Sci. J. Fac. Sci. Monoufia Univ., Vol. II, 1988.

FREE CYLINDRICAL COUETTE FLOW OF A RAREFIED GAS WITH HEAT TRANSFER, POROUS SURFACES AND ARBITRARY REFLECTION COEFFICIENT.

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ABSTRACT

Six nonlinear moments equations are used to replace the Boltzman equation describing the free flow of a rarefied gas between two fixed coaxial cylinders. The moments equations with the convenient boundary conditions-concerning heat transfer, porosity and reflection at the surfaces-are solved using small parameters method. The behaviour of the velocity, the density and the temperature is discussed.

I- INTRODUCTION:

The motion of a rarefied gas between two coaxial cylinders one fixed and the other rotates with constant angular velocity was studied by Galkin (1965) using the moments method for obtaining suitable solution for any Knudsen number. The heat transfer from a rarefied electron gas between two coaxial cylinders was investigated by Khidr and Abader(1976) this study revealed that, as the distance between the two cylinders decreases the rarefaction becomes more apparent, and at any degree of rarefaction there exists a minimum value for the density between the two cylinders. Abdel-Gaid and Khidr (1979) studied the problem of flow over a right circular cylinder-within the framework of the kinetic theory of gases-under constant electric field in the radial direction. The moments equations were solved by the small parameter method. The obtained solution showed that the behaviour of flow speed depends on those forces at infinity and was ineffective near the cylinder. Hady (1976) studied the motion of a rarefied gas between two coaxial circular porous cylinders, the inner is fixed while the outer is rotating with constant angular velocity, and the gas is of arbitrary degree of rarefaction. He considered the case, when the temperature difference between the two cylinders was small and the normal velocity to the porous surface was equal to a constant value.

In the present paper we consider the motion of a rarefied gas between two coaxial circular porous cylinders, the inner and the outer are fixed. The density and temperature are assumed to vary as the distance varies from the axis.

We adopt the case when the temperature difference between the cylinders is small and the accomodation coefficient ε is arbitrary.

II- THEORETICAL PREDICTIONS:

Considering a steady situation and free molecular flow, then the distribution function may be obtained from Boltzmann equation in cylindrical coordinates.

$$C_r \frac{\partial f}{\partial r} + C_\Theta\left(\frac{1}{r}\frac{\partial f}{\partial \Theta}\right) + C_z \frac{\partial f}{\partial Z} = 0, \qquad [1]$$

where, $f = f(r,\vec{C})$ is the distribution function.

It is assumed that the distribution function of the molecules reflected from any surface with Maxwellian distribution differs from the density, velocity and temperature of that surface.

Multiplying equation [1] by any arbitrary function $\phi_i = \phi_i(\vec{C})$ we get:

$$\frac{d}{dr}\left[r \int \phi_i C_r f d\vec{C}\right] - \int C_\Theta^2 f \frac{\partial \phi_i}{\partial C_r} d\vec{C} + \int C_r C_\Theta f \frac{\partial \phi_i}{\partial C_\Theta} d\vec{C} = 0, \qquad [2]$$

where $C_r = C_n \sin \Psi$, $C_\Theta = C_n \cos \Psi$.

For any quantity ϕ_i we have

$$\phi_i = \int \phi_i f d\vec{C} = \int_\alpha^{\Pi-\alpha} \int_0^\infty \int_{-\infty}^\infty \phi_i f_1 C_n dC_n dC_z d\Psi + \int_{\Pi-\alpha}^{2\Pi+\alpha} \int_0^\infty \int_{-\infty}^\infty \phi_i f_2 C_n dC_n dC_z d\Psi, \qquad [3]$$

where,

$$f_1 = \frac{n_1}{(2\Pi R T_1)^{3/2}} \left(1 + \frac{V_1}{RT_1} C_r\right) e^{-C^2/2RT_1}; \quad \alpha < \Psi < \Pi - \alpha,$$

$$[4]$$

$$f_2 = \frac{n_2}{(2\Pi R T_2)^{3/2}} \left(1 + \frac{V_2}{RT_2} C_r\right) e^{-C^2/2RT_2}; \quad \Pi-\alpha < \Psi < 2\Pi + \alpha,$$

$\cos \alpha = R_1/r$.

V_1 and V_2 are the suction velocities out of the cylinders.

The six unknowns n_1, n_2, T_1, T_2, V_1 and V_2 are considered functions of the radial distance.

Hence, we have

$$n = \int f d\vec{C}$$

$$= \frac{1}{2\Pi}\left[(\Pi-2\alpha)n_1 + (\Pi+2\alpha)n_2\right] + \frac{\cos\alpha}{\sqrt{4\Pi R}}\left[\frac{n_1 V_1}{\sqrt{T_1}} - \frac{n_2 V_2}{\sqrt{T_2}}\right],$$

$$V_r = \frac{1}{n}\int C_r f d\vec{C}$$

$$= \frac{1}{n}\{\sqrt{\frac{R}{2\Pi}}\cos\alpha\,(n_1\sqrt{T_1} - n_2\sqrt{T_2}) +$$

$$+ \frac{1}{2\Pi}[(\Pi-2\alpha+\sin 2\alpha)\,n_1V_1+(\Pi+2\alpha-\sin 2\alpha)n_2V_2]\} ,$$

$$T = \frac{1}{n}\{\frac{R}{2\Pi}[(\Pi-2\alpha+\sin 2\alpha)\,n_1T_1+(\Pi+2\alpha-\sin 2\alpha)n_2T_2] +$$

$$+\sqrt{\frac{R}{2\Pi}}(3\cos\alpha-\cos^3\alpha)(n_1V_1\sqrt{T_1}-n_2V_2\sqrt{T_2})\} .$$

where, n is the density, V_r is the mean velocity and T is the temperature.

For ϕ_i takes the values $1, C_r, C^2, C_r^2, C_\Theta^2$ and $C_rC_\Theta^2$ equation [2] gives:

$$[n_1T_1{}^{3/2}-n_2T_2{}^{3/2}]\cos\alpha+\frac{1}{\sqrt{2\Pi R}}\{(\Pi-2\alpha+\sin 2\alpha)n_1V_1+(\Pi+2\alpha-\sin 2\alpha)n_2V_2\} = \gamma_1/r, \qquad [5]$$

where, γ_1 is arbitrary,

$$\frac{d}{dr}\{[(\Pi-2\alpha+\sin 2\alpha)n_1T_1+(\Pi+2\alpha-\sin 2\alpha)n_2T_2] +$$

$$+\sqrt{\frac{2\Pi}{R}}[(3\cos\alpha-\cos^3\alpha)(n_1V_1T_1^{\frac{1}{2}}-n_2V_2T_2^{\frac{1}{2}})]\}+\frac{4\sin 2\alpha}{r}[n_1T_1-n_2T_2] +$$

$$+\frac{1}{r}\sqrt{\frac{2\Pi\mu}{R}}[(3\cos\alpha-2\cos^3\alpha)(n_1V_1T_1^{\frac{1}{2}}-n_2V_2T_2^{\frac{1}{2}})] = 0, \qquad [6]$$

$$[n_1T_1{}^{3/2}-n_2T_2{}^{3/2}]\cos\alpha+\frac{5}{4\sqrt{2\Pi R}}\{(\Pi-2\alpha+\sin 2\alpha)n_1V_1T_1+$$

$$+(\Pi+2\alpha-\sin 2\alpha)n_2V_2T_2\} = \gamma_2/r. \qquad [7]$$

where γ_2 is an arbitrary constant,

$$\frac{d}{dr}\{[(3\cos\alpha-\cos^3\alpha)(n_1T_1^{3/2}-n_2T_2^{3/2})]+\frac{1}{\sqrt{2\Pi R}}[(3\Pi-6\alpha+4\sin 2\alpha-\frac{\sin 4\alpha}{2})n_1V_1T_1$$

$$+(3\Pi+6\alpha-4\sin 2\alpha+\frac{\sin 4\alpha}{2})n_2V_2T_2]\}+\frac{1}{r}[(3\cos\alpha-2\cos^3\alpha)(n_1T_1^{3/2}-n_2T_2^{3/2})]$$

$$+\frac{2}{r}\;\frac{1}{\sqrt{2\Pi R}}\;[(\Pi-2\alpha+2\;\sin\,2\alpha-\frac{3}{4}\;\sin\,4\alpha)n_1V_1T_1+$$

$$+\;(\Pi+2\alpha-2\;\sin\,2\alpha\;+\frac{3}{4}\;\sin\,4\alpha)n_2V_2T_2]\;=\;0\;, \qquad [8]$$

$$\text{Cos}^3\alpha(n_1T_1^{3/2}-n_2T_2^{3/2})\;+\;\frac{1}{\sqrt{2\Pi R}}\;[(\Pi-2\alpha+\frac{\sin\,4\alpha}{2})n_1V_1T_1+$$

$$+\;(\Pi+2\alpha-\frac{\sin\,4\alpha}{2})n_2V_2T_2]\;=\;\gamma_3/r^3\;, \qquad [9]$$

γ_3 is an arbitrary constant.

and

$$\frac{d}{dr}\{[(\Pi-2\alpha+\frac{\sin4\alpha}{2})n_1T_1^2\;+\;(\Pi+2\alpha-\frac{\sin4\alpha}{2})n_2T_2^2]\;+$$

$$+\;8\sqrt{\frac{2\Pi}{R}}[(5\;\cos^3\alpha-\;3\;\cos^5\alpha)(n_1V_1T_1^{3/2}-n_2V_2T_2^{3/2})]\}+$$

$$+\;\frac{2(\sin\,4\alpha-\;2\;\sin\,2\alpha)}{r}\;[n_1T_1^2-n_2T_2^2]\;+$$

$$+\;\frac{24}{R}\sqrt{\frac{2\Pi}{R}}\;[(5\;\cos^3\alpha-\;4\;\cos^5\alpha)(n_1V_1T_1^{3/2}-n_2V_2T_2^{3/2})]=\;0. \qquad [10]$$

The boundary conditions on f leads to the following conditions in the case when the reflection coefficient is arbitrary.

$$f_1(R_1)\;=\;(1-\varepsilon)\;[-f_2(R_2)]\;+\;\varepsilon f_{s_1}\;\;,$$

$$f_2(R_2)\;=\;(1-\varepsilon)\;[-f_1(R_1)]\;+\;\varepsilon f_{s_2}\;\;.$$

where,

$$f_{s_1}\;=\;\frac{n_{s_1}}{(2\Pi RT_{s_1})^{3/2}}\;(1\;+\;\frac{a}{RT_{s_1}}\;C_r)\;e^{-C^2/2RT_{s_1}}\;\;,$$

$$f_{s_2}\;=\;\frac{n_{s_2}}{(2\Pi RT_{s_2})^{3/2}}\;(1\;+\;\frac{a}{RT_{s_2}}\;C_r)\;e^{-\;C^2/2RT_{s_2}}$$

and

$$\left\{\frac{3R}{2\Pi}(\Pi+2\alpha_1)n_2(R_2)T_2(R_2)-4\sqrt{\frac{R}{2\Pi}}\cos\alpha_1\; n_2(R_2)V_2(R_2)\sqrt{T_2}(R_2)\right\}=$$

$$=(1-\varepsilon)\left\{\frac{3R}{2\Pi}(\Pi+2\alpha_1)n_1(R_1)T_1(R_1)+4\sqrt{\frac{R}{2\Pi}}\cos\alpha_1\; n_1(R_1)V_1(R_1)\sqrt{T_1(R_1)}\right\}+$$

$$+\varepsilon\left\{\frac{3R}{2\Pi}(\Pi+2\alpha_1)\; n_{s_2}\; T_{s_2}-4\sqrt{\frac{R}{2\Pi}}\cos\alpha_1\; n_{s_2}\; a\sqrt{T_{s_2}}\right\}, \qquad [16]$$

where, $n_{s_2}=n_s$, $T_{s_2}=T_s$, $T_{s_1}=T_s\ (1+x)$. and R_1, R_2 are the radii of inner and outer cylinders and T_s is the temperature of the outer cylinder. The inner cylinder was considered fixed and its temperature differs from the temperature of the outer cylinder by $X\ T_s$. The quantity X is taken small, so that we can neglect its square.

If, we take in nondimensional form the quantities

$$V_1 = V_1^{\backprime} a, \quad V_2 = V_2^{\backprime} a, \quad n_1 = n_1^{\backprime} n_s, \quad n_s = n_2^{\backprime} n_s$$

$$r = r^{\backprime} R_2, \quad T_1 = T_1^{\backprime} T_s, \quad T_2 = T_2^{\backprime} T_s, \quad \cos\alpha = \frac{R_1}{r^{\backprime} R_2}$$

$$q = \frac{R_1}{R_2}, \quad \cos\alpha_0 = 1, \quad \alpha_0 = 0, \quad \cos\alpha_1 = q$$

The system of equations [5]-[16] in nondimensional form after dropping the prims will be

$$[n_1T_1^{\frac{1}{2}}-n_2T_2^{\frac{1}{2}}]+\eta[(\Pi-2\alpha+\sin2\alpha)n_1V_1+(\Pi+2\alpha-\sin2\alpha)n_2V_2]\; r=\gamma_1, \qquad [17]$$

$$\frac{d}{dr}\left\{[(\Pi-2\alpha+\sin2\alpha)n_1T_1+(\Pi+2\alpha-\sin2\alpha)n_2T_2]+\frac{\beta_1(3r^2-q^2)}{r^3}(n_1V_1T_1^{\frac{1}{2}}-n_2V_2T_2^{\frac{1}{2}})\right\}$$

$$+\frac{4\sin2\alpha}{r}(n_1T_1-n_2T_2)+\frac{\beta_1}{r}\frac{(3r^2-q^2)}{r^3}(n_1V_1T_1^{\frac{1}{2}}-n_2V_2T_2^{\frac{1}{2}})=0, \qquad [18]$$

$$[n_1T_1^{3/2}-n_2T_2^{3/2}]+\frac{5\eta}{4}\{(\Pi-2\alpha+\sin2\alpha)n_1V_1T_1+(\Pi+2\alpha-\sin2\alpha)n_2V_2T_2\}r=\gamma_2, \qquad [19]$$

$$\frac{d}{dr}\left\{\left[\frac{(3r^2-q^2)}{r^3}(n_1T_1^{3/2}-n_2T_2^{3/2})\right]+\eta\left[(3\Pi-6\alpha+4\sin2\alpha-\frac{\sin4\alpha}{2})n_1V_1T_1+\right.\right.$$

By multiplying the last two equations by $d\vec{C}$, $C_r d\vec{C}$ and $C^2 d\vec{C}$ respectively and integrating, we get:

$$[\sqrt{2\Pi R^3}(\Pi-2\alpha_o)+4\Pi R\cos\alpha_o \frac{V_1(R_1)}{\sqrt{T_1(R_1)}}]\, n_1(R_1) =$$
$$=(1-\varepsilon)[-\sqrt{2\Pi R^3}(\Pi-2\alpha_o)-4\Pi R\cos\alpha_o \frac{V_2(R_2)}{\sqrt{T_2(R_2)}}]\, n_2(R_2)$$
$$+\varepsilon[\sqrt{2\Pi R^3}(\Pi-2\alpha_o)+4\Pi R\cos\alpha_o \frac{a}{\sqrt{T_{s_1}}}]\, n_{s_1}, \quad [11]$$

$$[\sqrt{2\Pi R^3}(\Pi+2\alpha_1)-4\Pi R\cos\alpha_1 \frac{V_2(R_2)}{T_2(R_2)}]\, n_2(R_2) =$$
$$=(1-\varepsilon)[-\sqrt{2\Pi R^3}(\Pi+2\alpha_1)+4\Pi R\cos\alpha_1 \frac{V_1(R_1)}{T_1(R_1)}]\, n_1(R_1) +$$
$$+\varepsilon[\sqrt{2\Pi R^3}(\Pi+2\alpha_1)-4\Pi R\cos\alpha_1 \frac{a}{\sqrt{T_{s_2}}}\, n_{s_2}], \quad [12]$$

$$[2\Pi R^2\cos\alpha_o n_1(R_1)\sqrt{T_1(R_1)}+\sqrt{2\Pi R^3}(\Pi-2\alpha_o+\sin 2\alpha_o)n_1(R_1)V_1(R_1)]=$$
$$=(1-\varepsilon)[-2\Pi R^3\cos\alpha_o n_2(R_o)\sqrt{T_2(R_2)}-\sqrt{2\Pi R^3}(\Pi-2\alpha_o+\sin 2\alpha_o)n_2(R_2)V_2(R_2)]$$
$$+\varepsilon[2\Pi R^2\cos\alpha_o n_{s_1}\sqrt{T_{s_1}}+\sqrt{2\Pi R^3}(\Pi-2\alpha_o+\sin 2\alpha_o)a\, n_{s_1}], \quad [13]$$

$$\{2\Pi R^2\cos\alpha_1\, n_2(R_2)\sqrt{T_2(R_2)}-\sqrt{2\Pi R^3}(\Pi+2\alpha_1-\sin 2\alpha_1)n_2(R_2)V_2(R_2)\} =$$
$$=(1-\varepsilon)\{-2\Pi R^3\cos\alpha_1\, n_1(R_1\sqrt{T_1}(R_1)+\sqrt{2\Pi R^3}(\Pi+2\alpha_1-\sin 2\alpha_1)n_1(R_1)V_1(R_1)\}$$
$$+\varepsilon\{2\Pi R^2\cos\alpha_1\, n_{s_2}\sqrt{T_{s_2}}-\sqrt{2\Pi R^3}(\Pi+2\alpha_1-\sin 2\alpha_1)a\, n_{s_2}\}, \quad [14]$$

$$\{\frac{3R}{2\Pi}(\Pi-2\alpha_o)n_1(R_1)T_1(R_1)+4\sqrt{\frac{R}{2\Pi}}\cos\alpha_o\, n_1(R_1)V_1(R_1)\sqrt{T_1}(R_1)\} =$$
$$=(1-\varepsilon)\{\frac{-3R}{2\Pi}(\Pi-2\alpha_o)n_2(R_2)T_2(R_2)-4\sqrt{\frac{R}{2\Pi}}\cos\alpha_o\, n_2(R_2)V_2(R_2)\sqrt{T_2}(R_2)\}$$
$$+\varepsilon\{\frac{3R}{2\Pi}(\Pi-2\alpha_o)\, n_{s_1}T_{s_1}+4\sqrt{\frac{R}{2\Pi}}\cos\alpha_o\, n_{s_1}\, a\sqrt{T_{s_1}}\}, \quad [15]$$

$$+(3\Pi+6\alpha-4\sin2\alpha+\frac{\sin4\alpha}{2})\,n_2V_2T_2]\}+\frac{(3r^2-q^2)}{r^4}(n_1T_1^{3/2}-n_2T_2^{3/2})+$$

$$+\frac{2\eta}{r}[(\Pi-2\alpha+2\sin2\alpha-\tfrac{3}{4}\sin4\alpha)n_1V_1T_1+(\Pi+2\alpha-2\sin2\alpha+\tfrac{3}{4}\sin4\alpha)n_2V_2T_2]=0, \qquad [20]$$

$$\frac{q^3}{r^3}[n_1T_1^{3/2}-n_2T_2^{3/2}]+\eta[(\Pi+2\alpha+\frac{\sin4\alpha}{2})n_1V_1T_1+(\Pi+2\alpha-\frac{\sin4\alpha}{2})n_2V_2T_2]=\frac{\gamma_3}{r^3}, \qquad [21]$$

$$\frac{d}{dr}\{[(\Pi-2\alpha+\tfrac{1}{2}\sin4\alpha)n_1T_1^2+(\Pi+2\alpha-\tfrac{1}{2}\sin4\alpha)n_2T_2^2]+$$

$$+\frac{8\beta_1q^2}{r^5}(5r^2-3q^2)(n_1V_1T_1^{3/2}-n_2V_2T_2^{3/2})+\frac{2(\sin4\alpha-2\sin2\alpha)}{r}[n_1T_1^2-n_2T_2^2]+$$

$$+\frac{24\beta_1q^2}{r^6}[(5r^2-4q^2)(n_1V_1T_1^{3/2}-n_2V_2T_2^{3/2})]=0, \qquad [22]$$

where, $\eta=\dfrac{a}{q\sqrt{2\Pi RT_s}}$, $\beta_1=\sqrt{\dfrac{2}{RT_s}}\,a\,q.$

Under the boundary conditions

$$[1+4\eta q\frac{V_1(q)}{\sqrt{T_1(q)}}]\,n_1(q)=(1-\varepsilon)[-1-4.\eta q\frac{V_2(1)}{\sqrt{T_2(1)}}]n_2(1)+\varepsilon[1+\frac{4\eta q}{\sqrt{1+X}}], \qquad [23]$$

$$[(\Pi+2\alpha)-2\beta_1\frac{V_2(1)}{\sqrt{T_2(1)}}]n_2(1)=(1-\varepsilon)[-(\Pi+2\alpha_1)+2\beta_1\frac{V_1(q)}{\sqrt{T_1(q)}}]n_1(q)+$$

$$+[(\Pi+2\alpha_1)-2\beta_1], \qquad [24]$$

$$[n_1(q)\sqrt{T_1(q)}+\Pi q\eta n_1(q)V_1(q)]=(1-\varepsilon)[-n_2(1)\sqrt{T_2(1)}-\Pi q n_2(1)V_2(1)]+$$

$$+\varepsilon[\sqrt{1+X}+\Pi q\eta], \qquad [25]$$

$$[n_2(1)\sqrt{T_2}(1)-\eta(\Pi+2\alpha_1-\sin2\alpha_1)n_2(1)V_2(1)]=(1-\varepsilon)[-n_1(q)\sqrt{T_1(q)}+$$

$$+\eta(\Pi+2\alpha_1-\sin2\alpha_1)n_1(q)V_1(q)]+\varepsilon[1-\eta(\Pi+2\alpha_1-\sin2\alpha_1)], \qquad [26]$$

$$[\tfrac{3}{2}n_1(q)T_1(q)+4\eta q\,n_1(q)V_1(q)\sqrt{T_1(q)}]=$$

$$=(1-\varepsilon)[-\frac{3}{2}n_2(1)T_2(1)-4\eta q\; n_2(1)V_2(1)\sqrt{T_2(1)}]\; +$$

$$+\;\varepsilon[\frac{3}{2}(1+X)+4\eta q\sqrt{1+X}], \qquad [27]$$

$$[\frac{3}{2}(\Pi+2\alpha_1)n_2(1)T_2(1)-4q^2\eta n_2(1)V_2(1)\sqrt{T_2(1)}] =$$

$$=(1-\varepsilon)\{-\frac{3}{2}(\Pi+2\alpha_1)n_1(q)T_1(q)+4q^2\eta n_1(q)V_1(q)\sqrt{T_1(q)}\; +$$

$$+\;\varepsilon\{\frac{3}{2}(\Pi+2\alpha_1)-4q^2\eta\}. \qquad [28]$$

Equations [17]-[22] are nonlinear, X is small.

Now, we discuss the following two cases:

Case 1: If $V_1 = V_2 = 0$

In this case, we put.

$$n_1 = 1 + X\,n_1^{(1)} \quad , \quad n_2 = 1 + X\,n_2^{(1)}$$

$$T_1 = 1 + X\,T_1^{(1)} \quad , \quad T_2 = 1 + X\,T_2^{(1)},$$

and $\gamma_1 = 1 + X\,\gamma_1^{(1)}$

On substituting these values in equations [17],[18],[20] and [22]equating the coefficients of X on both sides in the resulting equations, we get:

$$n_1^{(1)}+\tfrac{1}{2}T_1^{(1)}-n_2^{(1)}-\tfrac{1}{2}T_2^{(1)} = \gamma_1^{(1)} \qquad [17]'$$

$$\frac{d}{dr}\{[(\Pi-2\alpha+\sin 2\alpha)(n_1^{(1)}+T_1^{(1)})+(\Pi+2\alpha-\sin 2\alpha)(n_2^{(1)}+T_2^{(1)})]\}$$

$$+\frac{4\sin 2\alpha}{r}[n_1^{(1)}+T_1^{(1)}-n_2^{(1)}-T_2^{(1)}]=0, \qquad [18]'$$

$$\frac{d}{dr}\{\frac{(3r^2-q^2)}{r^3}(n_1^{(1)}+\frac{3}{2}T_1^{(1)}-n_2^{(1)}-\frac{3}{2}T_2^{(1)})\}\; +$$

$$+\frac{1}{r}\{\frac{(3r^2-q^2)}{r^3}\,(n_1^{(1)}+\frac{3}{2}\,T_1^{(1)}-n_2^{(1)}-\frac{3}{2}\,T_2^{(1)}\}=0, \qquad [20]`$$

and

$$\frac{d}{dr}\,\{(\Pi-2\alpha+\tfrac{1}{2}\sin 4\alpha)(n_1^{(1)}2T_1^{(1)})+(\Pi+2\alpha-\tfrac{1}{2}\sin 4\alpha)(n_2^{(1)}+2T_2^{(1)})\}+$$

$$+\frac{2(\sin 4\alpha-2\sin 2\alpha)}{r}\,[n_1^{(1)}+2T_1^{(1)}-n_2^{(1)}-2T_2^{(1)}]=0. \qquad [22]`$$

The above equations are valid under the following boundary conditions:

$$n_1^{(1)}(q)=0 \quad , \quad n_2^{(1)}(1)=0$$
$$T_1^{(1)}(q)=\frac{1}{(2-\varepsilon)} \quad , \quad T_2^{(1)}(1)=\frac{(\varepsilon-1)}{(2-\varepsilon)} \qquad [29]$$

For α is sufficiently small ($\alpha \ll 1$), the solution of the system of equations [17]`,[18]`,[20]`and [22]`gives

$$n_1^{(1)}=\tfrac{1}{2}\,\{(\tfrac{3}{2}\,\gamma_1^{(1)}+2A_1-A_3)-\frac{r^2}{2(3r^2-q^2)}\,A_2\}\,, \qquad [30]$$

$$n_2^{(1)}=\tfrac{1}{2}\,\{(2A_1-A_3-\tfrac{3}{2}\,\gamma_1^{(1)}+\frac{r^2}{2(3r^2-q^2)}\,A_2\}\,, \qquad [31]$$

$$T_1^{(1)}=\tfrac{1}{2}\,\{(A_3-A_1-\gamma_1^{(1)})+\frac{r^2}{(3r^2-q^2)}\,A_2\}\,, \qquad [32]$$

and

$$T_2^{(1)}=\tfrac{1}{2}\,\{(\gamma_1^{(1)}-A_1+A_3)-\frac{r^2}{(3r^2-q^2)}\,A_2\}\,, \qquad [33]$$

where, A_1, A_2 and A_3 are the integration constants and can be determined by using the boundary conditions [29], where

$$\gamma_1^{(1)}=\tfrac{1}{2} \quad , \quad A_1=\frac{(5-q^2)(6+\varepsilon)-6(2-\varepsilon)(3-q^2)}{4(2-\varepsilon)(5-q^2)}\,,$$

$$A_2=\frac{6(3q^2)}{(5-q^2)} \quad , \quad A_3=\frac{(18-\varepsilon)(5-q^2)-18(2-\varepsilon)(3-q^2)}{4(2-\varepsilon)(5-q^2)}$$

Case 2: In this case we assume that V is of order of the Mach number, i.e.,

$$V_i = M\, V_{ir} \quad , \quad T_i = 1 + X\, T_i^{(1)} + M\, T_i^{(2)},$$
$$n_i = 1+Xn_i^{(1)} + Mn_i^{(2)} \quad , \quad \gamma_1 = 1 + X\, \gamma_1^{(1)} + M\, \gamma_1^{(2)}, \qquad [34]$$
$$\gamma_2 = M\ \gamma_2^{(2)}, \quad \gamma_3 = M\ \gamma_3^{(2)}, \quad \text{i.e. } M^2 \ll 1 \text{ and } XM \ll 1.$$

For α is sufficiently small, then $\sin\alpha = \alpha$, $q = 1$.

On substituting these values in equations [17]-[22] and on boundary conditions [23]-[28] and equating the coefficient of M on both sides of the resulting equations we get:

$$(n_1^{(2)}+\tfrac{1}{2}T_1^{(2)}-n_2^{(2)}-\tfrac{1}{2}T_2^{(2)})+\Pi\eta(V_{1_r}+V_{2_r})= \gamma_1^{(2)}. \qquad [35]$$

$$\frac{d}{dr}\{n_1^{(2)}+T_1^{(2)}+n_2^{(2)}+T_2^{(2)}\}+ \frac{\beta_1}{\Pi}\ \frac{(3r^2-1)}{r^4}\ (V_{1_r}-V_{2_r})=0, \qquad [36]$$

$$(n_1^{(2)}+ \frac{3}{2}T_1^{(2)}-n_2^{(2)}- \frac{3}{2}T_2^{(2)}+ \frac{5\Pi\eta}{4}\ .r(V_{1_r}+V_{2_r})= \gamma_2^{(2)} \qquad [37]$$

$$\frac{d}{dr}\ \{\frac{(3r^2-1)}{r^3}\ (n_1^{(2)}+ \frac{3}{2}T_1^{(2)}-n_2^{(2)}- \frac{3}{2}T_2^{(2)}+3\eta(V_{1_r}+V_{2_r})\} +$$
$$+[\ \frac{(3r^2-1)}{r^4}\ (n_1^{(2)}+ \frac{3}{2}T_1^{(2)}-n_2^{(2)}- \frac{3}{2}T_2^{(2)})+ \frac{2\Pi\eta}{r}\ (V_{1_r}+V_{2_r})]=0, \qquad [38]$$

$$\frac{1}{r^3}(n_1^{(2)}+ \frac{3}{2}T_1^{(2)}-n_2^{(2)}-\frac{3}{2}T_2^{(2)})+\eta\Pi(V_{1_r}+V_{2_r}) = \frac{\gamma_3^{(2)}}{r^3}\ , \qquad [39]$$

$$\frac{d}{dr}\ \{(n_1^{(2)}+2T_1^{(2)}+n_2^{(2)}+2T_2^{(2)})+ \frac{8\beta_1}{\Pi r^5}\ (5r^2-3)(V_{1_r}-V_{2_r})\} +$$
$$+ \frac{24\beta_1}{\Pi r^6}\ [(5r^2-4)(V_{1_r}-V_{2_r})] = 0 \qquad [40]$$

in the boundary conditions [41]-[46] and equating the terms free of q on both sides and equating the terms containing q on both sides, one obtains a system of boundary conditions.

If, we consider $n_2^{(2)}$, $T_2^{(2)}$ depend only on ρ and ρ^2, then, $C_3 = C_3' = 0$.

The solution of the resulting system of equations gives:

$$C_1 = 7\gamma_3^{(2)} - \frac{15}{2}\alpha_2^{(2)} + \frac{3}{2}\alpha_1^{(2)}, \quad C_2 = (242\alpha_2^{(2)} - \frac{233}{2}\alpha_3^{(3)}),$$

$$C_3 = 0, \quad C_4 = (238\gamma_2^{(2)} - \frac{159}{2}\gamma_3^{(2)}),$$

$$C_5 = -(3177\gamma_2^{(2)} + 3669\gamma_3^{(2)}, \quad C_6 = -(3679\gamma_2^{(2)} + 3737\gamma_3^{(2)},$$

$$C_1' = (\gamma_3^{(2)} - \gamma_1^{(2)}), \quad C_2' = (64\gamma_3^{(2)} - \frac{455}{2}\gamma_2^{(2)},$$

$$C_3' = 0, \quad C_4' = (104\gamma_3^{(2)} - \frac{465}{2}\gamma_2^{(2)},$$

$$C_5' = (3523\gamma_2^{(2)} + 3837\gamma_3^{(2)}, \quad C_6' = 3387\gamma_2^{(2)} + 3763\gamma_3^{(2)},$$

$$C_1'' = \frac{(\frac{99}{10}\gamma_2^{(2)} - 11\gamma_3^{(2)})}{\Pi\eta}, \quad C_2'' = \frac{(31\gamma_3^{(2)} - 36\gamma_2^{(2)})}{\Pi\eta},$$

$$C_3'' = \frac{370\gamma_2^{(2)} - 6\gamma_2^{(2)})}{\Pi\eta}, \quad C_4'' = \frac{(41\gamma_3^{(2)} - 36\gamma_2^{(2)})}{\Pi\eta},$$

$$C_5'' = \frac{-370\gamma_2^{(2)} - 723\gamma_3^{(2)}}{\Pi\eta}, \quad C_6'' = \frac{627\gamma_3^{(2)} - 20/\gamma_2^{(2)}}{\Pi\eta}$$

where $\gamma_1^{(2)}$, $\gamma_2^{(2)}$ and $\gamma_3^{(2)}$ can be written in the form

$$\gamma_1^{(2)} = \frac{8(\Pi\eta - 8\eta - 1) + \Pi\varepsilon(3 - 8\eta)}{2(2-\varepsilon)(3\Pi - 8)},$$

$$\gamma_2^{(2)} = \frac{192\eta(\Pi - 4) + 5\Pi\varepsilon(3 - 8\eta)}{8(2-\varepsilon)(3\Pi - 8)},$$

$$\gamma_3^{(2)} = \frac{16\eta(3\Pi - 2) + \Pi\varepsilon(3 - 8\eta)}{2(2-\varepsilon)(3\Pi - 8)},$$

The above equations are valid under the following boundary conditions:

$$[n_1^{(2)}+3\eta V_{1_r}(q)]=(1-\varepsilon)[-n_2^{(2)}(1)-4\eta V_{2_r}(1)]-\varepsilon(2\eta), \quad [41]$$

$$[\Pi n_2^{(2)}(1)-2\beta_1 V_{2_r}(1)]=(1-\varepsilon)[-\Pi n_1^{(2)}(q)+2\beta_1 V_{1_r}(q), \quad [42]$$

$$[(n_1^{(2)}+\tfrac{1}{2}T_1^{(2)}+\Pi\eta V_{1_r}(q)]=(1-\varepsilon)[-(n_2^{(2)}+\tfrac{1}{2}T_2^{(2)})-\Pi\eta V_{2_r}(1)+\varepsilon/2, \quad [43]$$

$$[(n_2^{(2)}+\tfrac{1}{2}T_2^{(2)}-\Pi\eta V_{2_r}(1)]=(1-\varepsilon)[-(n_1^{(2)}+\tfrac{1}{2}T_1^{(2)})+\Pi\eta V_{1_r}(q)], \quad [44]$$

$$[\tfrac{3}{2}(n_1^{(2)}+T_1^{(2)})+4\eta V_{1_r}(q)]=(1-\varepsilon)[-\tfrac{3}{2}(n_2^{(2)}+T_2^{(2)})-4\eta V_{2_r}(1)]+\varepsilon(2\eta), \quad [45]$$

and

$$[\tfrac{3}{2}\Pi(n_2^{(2)}+T_2^{(2)})-4\eta V_{2_r}(1)]=(1-\varepsilon)[-\tfrac{3}{2}\Pi(n_1^{(2)})+T_1^{(2)}+4\eta V_{1_r}(q)]. \quad [46]$$

On substituting $r = 1 - \rho$, $\rho^3 << 1$

$n_1^{(2)} = C_1 + C_2\rho + C_5\rho^2$, $n_2^{(2)} = C_3 + C_4\rho + C_6\rho^2$,

$T_1^{(2)} = C_1' + C_2'\rho + C_5'\rho^2$, $T_2^{(2)} = C_2' + C_4'\rho + C_6'\rho^2$,

$V_{1_r} = C_1'' + C_2''\rho + C_5''\rho^2$, $V_{24} = C_3'' + C_3''\rho + C_6''\rho^2$.

in equations [35]-[40] by equating the terms free of ρ on both sides, equating the terms containing ρ on both sides and equating the terms containing ρ^2 on both sides and by solving the resulting system of equations, and on substituting

$\rho = 0$, $\rho = 1-q$

$n_1^{(2)} = C_1 + C_2(1-q)$, $T_1^{(2)} = C_1' + C_2'(1-q)$, $V_{1_r} = C_1'' + C_2''(1-q)$

$n_2^{(2)} = C_3$, $T_2 = C_3'$, $V_{2_r} = C_3''$, then

$$n_1^{(2)}=(7\gamma_3^{(2)}-\frac{15}{2}\gamma_2^{(2)}+\frac{3}{2}\gamma_1^{(2)})+(242\gamma_2^{(2)}-\frac{233}{2}\gamma_3^{(2)})\rho-(3177\gamma_2^{(2)}+3669\gamma_3^{(2)})\rho^2,$$

$$n_2^{(2)}=(238\gamma_2^{(2)}-\frac{159}{2}\gamma^{(2)})\ \rho-(3679\gamma_2^{(2)}+3727\gamma_3^{(2)})\rho^2,$$

$$T_1^{(2)}=(\gamma_3^{(2)}-\gamma_1^{(2)})+(64\gamma_3^{(2)}-\frac{455}{2}\gamma_2^{(2)})\rho+(3525\gamma_2^{(2)}+3837\gamma_3^{(2)})\rho^2.$$

$$T_2^{(2)}=(104\gamma_3^{(2)}-\frac{465}{2}\gamma_2^{(2)})\rho+(3387\gamma_2^{(2)}+3763\gamma_3^{(2)})\rho^2,$$

$$V_{1_r}=\frac{1}{\Pi\eta}\{(\frac{99}{10}\gamma_2^{(2)}-11\gamma_3^{(2)})+(31\gamma_2^{(2)}-\frac{3}{10}\gamma_3^{(2)})\rho-(370\gamma_2^{(2)}+723\gamma_3^{(2)})\rho^2\},$$

and

$$V_{2_r}=\frac{1}{\Pi\eta}\{(7\gamma_3^{(2)}-6\gamma_2^{(2)})+(41\gamma_3^{(2)}-36\gamma_2^{(2)})\rho-\{(627\gamma_3^{(2)}-201\gamma_2^{(2)})\rho^2\}.$$

For $r=1-\rho$, $q=1$, $\rho^3 << 1$, the zeroth approximation of the density and the temperature are

$$n=1+\frac{3X\alpha}{4\Pi}\rho,$$

$$T=\frac{1}{n}[1+\frac{X}{2}(\frac{\varepsilon}{2-\varepsilon})],$$

and the first approximation of the density, mean velocity and the temperature are

$$n=1+\frac{3X\alpha}{4\Pi}\rho+M[\frac{3}{4}\gamma_1^{(2)}+\frac{25}{4}\gamma_2^{(2)}-8\gamma_3^{(2)})+\frac{(2923\gamma_2^{(2)}-1333\gamma_3^{(2)})}{10}\rho]-$$

$$-\alpha_1\frac{M}{\Pi}[(7\gamma_3^{(2)}-\frac{15}{2}\gamma_2^{(2)}+\frac{3}{2}\gamma_1^{(2)})+(4\gamma_2^{(2)}-32\gamma_3^{(2)})\rho]; \quad [47]$$

$$V_r=\frac{\eta}{2\Pi n}\{(x/16)[8-\rho]+M[(\frac{15}{2}\gamma_3^{(2)}-\frac{15}{2}\gamma_2^{(2)}+\gamma_1^{(2)})+(\gamma_1^{(2)}-\gamma_2^{(2)}-\frac{89}{2}\gamma_3^{(2)})\rho]\}+$$

$$+\frac{M}{2\Pi\eta n}[(\frac{39}{10}\gamma_2^{(2)}-4\gamma_3^{(2)})+(\frac{407}{10}\gamma_3^{(2)}-5\gamma_2^{(2)})\rho] \quad [48]$$

$$T=\frac{1}{n}\{1+\frac{X}{2}[\frac{\varepsilon}{2-\varepsilon}]+M[(\frac{1}{4}\gamma_1^{(2)}+\frac{25}{2}\gamma_2^{(2)}-\frac{15}{2}\gamma_3^{(2)})+$$

$$+(\frac{527}{10}\gamma_2^{(2)}-\frac{378}{10}\gamma_3^{(2)})\rho]\}, \quad [49]$$

where η is the coefficient related to the velocity through the porous surface.

CONCLUSION:

The numerical investigation to the above results are illustrated in figures (1)-(4). The analysis of these results leads to:

(i) From Fig. (1), we see that the density decreases with the increase of ρ (distance between the two cylinders) for constant η (coefficient related to velocity through the porous surface) and reflection coefficient ε .

(ii) From Fig. (2), we see that the density decreases as ρ increases for constant η and ε , and it increases as η increases for constant ρ .

(iii) The magnitude of velocity increases with the increase of ρ for constant ε and it increases with the increase of ε for constant η as it is seen from Fig. (3).

(iv) From Fig. (4) the temperature at any point between the two cylinders increases as η increases for constant ε and η , and it increases as ε increases for constant η .

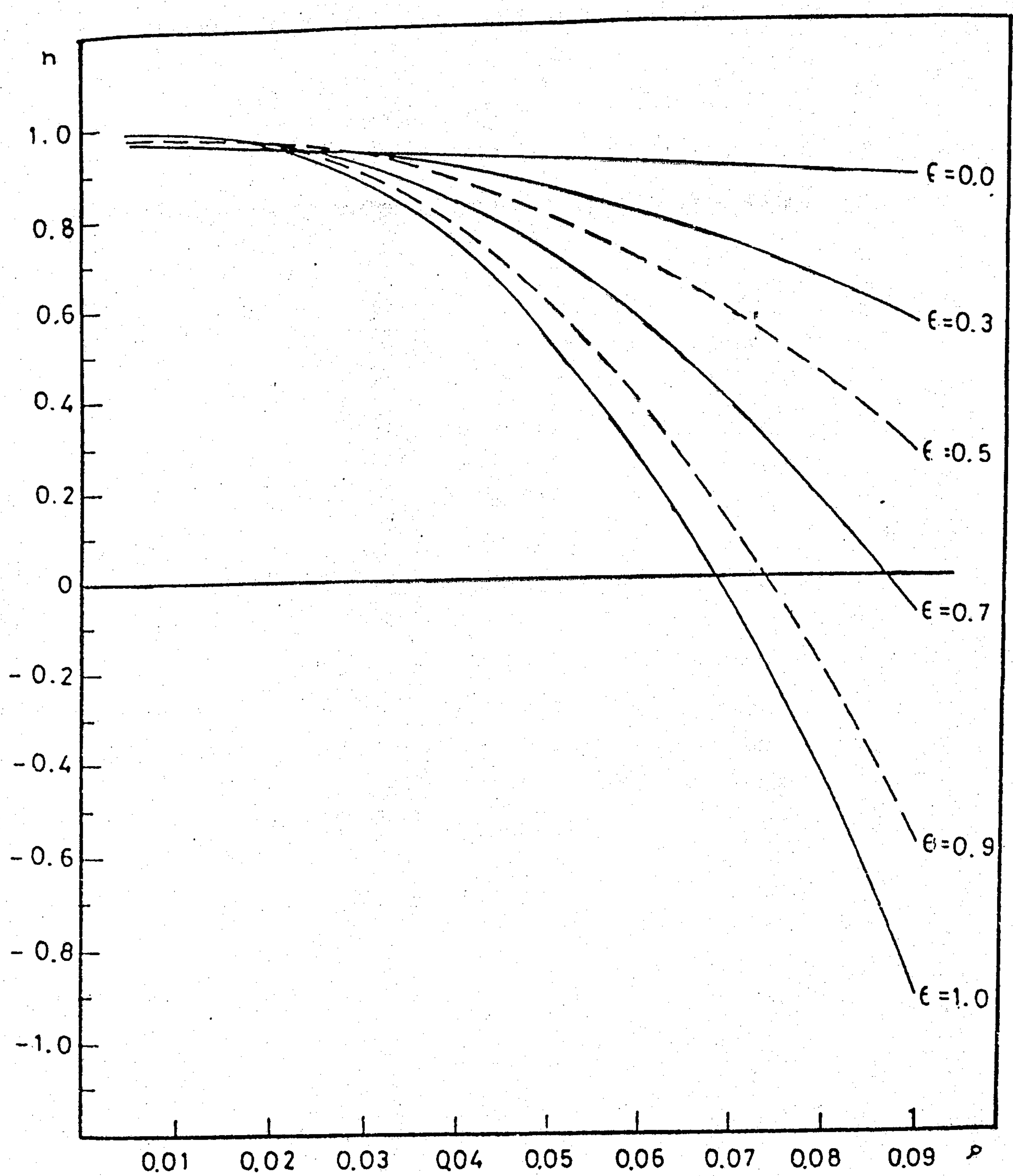

Fig. (1) Variation between density and distance ρ for constant (η = 0.01).

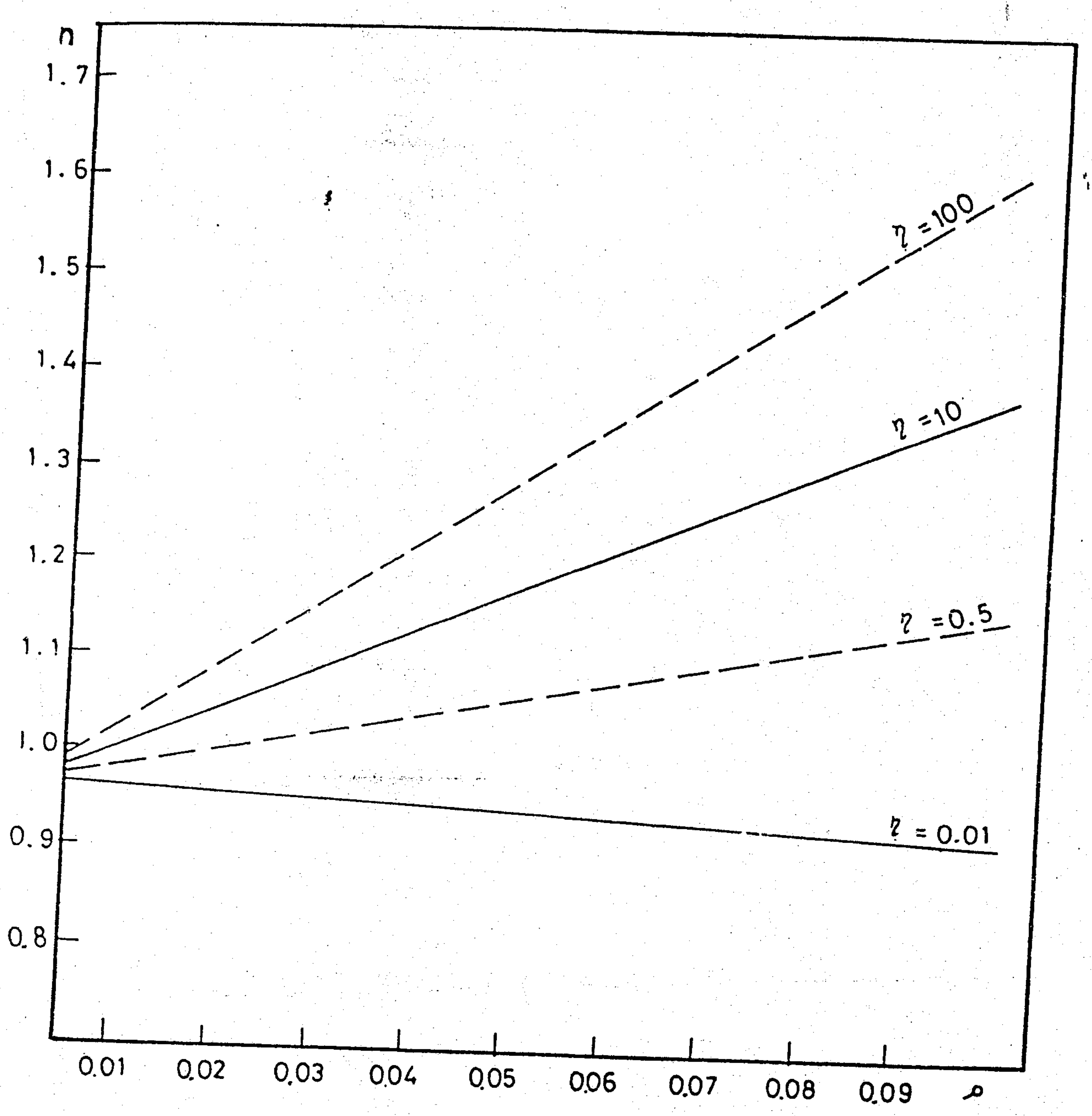

Fig. (2) Variation between density and distance ρ for constant (ϵ = 0.8).

