Design of circularly symmetric 2-D FIR filters using a novel technique in calculating the coefficients of McClellan Transformation

تصميم مرشحات رقمية ثنائية لأبعاد من صنف استجابة النبضة المحدودة ذات تماثل

دائري باستخدام طريقة رائدة لحساب معاملات تحويل ماككليلان.

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الملخص :-

يقدم هذا البحث طريقة تحليلية رائدة لتصميم مرشحات رقمية ثنائية الأبعاد من صنف النبضة المحددة وذات تماثل دائري وطور صفري باستخدام تحويل ماككليلان. في هذه الطريقة يتم حساب معاملات تحويل ماككليلان بأسلوب مباشر عبر رسم المنحنى الدائري ثنائي الأبعاد للقطع على قطع مدى التمرير الترددي لأحادي الأبعاد وبنفس الوقت تعريف دالة اشتقاقية معينة والتي سترغم بدورها لتكون مساوية للصفر على ذلك المنحنى . وبعد هذا تجري مقارنة المعاملات في الدالة لاشتقاقية الناتجة منها تحسب المعاملات المطلوبة بسهولة وبدون الحاجة للمقايسة في الحساب . حسبت معاملات ماككليلان لقيم مختلفة من نصف قطر تردد قطع مدى التمرير ثاني الأبعاد وكان بالإمكان خلال الحسابات ادخال تقريب جيد النتائج بحيث تبقى المنحنى دائرياً . تبين أن هذا التقريب بالحسابات هو الأفضل ما بين توخي الدقة وتبسيط صعوبة بناء نظام أدواتيا . إن استخدام هذا التقريب يقود إلى علاقات مناسبة لتطبيقات الزمن الفعلي مما يمكن من استخدامها على معالج إشارات ذي غرض عام . اختبرت طرقة التصميم المذكورة بنجاح من خلال أمثلة تصاميم متعددة وبالمقارنة مع طرق سبق نشرها بهذا الخصوص .

Abstract

In this paper, a novel analytic technique for the design of *2-D* zero phase FIR circularly-symmetric filters is presented using the McClellan transformation. By this technique, a direct computation of the coefficients of McClellan transformation is derived by mapping the *2-D* circular cutoff contour onto the *1-D* passband cut off and defining some deviation function, which is then forced to be zero for that contour. Then by comparing all the parameters in the resulting deviation function, one can easily determine the coefficients of McClellan transformation. This technique is formulated to assure that no scaling is needed. The McClellan coefficients are determined for different values of *2-D* passband cutoff radius. The formulas for the coefficients in the presented technique may be simplified while a good approximation of the circular contour is maintained. The approximated version of the coefficients formulas appears to be the best to solve the problem of both accuracy and hardware implementation complexity. Using this approximation, one can get coefficients formulas which are suitable for real-time applications and can be easily implemented on a general-purpose signal processor.

The design procedure is verified by several design examples and by comparison with previously published results.

Keywords: 2-D FIR Filters, Circularly Symmetric, McClellan Transformation, Coefficient Calculation, Zero Phase, Low pass, Band Pass

Introduction

One of the most important classes of 2-D digital filters   is the circularly-symmetric digital filters which are used for smoothing, sharpening, enhancement, and restoration of images and for radar ,sonar, and satellite data analysis [1], [2].

It was proved that McClellan transformation is a very useful tool for the design of circularly-symmetric filters with low cutoff radius because it is simple, fast and efficient implementations exist for the resulting filters [3], [4]. The design, using this transformation, is preferred via the FIR approaches over the IIR counterparts since the resulting 2-D zero-phase IIR filters are unstable and need to be factorized into two half-plane or four quarter-plane filters which are stable [5]. On the other hand the resulting 2-D FIR stable filters results in an exact linear-phase and they do not need any extra phase compensation.

Several linear and nonlinear optimization techniques [3], [4], and [6] are generally used for the selection of McClellan coefficients. Such techniques require a large computational effort. The simplest transformation for designing 2-D approximately circularly symmetric FIR digital filters was originally proposed by McClellan himself. This transformation is given by [7]

$$\cos \omega = f(\omega_1, \omega_2) = -\frac{1}{2} + \frac{1}{2}\cos \omega_1 + \frac{1}{2}\cos \omega_2 + \frac{1}{2}\cos \omega_1 \cos \omega_2 \qquad (1.a)$$

which can be written in a general form as

$$\cos \omega = A + B \cos \omega_1 + C \cos \omega_2 + D \cos \omega_1 \cos \omega_2 \qquad (1.b)$$

where ω is the 1-D frequency variable and (ω_1, ω_2) is the 2-D frequency variables tuple, and A,B,C and D are coefficients to be determined.

For this transformation, the contours do not give a good approximation for the cutoff circle specially for higher cutoff frequencies. A power series expansion method is used by Fettweis [8] to choose the coefficients for circular contour, but this method has two disadvantages. First, the circular contour is approximated only for values of $\omega = \pi$ irrespective of the cutoff boundary of the 2-D filter. The second disadvantage is that the entire region in the (ω_1, ω_2) plane outside the contour for $\omega = \pi$ will not map onto any point on the ω-axis. This makes the stopband response takes a different shape other than of the 1-D transformed filter.

An analytic technique was proposed by Hazard and Reddy [9]. It involves coefficients formulas defined in the form of $\sin^2(x/2)$ and $\sin^2(x/2\sqrt{2})$. Another analytic technique was presented later by Kwan and Chan [10]. It involves coefficients formulas defined in the form of $j_0(x)$, $j_0(2x)$, $j_0(\sqrt{2} x)$, $j_0(\sqrt{5} x)$, $j_0(\sqrt{8} x)$, and $\cos(x)$. So, to implement coefficients, formulas of the above two techniques in hardware, we require either a ROM lookup table or a numeric data processor. An extremely simple approximate version of the formulas in [9] was then presented by Nagamuthu and Swamy [11]. Such

approximated formulas are suitable for real-time application since they are given directly in terms of the *2-D* frequency specifications. They are very simple, but they do not give good approximation for higher values of cutoff frequencies. Some other analytic techniques are also treated in [11] but all of them suffer from the bad approximation of the cutoff circle.

In this paper, a novel analytic technique for the design of the *2-D* zero-phase FIR circularly-symmetric filters is presented using the McClellan transformation. By this technique, a direct computation of the coefficients of McClellan transformation is derived by mapping the *2-D* circular cutoff contour onto the *1-D* passband cutoff frequency and defining some deviation function, which is then forced to be zero for that contour. Then by comparing all the parameters in the resulting deviation function, one can easily determine the coefficients of McClellan transformation. This technique is formulated to assure that no scaling is needed, the McClellan coefficients are determined for different values of *2-D* passband cutoff radius. The resulting errors are shown to be better than those given in [7] and [11] and of the same order of those given in [9] and [10].The formulas for the coefficients in our technique may also be simplified while a good approximation of the circular contour is maintained which is not the case for those formulas given in [9] when simplified in [11], the approximated version of the coefficients formulas of this technique appears to be the best to solve the problem of both accuracy and hardware implementation complexity. Using this approximation, one can get coefficients formulas which are suitable for real-time applications and

can be easily implemented on a general-purpose signal processor.

Section 2 contains the proposed technique. The design procedure is illustrated in Section 3 by a typical example. In Section 4, the design of *2-D* FIR band-pass circularly-symmetric filters is invoked by an example. Section 5 shows a comparative study for different known design techniques, and the proposed technique.

2. The proposed technique

Since McClellan transformation, used here, is of quadrantal symmetry, the analysis can be limited to the first quadrant. Let ω_c be the *2-D* cut-off pass-band edge. For a circularly-symmetric filter, the pass-band boundary is a circle of radius R so that

$$R = \omega_c \qquad (2)$$

For $\omega_c \in [0, \pi]$, different values of the coefficients can be selected in order to produce circular filter with arbitrary cutoff pass-band edge. If ω_0 is the cutoff frequency of the *1-D* prototype zero-phase LPF, then $\omega = \omega_0$ should be mapped onto the cutoff radius of $R = \omega_c$.

If both *1-D* prototype and *2-D* circular filters are chosen to be of LP type, then the global maximum will be attained at the point (0,0) and the global minimum will be attained at the point (π, π). Thus, one can choose the point $\omega = 0$ to maps onto (0,0) with f(0,0)=1, resulting in

$$1 = A + B + C + D \qquad (3)$$

while the point $\omega = \pi$ can be chosen to map onto (π, π) with $f(\pi, \pi) =$

-1, resulting in

$$- l = A - B - C + D \qquad (4)$$

From (3) and (4), the following relations can be written in such away
First by adding Eq.(4) to Eq(3) results in

$$A = - D \qquad (5.a)$$

Second by subtracting Eq.(4) from Eq.(3) results in

$$B = l - C \qquad (5.b)$$

Thus, the 1st order McClellan transformation usually expressed as [7]:

$$\cos \omega = f (\omega_1, \omega_2) = A + B \cos \omega_1 + C \cos \omega_2 + D \cos \omega_1 \cos \omega_2 \qquad (6)$$

can be reduced here to the following form:

$$f(\omega_1, \omega_2) = [-D + (1 - C) \cos \omega_1] + [C + D \cos \omega_1] \cos \omega_2 \qquad (7)$$

and the design subproblem can now be reduced to the determination of l-D filter of cutoff frequency ω_0, and to the evaluation of the C and D coefficients.

It should be noted that using the above substitutions does not assure the similarity of the frequency response of the original l-D FIR filter with that of the 2-D filter along the ω_1-axis ($\omega_2 = 0$) and along the ω_2-axis ($\omega_1 = 0$). Such a problem may be partially solved by forcing the points ($\omega_0, 0$) and ($0, \omega_c$) from the (ω_1, ω_2) plane to map onto $\omega = \omega_0$.   For   the

$\omega = \omega_0 \leftrightarrow (\omega_1, \omega_2) = (\omega_c, 0)$ mapping, we have

$$\cos \omega_0 = [-D + (1 - C) \cos \omega_c] + [C + D \cos \omega_c]$$

or

$$C - D = \frac{\cos \omega_0 - \cos \omega_c}{1 - \cos \omega_c} \qquad (8)$$

and for the $\omega = \omega_0 \leftrightarrow (\omega_1, \omega_2) = (0, \omega_c)$ mapping, we have

$$\cos \omega_0 = [-D + (1 - C)] + [C + D] \cos \omega_c$$

or

$$C + D = \frac{1 - \cos \omega_0}{1 - \cos \omega_c} \qquad (9)$$

Adding Eq.(8) and Eq. (9), results in

$$2 C = 1 \text{ or } C = 0.5 \qquad (10)$$

then,

$$D = \frac{1 - \cos \omega_0}{1 - \cos \omega_c} - 0.5 \qquad (11)$$

It should be noted, here, that this value of C agrees with the C value of the optimal method of Mersereau, in [12], and also with the coefficients values of the analytic method of Hazra and Reddy in [9]. If ω_0 is chosen to be equal to ω_c (i.e. , ω_0 is not one of the design parameters which is the case for the original McClellan transformation for circularly-symmetric filters given in [7], then the D coefficient can be easily determined from Eq.(11) as

$$D = 0.5 \qquad (12)$$

Since ω_0, in our design technique, is not one of the specifications (it is one of the parameters to be determined), the D

coefficient can never take the constant value given in eq.(12) for different values of ω_c, thus other formulas for D and ω_0 are needed.

Using Eq.(9), then Eq.(7) can be reduced to

$$f(\omega_1,\omega_2)=0.5\left\{\begin{matrix}[\cos\omega_1 -2D]+\\ [1+2D\cos\omega_1]\cos\omega_2\end{matrix}\right\} \qquad (13)$$

An obvious way to formulate the design problem is to define deviation function $d(\omega_1,\omega_2,\omega)$ which is given by [13],[14]

$$d(\omega_1,\omega_2,\omega)=f(\omega_1,\omega_2)-\cos\omega \qquad (14)$$

As mentioned before $\omega=\omega_0$ should be mapped onto the cutoff passband radius of $R=\omega_c$; i.e., onto the cutoff contour

or $\omega_1^2+\omega_2^2=\omega_c^2$

$$\omega_2=\sqrt{\omega_c^2-\omega_1^2}=g(\omega_1,\omega_c) \qquad (15)^*$$

Therefore, one can put

$$d(\omega_1,\omega_2,\omega_0)=0 \qquad (16)$$

This sign $=$ stands for identical equalization.

Substituting for ω_2 from Eq.(15) into Eq.(16), yields

$$d(\omega_1,g(\omega_1,\omega_c),\omega_0)=0$$

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* the negative root is not of concern, here, since our analysis can be limited to the first quadrant becoues of the quadrantal symmetry of the used McClellan Transformation.

or

$$f(\omega_1,g(\omega_1,\omega_c))-\cos\omega_0=0$$

Substituting from (13)

$$0.5\left\{\begin{matrix}[\cos\omega_1-2D]+\\ [1+2D\cos\omega_1]\\ \cos\left(\sqrt{\omega_c^2-\omega_1^2}\right)\end{matrix}\right\}-\cos\omega_0=0 \qquad (17)$$

Eq. (17) can be simplified by using the power series expansion of $\cos u$; i.e.,

$$\cos u=1-\frac{u^2}{2!}+\frac{u^4}{4!} \qquad (18)$$

and the following formula may be obtained by equating the constant term in (17) to zero.

$$D=\frac{\dfrac{\omega_0^2}{2!}+\dfrac{\omega_0^4}{4!}+-\cdots\cdots}{\dfrac{\omega_c^2}{2!}+\dfrac{\omega_c^4}{4!}+-\cdots\cdots}-0.5 \qquad (19)$$

This equation is the same as Eq.(11), but it is written in a power series form of eq.(18).

By equating the ω_1^2 term in Eq.(17) to zero, one can get the following formulas

$$D=\frac{0.5\ S}{1-S-\cos\omega_c} \qquad (20)$$

where

$$S=\sum_{m=1}^{\infty}(-1)^{m+1}\frac{\omega_c^{2m}}{(2m+1)!} \qquad (21)$$

A good approximation for Eq.(21) is obtained if it is written as

$$S=\sum_{m=1}^{M}(-1)^{m+1}\frac{\omega_c^{2m}}{(2m+1)!} \qquad (22)$$

where *M=4*

The only design parameter left to be determined is ω_0. It can be easily calculated, now by using Eq.(9) as

$$\omega_0 = \cos^{-1}\left[1 - p\left(1 - \cos \omega_c\right)\right] \qquad (23)$$

where

$$P = C + D = 0.5 + D \qquad (24)$$

Examination of Eq.(20) implies that

$$D \leq 0.5 \text{ for all } \omega_c \in (0, \pi) \qquad (25)$$

Therefore, from eq.(24)

$$P \leq 1 \qquad (26)$$

For $P < 1$, eq.(23) yields that $\omega_0 < \omega_c$ which is the same case for the optimal technique of [4] and the analytic techniques of [9] and [10], while, for $P = 1$, then from eq.(23), $\omega_0 = \omega_c$. This is the case for the original McClellan transformation for circularly-symmetric filters of [7]. Another design technique reported in [11], uses $P = 0.5$, and $\omega_0 = \omega_c$. That is called the scaled approximation solution.

3. Design procedure and example:

In this section we will first apply the present technique to the design of $2 - D$ zero-phase FIR circularly-symmetric filters. Then we will examine this technique by taking an illustrative example.

To design a $2 - D$ zero-phase FIR circularly-symmetric filter having a cut-

off edge ω_c, one must proceed according to the following steps:

1) From the desired cutoff edge frequency ω_c and by using Eq.(10), (20), and (5 a&b), one can compute the appropriate values of the coefficients of McClellan transformation C, D, A, and B, respectively.

2) Using Eq.(24) and then (23), one can define the cutoff frequency ω_0 of the *1-D* prototype filter.

3) Using some powerful *1-D* FIR filter design method, one can design a zero-phase low pass FIR filter with cutoff frequency ω_0.

4) Using   the   fm
$$H\left(e^{jw1}, e^{jw2}\right) = h(o) + \sum_{n=1}^{N} 2h(n)$$
$$Tn\left(f\left(\omega_1, \omega_2\right)\right) \qquad (27)$$

One can express the frequency response of the *2-D* filter with the replacement of $f(\omega_1, \omega_2)$ in the frequency response by the McClellan transformation of Eq.(6).

The above design procedure is illustrated by the following example:

Example 1:

A *2-D* zero-phase FIR circularly-symmetric low pass filter is to be designed with a passband cutoff edge of $\omega_c = 0.25 \ \pi \ rad.$

The resulting McClellan transformation coefficients arc

$A = -D = -0.2579878$ and $B = 1 - c = 0.5$.

The McClenllan transformation can now be written as

$\cos \omega = f(\omega_1, \omega_2) = -0.2570678 +$

$0.5 \cos \omega_1 + 0.5 \cos \omega_2 +$

$0.2579678 \cos \omega_1 \cos \omega_2$

A 25-point *I-D* low pass FIR filter is designed with a cutoff frequency $\omega_0 = 0.2162362 \, \pi$ rad., using the program in [15]. The resulting *I-D* impulse response is given by

$h(0) \quad = 2.582495 \ E-01$
$h(1) \quad = 2.301630 \ E-01 = h(-1)$
$h(2) \quad = 1.570708 \ E-01 = h(-2)$
$h(3) \quad = 6.720543 \ E-02 = h \ (-3)$
$h(4) \quad = -7.768675 \ E-03 = h(-4)$
$h(5) \quad = -4.870018 \ E-02 = h(-5)$
$h(8) \quad = -4.670227 \ E-02 = h(-8)$
$h(7) \quad = -2.150187 \ E-02 = h(-7)$
$h(8) \quad = 8.495199 \ E-03 = h(-8)$
$h(9) \quad = 2.729922 \ E-02 = h(-9)$
$h(10) = 3.077441 \ E-02 = h(-10)$
$h(11) = 2.699446 \ E-02 = h(-11)$
$h(12) = -2.099424 \ E-02 = h(-12)$

substituting for *h(n)* in the *2-D* frequency response of Eq.(27), with M = 12, the frequency response of the *2-D* filter can be expressed with the replacement of $f(\omega_1, \omega_2)$ by the McClellan transformation of Eq.(6). The magnitude of the frequency response of the designed *2-D* filter is shown in Fig.1 with its contours shown in Fig.2

4. *2-D* BPF filter design:

Kwan and Chan [10] have presented a technique for the calculation of the 1st order McClellan transformation. This technique defines a general and complete expression for the error E of approximated circular contour with a radius of the critical frequency ω_c as

$$E = F_1(\omega_c) + D F_2(\omega_c) + D^2 F_3(\omega_c) \quad (28)$$

where

$$F_1(\omega_c) = 0.25 \left[j_0(2\omega_c) + 1 \right] + 0.25 \left[\cos \omega_c + 1 \right]^2 + 0.5 f_0(\sqrt{2} \, \omega_c) - \left[\cos \omega_c + 1 \right] j_0(\omega_c)$$

$$(29)$$

$$F_2(\omega_c) = j_0(\sqrt{5} \, \omega_c) + j_0(\omega_c) - 2 \cos \omega_c \, j_0(\omega_0) - \left[\cos \omega_c + 1 \right] j_0(\sqrt{2} \, \omega_c) + \left[\cos \omega_c + 1 \right] \cos \omega_c$$

$$(30)$$

and

$$F_3(\omega_c) = 0.25 \left[j_0(\sqrt{8} \, \omega_c) + 2 j_0(2 \omega_c) + 1 \right] + \cos^2 \omega_c - 2 \cos \omega_c \, j_0(\sqrt{2} \, \omega_c)$$

$$(31)$$

$J_0(z)$ is the Bessel function of the 1st kind, zero order in the value z. For low-pass (LP) and high-pass (HP) filter design, the optimal solution for D can be obtained from Eq.(28) by setting

$$\frac{\partial E}{\partial D} = 0 \qquad (32)$$

Hence, it is obtained that

$$D = \frac{F_2(\omega_c)}{2 F_3(\omega_c)} \qquad (33)$$

For band-pass (BP), band-stop (BS), and multi-band filters, there may be

K critical frequencies $\left(\omega_{ci}, i = 1,2,...,K \right)$. The corresponding error function E can then be expressed as

$$E = \sum m_i \left[F_1\left(\omega_{ci}\right) + D F_2\left(\omega_{ci}\right) + D^2 F_3\left(\omega_{ci}\right) \right]$$

(34)

where each m_i is a positive weighting factor for the corresponding value of i=1,2,...,K The optimal solution can then be obtained from eq.(34) by applying the condition in eq.(32) to yield

$$D = \frac{-\sum_{i=1}^{K} F_2\left(\omega_{ci}\right)}{2\sum_{i=1}^{K} F_3\left(\omega_{ci}\right)}$$

(35)

Example 2:

To design a 2-D zero-phase FIR band-pass circularly-symmetric filter having the 2-D cutoff frequencies $\omega_{ci} = 0.25\,\pi\ rad.$, $\omega_{02} = 0.5\,\pi\ rad.$, the following steps may be followed:

1) by mapping the two 1-D cutoff frequencies ω_{01}, and ω_{02} onto the points $(\omega_{c1},0)$ and $(\omega_{c2},0)$ in the 2-D plane, respectively, it can be easily shown that

$$\omega_{01} = \omega_{c1} \text{ and } \omega_{02} = \omega_{c2} \qquad (36.\ a\&b)$$

Hence, for this example $\omega_{01} = 0.25\,\pi\ rad.$ and $\omega_{02} = 0.5\,\pi\ rad.$

2) D coefficient can be determined from Eq.(35) using values of ω_{c1}, ω_{c2}. The weighting values m_1

and m_2 in Eq.(34) are chosen to be 1.

From Eq.(10), the coefficient C can be calculated, while A and B coefficients can then be calculated using Eq(5.a&b). in this example.

$D = -A = -0.284063, \quad C = 1 - B = 0.5$

3) Using the design steps as in [15] for 1-D FIR filter, a zero-phase band-pass FIR with cutoff frequencies $\omega_{01} = 0.25\,\pi\ rad.$ and $\omega_{02} = 0.5\,\pi\ rad.$can be designed . A 23-point 1-D band-pass FIR filter is designed and the resulting 1-D impulse response $h(n)$ is given by

$h(0)\ =\ 2.891397\ E-01$
$h(1)\ =\ 1.472599\ E-01\ = h(-1)$
$h(2)\ =\ -1.137585\ E-01\ = h(-2)$
$h(3)\ =\ -2.141851\ E-01\ = h(-3)$
$h(4)\ =\ -9.967208\ E-02\ = h(-4)$
$h(5)\ =\ 4.616572\ E-02\ = h(-5)$
$h(6)\ =\ 7.473487\ E-02\ = h(-6)$
$h(7)\ =\ 2.301105\ E-02\ = h(-7)$
$h(8)\ =\ -7.541947\ E-03\ = h(-8)$
$h(9)\ =\ 2.406392\ E-03\ = h(-9)$
$h(10)\ =\ 1.567426\ E-02\ = h(-10)$
$h(11)\ =\ -4.657915\ E-03\ = h(-11)$

4) Substituting for h(n) in the 2-D frequency response of eq. (27), with N=11, the frequency response of the 2-D filter can be expressed with the replacement of $f\left(\omega_1,\omega_2\right)$ by the McClellan transformation of eq, (6). The magnitude of the frequency response of the designed 2-D filter is shown in Fig.3 with its contours shown in Fig.4

5.   A comparative study for circular filter techniques:

In order to examine the present technique quality, a large number of contours for different values or $R = \omega_c$ have been computed using the technique proposed here and three other techniques, namely, the original McClellan transformation in [7], the scaled approximated technique in eq.(11), and the technique proposed by Hazra and Reddy in eq.(9), The following error criteria is used to measure the performance of all the compared techniques

$$E = \frac{R' - R}{R} \qquad (37)$$

where

$$R' = \sqrt{{\omega_1}^2 + {\omega_2}^2} \qquad (38)$$

The coefficients of Eq.(6), the cutoff boundary ω_c, the maximum absolute value of error E_{max} and the root-mean-square value of error E_{RMS}, are given in Table. 1 for the different techniques under comparison. It can be observed that, the present technique and that of [9] (which has the same results as the optimization procedure of [4]) give nearly identical coefficients values. E_{max} and E_{RMS} values are of the same order. It can be also seen that the present technique gives smaller errors than those given in both techniques of [7] and [11].

