
$y$	1	5	10	12	15

Calculate the sum of squares of the residuals in each case and test which curve is more suitable to the data.

(iii) If the function  $f(x, y, z)$  is given by

$$f(x, y, z) = \frac{x+y}{z} + \sin x + \ln y$$

where:  $x = 2 \pm 0.02$ ,  $y = 4 \pm 0.04$  and  $z = 6 \pm 0.06$

Find the maximum possible error in the function  $f(x, y, z)$ .

(10 Marks)

(C) Show that the function  $f(z) = z^2 \bar{z}$  is not analytic anywhere.

(2 Marks)

**Question 4 ( 15 MARKS)**

(A) Evaluate  $\oint_C \frac{dz}{z-a}$  where  $C$  is any simple closed curve and  $z = a$  is

(i) outside  $C$ ,(ii) inside  $C$ .

(5 Marks)

(B) Use Rung-Kutta 4<sup>th</sup> order method to solve the diff. eq.  $\frac{dy}{dx} = y - x^2 + 1$  to

obtain the value of  $y$  at  $x = 0.4$  knowing that  $y(0) = 0.5$  (take  $h=0.4$ ). (5 Marks)

(C) i) Prove that the function  $u = x^2 - y^2$  is harmonic function.

ii) Find a function  $v$  such that  $f(z) = u + iv$  is analytic.

iii) Express  $f(z)$  in terms of  $z$ .

(5 Marks)

This exam measures the following ILOs

Question Number	Q1-a	Q2-d	Q2-b		Q4-a	Q4-b	Q2-c		Q1-b	Q2-a	Q3-b
Skills	Q4-b	d2-i	Q3-b		Q3-a	Q3-c			Q3-d	Q4-c	
	Knowledge & understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky