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Dynamic Investigations of Centrifugal Pump Driven by Rotary Cultivator.

BY

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ABSTRACT :

This work describes an dynamic investigations of Centrifugal pump driven by rotary cultivator. It is necessary to study the dynamic of this system after the wide application of agricultural mechanization in Egypt. The dynamic behavioour of the system calculated by two methods, the first is simulating the real system into two degrees of freedom and in the second it's calculated for the real system of six degrees of freedom.

Keywords:
Dynamics, Design, Application of matrix theory to vibration.

1. Theoretical Analysis :

In order to estimate the dynamic characteristics of centrifugal pump rotated by rotary cultivator, The following assumptions will be taken into consideration, the system is assumed to perform small torsional vibration, shafts on which the gear disks and flywheel are assumed light of uniform mass and uniform stiffness, the coupling and the mesh gears are ideal without slipping, the internal damping and friction at bearing and contact surfaces are negligible rotary cultivator clutch system is considered of high rigidity and of large mass and the excitation is assumed of periodic nature while the ressistance of imPeller is assumed nearly constant.

1.1- Calculation for a Simulating System :

Theoretical calculations in the first were determined by simulating the actual system shown in Fig.(1) into equivalent mass system shown in Fig.(2), The equivelent mass moment of inertia besides th equivalent stiffness of the simulated system are tabulated in Table (1).

Table (1)

No.	Moment of Inertia (J) Kg.Cm.Sec^2	Torsional Stiffness (C) Kg. Cm/rad.
1	72.3	1.6×10^5
2	275.75	5.1×10^5

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1.1.1- Equation of motion :

Equation of motion of the simulated system can be determined by using lagrange's equations 1,2,3 which can b written in the following matrix form :

$$\begin{bmatrix} J_1 & 0 \\ C & J_2 \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ -M_2 \end{bmatrix} \quad \ldots\ldots(1)$$

2.1.1- The natural frequencies ;

To determined the natural frequencies of the system, we must calculate the inverse of the dynamic matrix :

$$\left[Y \right]^{-1} = \left[m \right]^{-1} \cdot \left[C \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= \frac{1}{J_1 \cdot J_2} \begin{bmatrix} J_2 \; (C_1 + C_2) & -J_2 \; C_2 \\ -J_1 \; C_2 & J_1 \; C_2 \end{bmatrix}$$

The characteristic equation is

$$\left[\left[Y \right]^{-1} - W^2 \left[I \right] \right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From equation (3) it can be obtain the natural frequency w,
Where : W_1 = 36.3 rad/sec.
W_2 = 99.0 rad/sec.

2. The Mathematical Model for the Real System :

In the Practical engineering the actual continuous system is ferquently aPProximated by the behavior of the discrete model. The system is simulated as six degrees of freedom closed couPled system. The equivalent is formed from massless elastic shafts with two free ends. The inertia of the shafts is measured at six stations through the system as shown in Fig.(3). Also the elasticity of the torsional shaft are modeled by a set of torsional sPrings.

1.2- Determination of the Equivalent Stiffness :

The equivalent stiffness of each segment is given by :

$$K_i = \frac{G \cdot J_P}{\ell_i}$$

Where : G : Shear modulus of elasticity,
J_P : Polar moment of inertia for esch segment,
ℓ_i : Length of each segment in Cm.

The following table (2) rePresents the calculated stiffness for each segment of the system.

Table (2)

Bart	Equivalent Stiffness Kg. Cm. rad
1	272271
2	204203
3	2042035
4	1021017
5	453785
6	204203

2.2- Detrimation of the Equivalent Masses :

The equivalent mass of each segment given by :

$$M_i = \rho \cdot J_P \cdot \ell_i$$

Where :

ρ = density of shaft material,

J_P = Polar moment of inertia of shafts,

ℓ_i = Length of each shaft.

The following table (3) rePresents the calculated equivalent mass of each shaft :

Table (3)

Part	Equivalent Mass Kg. Cm.^{2}
1	23.56
2	31.40
3	3.14
4	3.14
5	14.13
6	76.79

And the equivalent mass of each disk is given as follow :

$$I_1 = I_{Engine} , \qquad I_2 = D_{coup.} + \frac{M_1}{2} + \frac{M_2}{2} ,$$

$$I_3 = D_3 + \frac{M_2}{2} + \frac{M_3}{2} , \qquad I_4 = D_4 + \frac{M_3}{2} + \frac{M_4}{2} ,$$

$$I_5 = D_5 + \frac{M_4}{2} , \qquad I_6 = D_6 + \frac{M_5}{2} ,$$

$$I_8 = D_8 + \frac{M_6}{2} ,$$

$$I_9 = D_P + \frac{M_6}{2} .$$

Where : D_i = moment of inertia of the disks.

The following table (4) rePresents the equivalent mass of each disk of the system :

Table (4)

Part	Equivalent Mass Kg. Cm^2
1	3526×10^6
2	4367.28
3	57.52
4	29.84
5	11.10
6	295.13
7	295.13
8	19.53
9	32587.7

3.2- **Determination of kinatic and Potential Engergy of the system :**

Kinatic Energy = K.E. = $\sum_{i=1}^{n} \frac{1}{2} I_i \dot{q}_i^2$

Where q = vector of virtual generalized coordinates.

The following equation rePresents the kinatic energy of the system as shown;

$$K.E. = \frac{1}{2} I_1 \dot{\phi}_1^2 + \frac{1}{2} I_2 \dot{\phi}_2^2 + \frac{1}{2} I_3 \dot{\phi}_3^2 + \frac{1}{2} I_4 \dot{\phi}_4^2 + \frac{1}{2} I_5 \dot{\phi}_5^2 + \frac{1}{2} I_6 \left(\frac{Z_5}{Z_6}\right)^2 \dot{\phi}_5^2 + \frac{1}{2} I_7 \dot{\phi}_6^2 + \frac{1}{2} I_8 \left(\frac{Z_7}{Z_8}\right)^2 \dot{\phi}_6^2 + \frac{1}{2} I_9 \dot{\phi}_7^2$$

We consider the value of equivelent mass I_1 as arigid body with infinity mass moment of inertia.

From the Previous equation the mass matrix can be wirtten in the following form :

$$\underset{\sim}{m} = \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_5+I_6(i_{56})^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_7+I_8(i_{78})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_9 \end{bmatrix}$$

Where : $i_{56} = \frac{Z_5}{Z_6}$, $i_{78} = \frac{Z_7}{Z_6}$

$$m = \begin{bmatrix} 4367.28 & 0 & 0 & 0 & 0 & 0 \\ 0 & 57.52 & 0 & 0 & 0 & 0 \\ 0 & 0 & 29.84 & 0 & 0 & 0 \\ 0 & 0 & 0 & 60.94 & 0 & 0 \\ 0 & 0 & 0 & 0 & 461.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 32587.7 \end{bmatrix}$$

From lagrang's equations the Potential energy of the system can be written as follows :

$$P.E. = \tfrac{1}{2}\Big\{ K_1 \Phi_2^2 + K_2 (\Phi_3 - \Phi_2)^2 + K_3(\Phi_4 - \Phi_3)^2 + K_4(\Phi_5 - \Phi_4)^2 + K_5 \left[\Phi_6 - \Phi_5\left(\frac{Z_5}{Z_6} \right)\right]^2 + K_6\left[\Phi_7 - \Phi_6 \left(\frac{Z_7}{Z_8} \right)\right]^2$$

From the Previous equation the stiffness matrix can be written in the following form :

$$K = \begin{bmatrix} K_1+K_2 & -K_2 & 0 & 0 & 0 & 0 \\ -K_2 & K_2+K_3 & -K_3 & 0 & 0 & 0 \\ 0 & -K_3 & K_3+K_4 & -K_4 & 0 & 0 \\ 0 & 0 & -K_4 & K_4+K_5(i_{56})^2 & -K_5(i_{56})^2 & 0 \\ 0 & 0 & 0 & -K_5(i_{56})^2 & K_5+K_6(i_{78})^2 & -K_6(i_{78})^2 \\ 0 & 0 & 0 & 0 & -K_6(i_{78})^2 & K_6 \end{bmatrix}$$

$$K = \begin{bmatrix} 476476 & -204203 & 0 & 0 & 0 & 0 \\ -204203 & 2246238 & -2042035 & 0 & 0 & 0 \\ 0 & -2042035 & 3063052 & -1021017 & 0 & 0 \\ 0 & 0 & -1021017 & 1097661 & -186487 & 0 \\ 0 & 0 & 0 & -186487 & 2194819 & -596273 \\ 0 & 0 & 0 & 0 & -596273 & 204203 \end{bmatrix}$$

4.2- Equation of motion of the system :

The equation of motion for the free undamPed system can be written in the following matrix form :

$$[m][\ddot{q}] + [k][q] = [0]$$

Where $\ddot{q}$ are the vectors of generalized accelerations.

5.2- Natural frequencies and natural modes of the system :

Since [K] is singular matrix, the eigen Problem can be written as follow :

$$\left[[D] - u \; [I] \right] \; [V] = 0$$

Using comPuter Programe for solving the Previous equation

Where $\left[[D] = [m]^{-1} \cdot [K] \right.$,

$$U_i = W_i^2$$

and $[V] = \left[V_1 , V_2 \; \ldots\ldots\ldots\ldots\ldots\ldots , V_6 \right]$

The following table (5) rePresents the natural frequency and natural modes of the real system :

Table (5)

Natural Frequency	330,6	196.95	108.90	67.79	10.24	1.09
Natural Modes	0.00000	0.00016	0,00000	0.00000	0.01513	0.00000
	-0.00893	-0.13140	-0,00606	-0.0068	0.00139	0.00000
	0.17666	-0.00983	-0.04651	0.00657	0.00014	0.00013
	-0.03244	0.00794	-0.12225	-0.01870	0.00013	0.00038
	0.00013	-0.00000	0.00700	0.04599	0.00000	0.00154
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00554

6.2- ResPonse of the System :

When the system excited by a harmonic excitation $T_0 \sin \Omega t$, the equation of motion of the system is :

$$[m] \; [\ddot{q}] + [k] \; [q] = [T_0 \sin \Omega t]$$

The resPonse of the system can be calculated as a forced system, then the imPedence matrix take the following form:

$$[z] = [k] - w^2 \; [m]$$

The resPonse of the system can be calculated from the following matrix by using comPuter Programe :

$$[r] = [z]^{-1} \cdot [T_0 \sin \Omega t]$$

Where r = the vector of amPlitude resPonse. The following table (6) rePresents the resultant resPonse values of the system form the calculations of the comPuter Programe ;

No.	r_1	r_2	r_3	r_4	r_5	r_6
resPonse Value μm/sec.	0.150581	0.713334	0.78071	2.33964	13.71929	50.3906

3- ExPerimental Investigations :

ExPerimental investigations are carried out on the low Pressure centrifugal PumP(4/3 110 m^3/hr) deriven by rotary cultivator (14 HP. 3000 r.P.m) shown as bolck digram in Fig. (4) and PhotograPhically in Fig. (5).

1.3- Measuring Instruments :

The torsional vibration characteristics can be exPressed by the following measurements

1- Torsional vibration measurements (Kg. Cm)
2- Rotating sPeed measurements (r.P.m)

The measuring instruments used in this investigation are shown in Fig. (5)

2.3- ExPerimental Procedure :

Before carring out the test, the station had been checked thoroughly for leaks.

To study the (T-R.P.M, Q - R.P.M) of the PumP, the following Procedure were carried out at (1000 r.P.m.) of the rotary cultivator sPeed,

a- Start of the rotary cultivator,
b- Start the PumP from the rotary cultivator through its clutch,
c- The following readings were recorde :

i- Torque (T) Kg. mt
ii- Discharge (Q) m^3 /hr
iii- R. P. M.

The other charts of the measured (T-R.P.M), (Q-R.P.M) quantities were carried out by increasing the fuel rate of the rotary cultivator, until the maximum sPeed was reached(maximum sPeed, 3000 r. P. m.)

3.3- Torsional Vibration Measurements :

Torsional oscillation has been measured by means of the mercury electric-sliPPing torque transducer (1) which is connected with the sPindle of rotary cultivator and the inPut shaft of the centrifugal PumP .

The signal is fed into the carrier frequency bridge (2) through a low noise cable, the bridge dial readings rePresenting the measured torque is recorded on the chart of the Programmable striP chart recorder (5) .

4.3- Rotating SPeed Measurements :

The angular sPeed of the rotor system is measured by using the Portable digital multitachometer (6) which can be used by aPPlying its rubber friction couPling at the axial centre of the rotor.

5.3- CaPacity Measurements :

Fig. (5) illustrate a Practical method for measuring the flow rate of the PumP according to Newton's low of gravity. This laboratory method was used in this investigation.

$$Q = k \cdot d^2 \cdot y$$

Where

$$K = \frac{32H}{g}$$

Q	:	Discharge	$cm^3/sec.$
d	:	Discharge Pipe diameter	in cm.
y	:	The horizantal distance	in cm.
H	:	30 cm.	
g	:	980.66	cm/sec^2

4- ExPerimental Results :

The exPerimental results may be dicided into two Parts :

1- Dynamic behaviour of the rotating Parts.
2- Effect of sPeed on the PumP caPacity.

1- Dynamic behaviour of the Rotating Parts :

The measured torques according to the oPerating time of the system are illustrated in Fig. (7) at 2700 r.P.m. , Fig. (8) shows the recorded chart at 2700 r. P. m.

The dynamic torque at each sPeed and its corresPonding static torque are graPhically Plotted in Fig. (9).

The dynamic factor can be comPuted from the exPerimental results of Fig. (10).

Fig.(11) shows the torsional vibration amPlitudes during the sPeed range from 1100 to 3000 r.P.m. of the system. It gives a linear decrement character starting from 9 into 2. 3Kg·mt.

2- Effect of SPeed on the PumP CaPacity :

The measured discharge at each sPeed are graPhically illustrate in Fig. (12) The PumP discharge increase with the increase of the sPeed until it reaches its maximum value of $110 m^3/hr$ at 2000 r. P. m.

5- ComParison Between Theoretical and ExPerimental Results :

From the analysis of the Previous results, it is clear that, the calculations of the real system give a good agreement with the exPerimental results, the deviation in this results is nearer than the first case when simulating the system into two degrees of freedom. This deviation is equal 5% nearly. This variation of the results because the exPerimental results dePends on the accuracy of insturements and it's calibaration.

From Fig's (8,13) it is clear that, the deviation between theoretical and exPerimental driving resPonse can be neglected, because it's has a small value not exceed 4% and its give a good agreement with the exPerimental results in the steady state case.

The mean value of the torsional vibration is 18.5 Kg.m, when the transiant value is 52.4Kg.m, then the dynamic factor Kd is = 2.83, that must taken in consideration when design such machine, in the invornemental condition of EgyPt.

Conclusion :

From the analysis of the Previous results, it can be concluded that :

1- The calculations of the real system give a good agreement with the exPerimental results comPairing with the case of simulating the system.

2- The calculated critcal sPeeds of the system are 350, 950r.P.m.

3- The oPtimum oPerating sPeed is 2000 r.P.m. which, more than the two critical sPeeds gives the maximum discharge of the PumP. (110 m^3/hr). This discharge is greater than the maximum discharge obtained by the largest water wheel cPerated by animals.

4- Suction head must be 2.5 mt which is equivelent to maximum diameter of the water wheel.

5- The dynamic factor at which the system is safely oPerated is 2.77 for the system under study. (F.T.O) of the rotary cultivator sPlinned joint, certifugal PumP, suction ruber tube, the set jointed to the cultivator as centilever .

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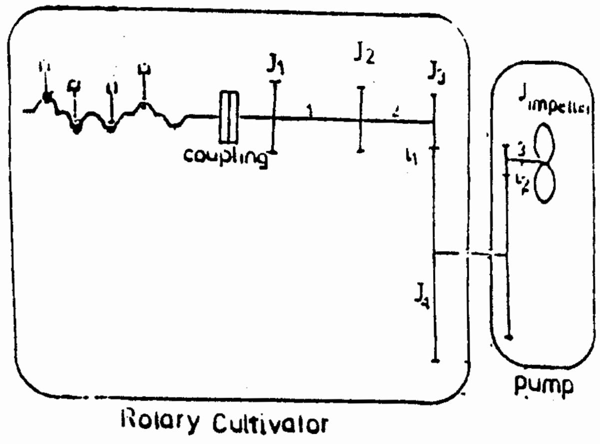


FIG.(1) THE PUMP DRIVEN BY ROTARY CULTIVATOR.

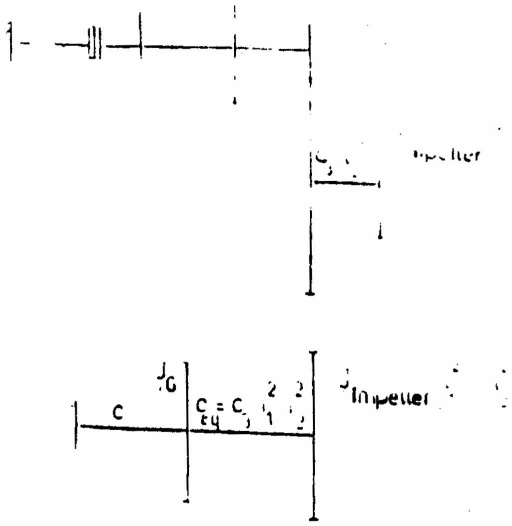


FIG.(2) THE EQUIVALENT SYSTEM.

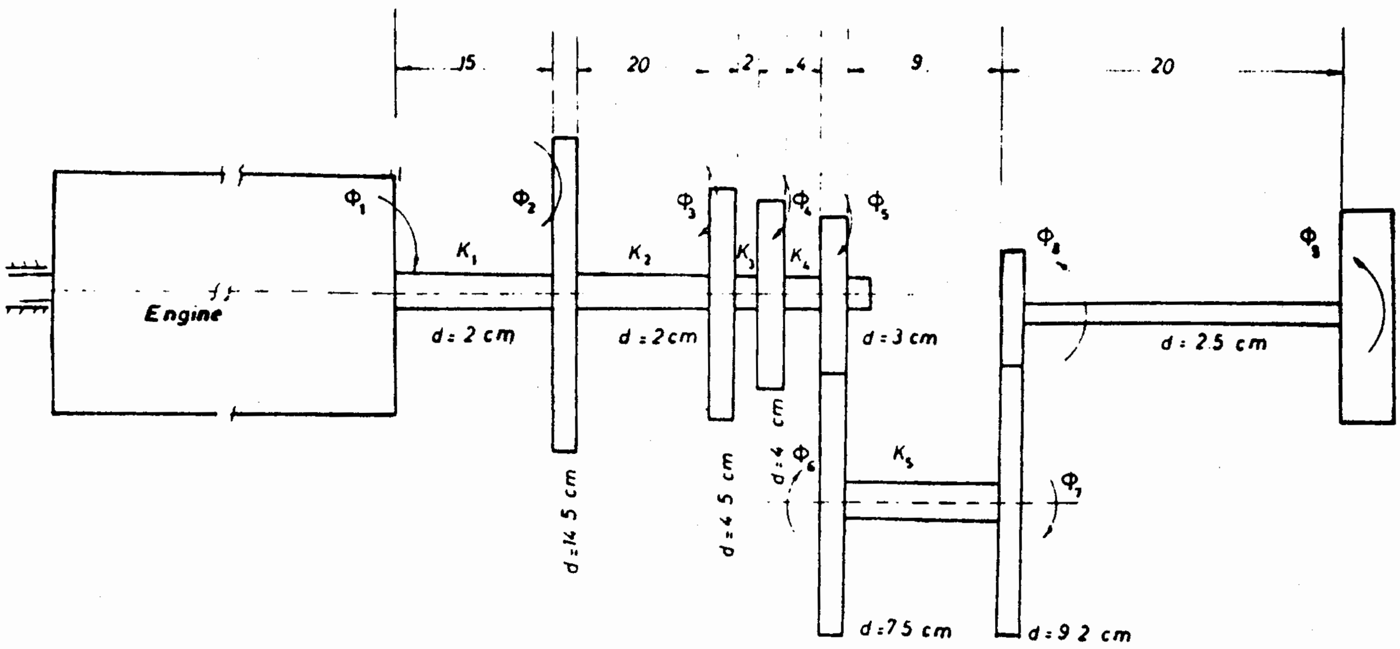
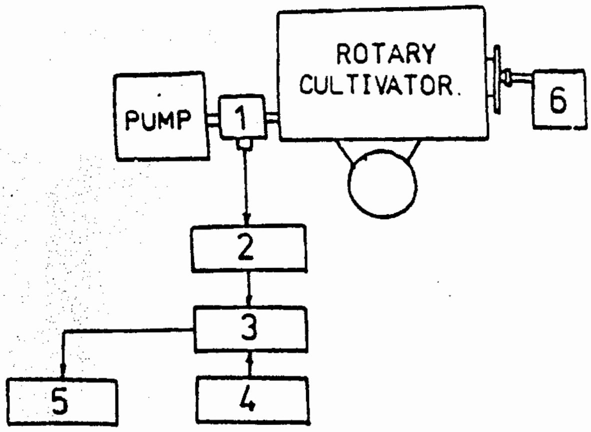


FIG (3) SYSTEM UNDER STUDY



1- Torque Transducer.

2- Bridge.

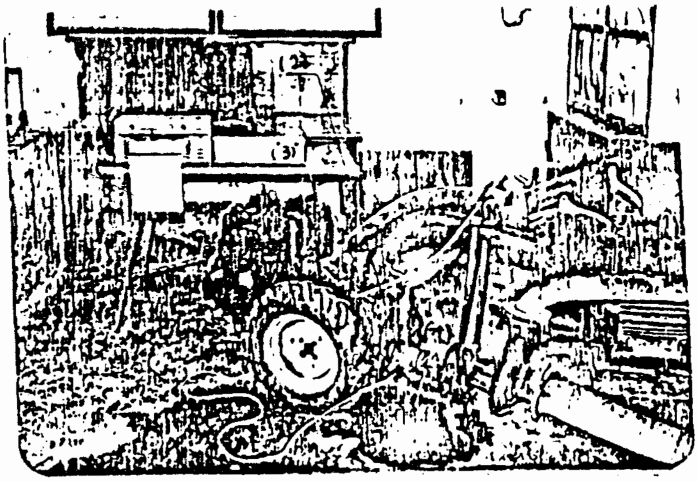
3- Amplifier.

4- Offset.

5- Recorder.

6- RPM Tachometer.

FIG(4) BLOCK DIAGRAM OF THE MEASURING INSTRUMENTS



1 - Torque Transducer

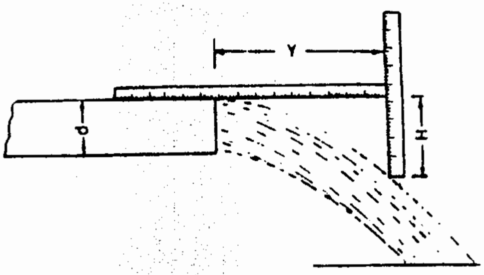
2 - Bridge.

3 - Amplifier

4 - Offset

5 - Recorder.

FIG (5) MEASURING SYSTEM



Y Horizontal distance cm

d Discharge pipe diameter cm

H 30 cm

FIG (6) Approximate Capacity GPM For Full Flowing Horizontal Pipes.