From the above comparison, it is obvious to choose either the present or those given in [9] and [10], However, a drawback in the technique of Hazra and Reddy of [9] exists, where it involves

parameter's formulas defined in the form of $\sin^2(x/2)$ and $\sin^2(x/2\sqrt{2})$. So, to implement these formulas in hardware, we require either a ROM lookup tables or a numeric data processor. An approximated version of these formulas is given in [11] with

$$\omega_0 = (\sqrt{s}/2)\omega_c \qquad (39.\text{a})$$
$$-A = D = (1/4)\left[1 + (\omega_c^2/8)\right] \qquad 39.\text{b}$$

and

$$B = 1 - C = 0.5 \qquad (39.\text{c})$$

The formulas (39.a,b and c) help in reducing the real-time implementation complexity, but they fail to give good accuracy even for lower values of ω_c as shown in Table 2. On the other hand, our technique can give extremely simple approximated formulas for coefficients which are also suitable for real-time applications and give a better accuracy, even for high values or $\omega_c < 0.9\ \pi$, as compared with formulas (39.a,b and c). See Table. 2 for the comparison. The approximated formulas for our technique are derived by putting M = 2 in Eq.(28). In Eq.(20) we also put

$$\cos \omega_c \cong 1 - \frac{\omega_c^2}{2!} + \frac{\omega_c^4}{4!} - \cdots + \qquad (40)$$

These formulas are

$$-A = D = (1/8)\left\{1 - \left[10/(\omega_c^2 - 10)\right]\right\} \qquad (41.\text{a})$$

$$B = 1 - C = 0.5 \qquad (41.\text{b})$$

and

$$\omega_0 = \sqrt{P}\ \omega_c \qquad (41.\text{c})$$

From the above discussion, it can be concluded that for M = 2, an approximated formula can be met for

lower complexity in implementation but with more errors than that for M = 4 which gives higher complexity in implementation. This arises again the problem of choice between complexity and accuracy.

Conclusions

As a conclusion for this paper, a novel analytic technique has been developed for the design of 2-D zero phase FIR circularly-symmetric filter via the McClellan transformation. A direct computation of the coefficients has been presented by mapping the cutoff frequency of the prototype 1-D zero-phase FIR filter onto the 2-D circular cutoff contour. A suitable deviation function has then been defined and forced to be zero for that contour. Determination of the McClellan coefficients has easily been done by comparing all the parameters in the resulting deviation function. The proposed technique has been formulated to have no scaling. The accuracy of such technique is proved to be as those of [9] and [10], and better than those of [7] and [11]. However, the accuracy of the present technique becomes better than that of the technique in [9] when both techniques are approximated for real-time adaptive filter implementations.

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Table .1 Comparison of results given by various techniques.

ω_c	The technique	$D = -A$	$C = 1-B$	ω_0	E_{max}	E_{RMS}
0.25 Π	present	0.257968	0.5	0.2162362 Π	2.696×10^{-6}	1.469×10^{-6}
	of [9]	0.257893	0.5	0.2162251 Π	7.535×10^{-7}	4.156×10^{-7}
	of [4]	0.25	0.5	0.25 Π ⊗	1.655×10^{-1}	1.652×10^{-1}
	of [7]	0.5	0.5	0.25 Π ⊗	6.644×10^{-3}	4.227×10^{-3}
0.5 Π	present	0.285395	0.5	0.4311537 Π	2.256×10^{-4}	1.236×10^{-4}
	of [9]	0.283967	0.5	0.4306883 Π	5.670×10^{-5}	3.507×10^{-5}
	of [4]	0.25	0.5	0.5 Π ⊗	2.163×10^{-1}	2.073×10^{-1}
	of [7]	0.5	0.5	0.5 Π ⊗	2.971×10^{-2}	1.877×10^{-2}
0.75 Π	present	0.347334	0.5	0.6473261 Π	4.637×10^{-3}	2.551×10^{-3}
	of [9]	0.337338	0.5	0.6412837 Π	1.188×10^{-4}	7.352×10^{-4}
	of [4]	0.25	0.5	0.75 Π ⊗	4.580×10^{-1}	3.508×10^{-1}
	of [7]	0.5	0.5	0.75 Π ⊗	8.495×10^{-2}	5.269×10^{-2}
Π	present	0.497800	0.5	0.97001329 Π	2.220×10^{-1}	1.230×10^{-1}
	of [9]	0.443128	0.5	0.8467031 Π	3.188×10^{-2}	2.083×10^{-2}
	of [4]	0.25	0.5	Π ⊗	4.072×10^{-1}	1.921×10^{-1}
	of [7]	0.5	0.5	Π Π ⊗	4.071×10^{-1}	1.920×10^{-1}

⊗ one of the specification and not a design parameter.