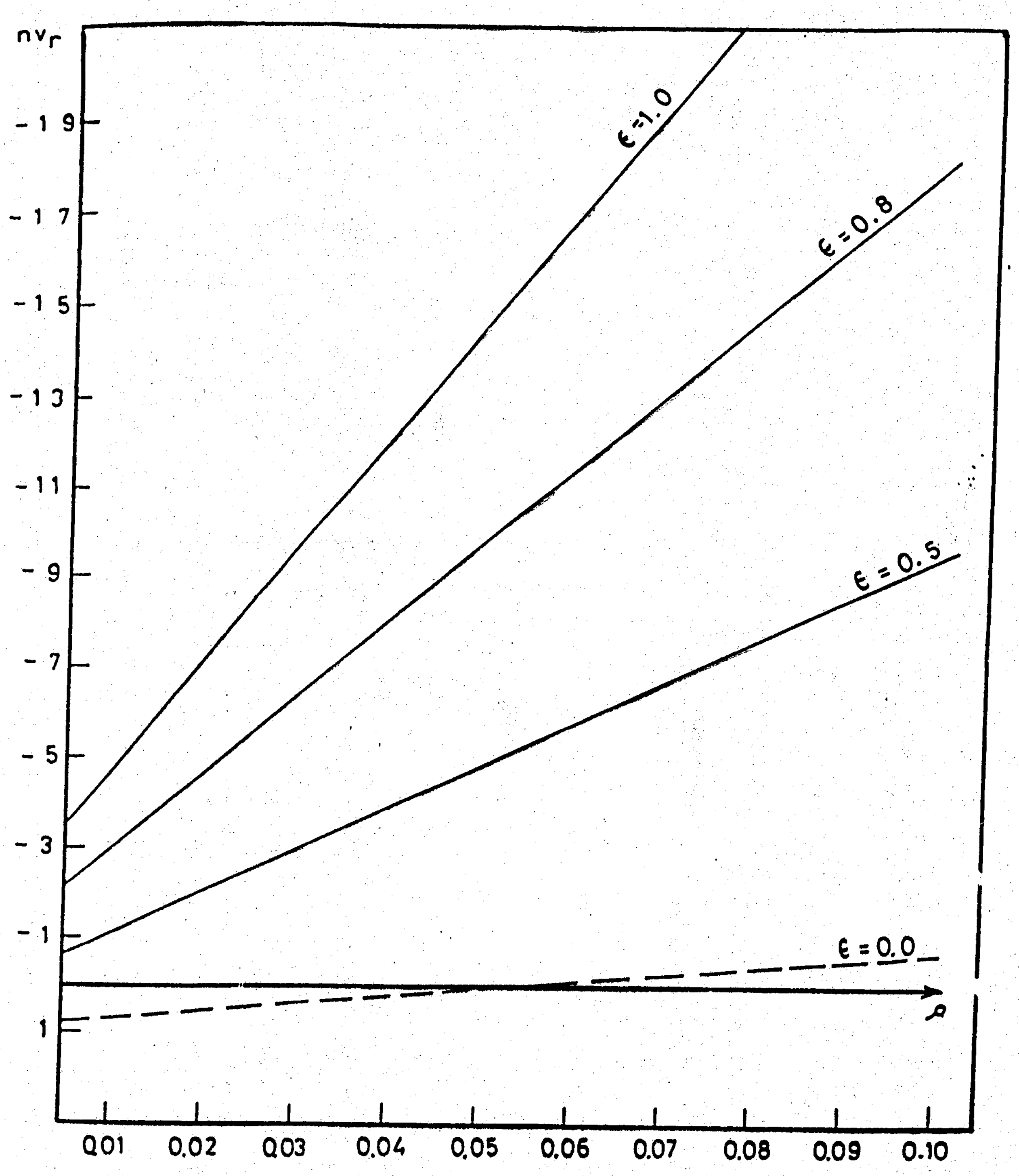

Fig. (3) Variation between nV_r and distance ρ for constant (η = 0.01).

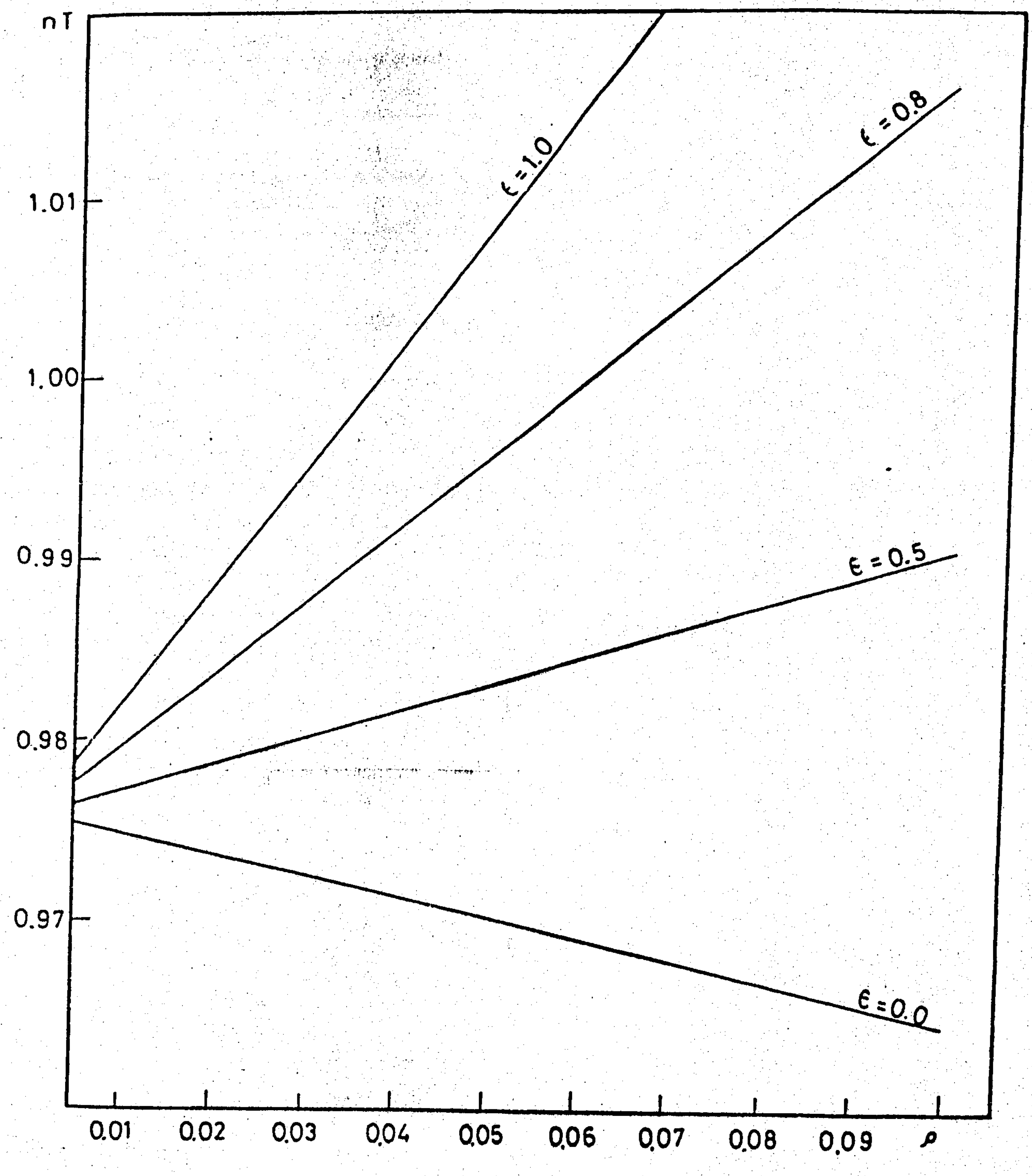

Fig. (4) Variation between n T and distance ρ for constant (η = 0.01).

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