Ref.[10] is not included in the above comparison, since the errors via the technique in [10] are of the same orders of those in [9].

Table. 2 Comparison of results given by the present technique [approximated as in (41)] and the technique of [9] [approximated as in (39)] .

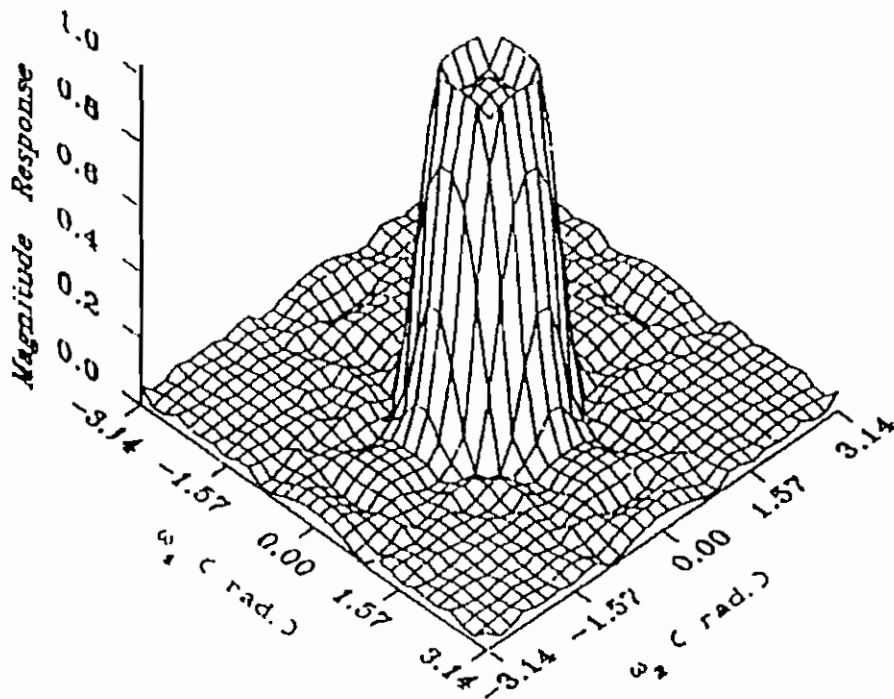| ω_c	The technique approximation. in	$D = -A$	$C = 1-B$	ω_0	E_{max}	E_{RMS}
0.25 Π	(41)	0.258217	0.5	0.2176893 Π	1.158×10^{-5}	7.079×10^{-6}
	(39)	0.269275	0.5	0.2165060 Π	6.521×10^{-3}	6.314×10^{-3}
0.5 Π	(41)	0.290945	0.5	0.4448755 Π	1.098×10^{-3}	6.782×10^{-4}
	(39)	0.327106	0.5	0.4330127 Π	2.772×10^{-2}	2.437×10^{-2}
0.75 Π	(41)	0.406003	0.5	0.7138815 Π	3.303×10^{-2}	2.056×10^{-2}
	(39)	0.423489	0.5	0.6495190 Π	7.437×10^{-2}	5.681×10^{-2}

Fig.1 Magnitude response of the 2-D LP FIR circularly-symmetric filter with $\omega_c = 0.25\,\pi$ rad. which corresponds to a 25 – point 1-D LP FIR filter with $\omega_0 = 0.2162362\,\pi$ rad.
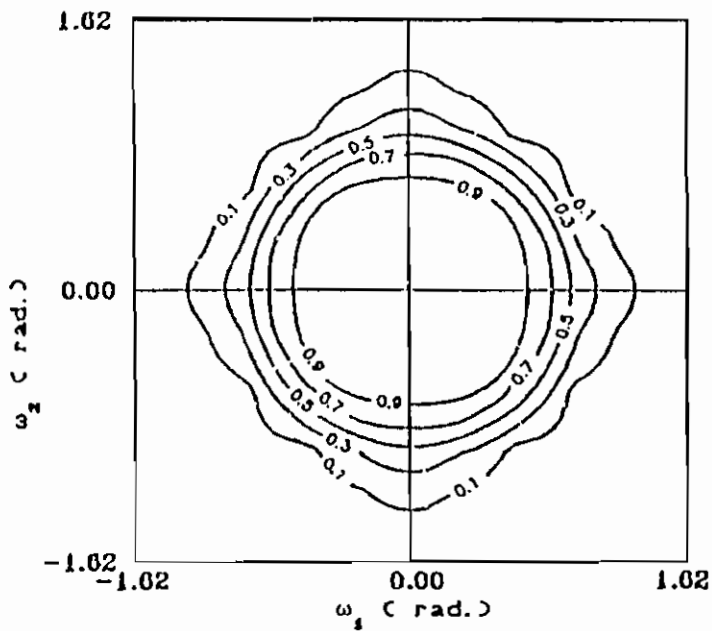

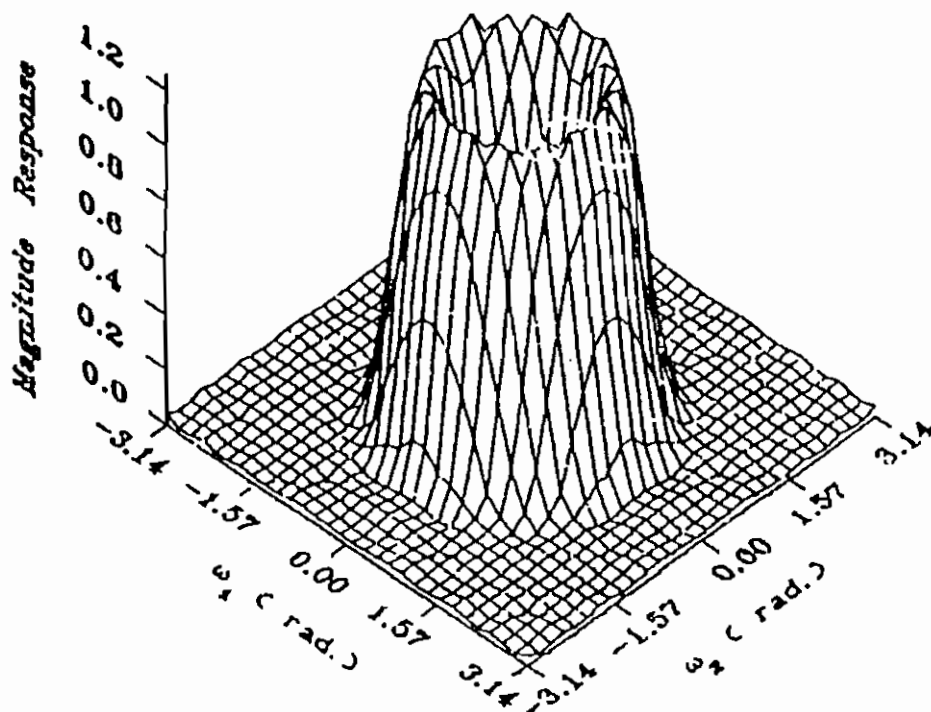**Fig.2** Contour map of the magnitude response in Fig.1.

Fig.3 Magnitude response of the 2-D BP FIR circularly-symmetric filter
with $\omega_{c1} = 0.25\,\pi$ $rad.$, $\omega_{c2} = 0.5\,\pi$ $rad.$ which corresponds to a
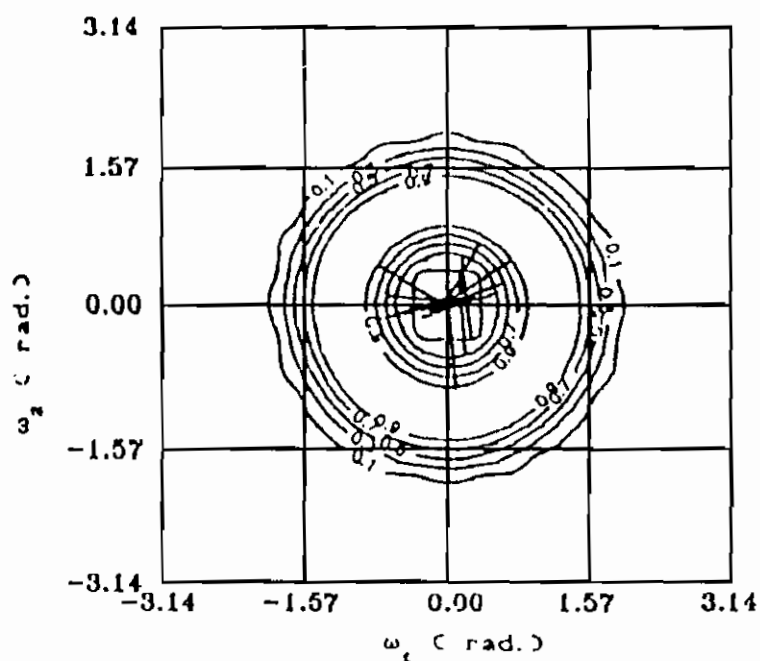23–point 1-D BP FIR filter with $\omega_{01} = \omega_{c1}$ and $\omega_{02} = \omega_{c2}$.

Fig.4 Contour map of the magnitude response in Fig.3