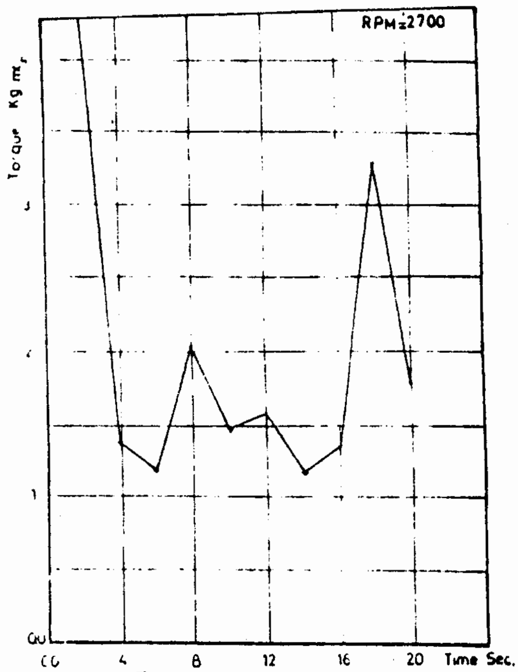
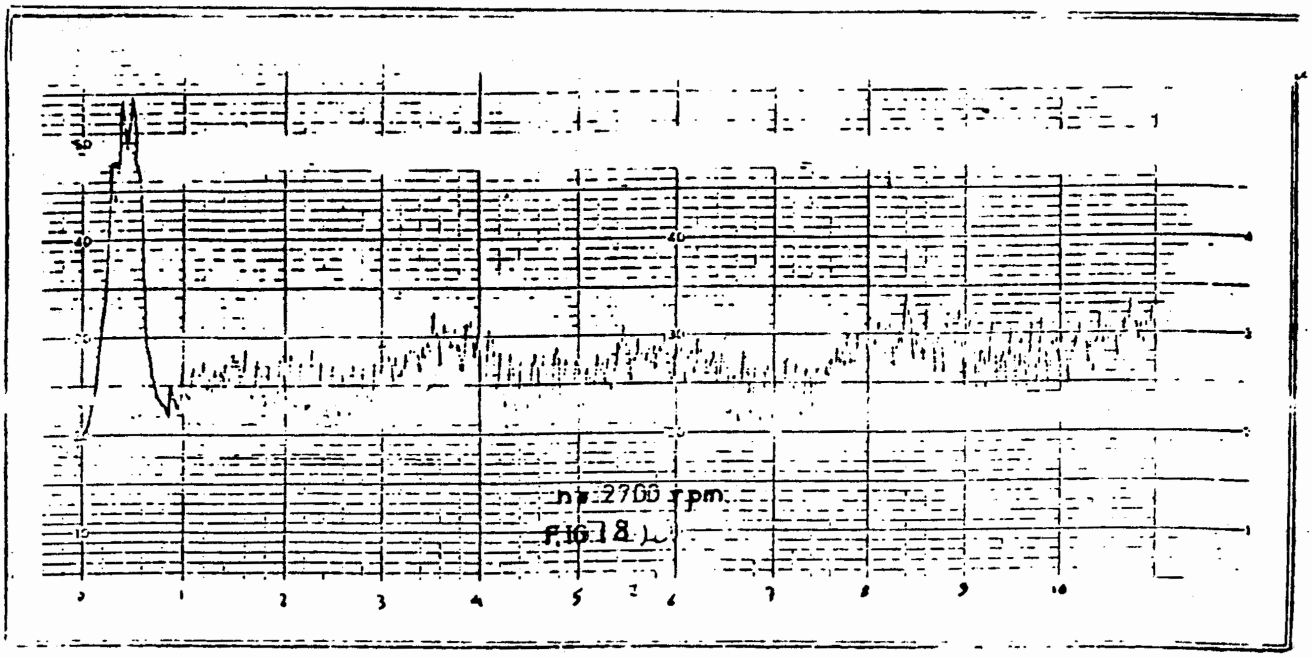


FIG (7) The measured Torque according to operating time



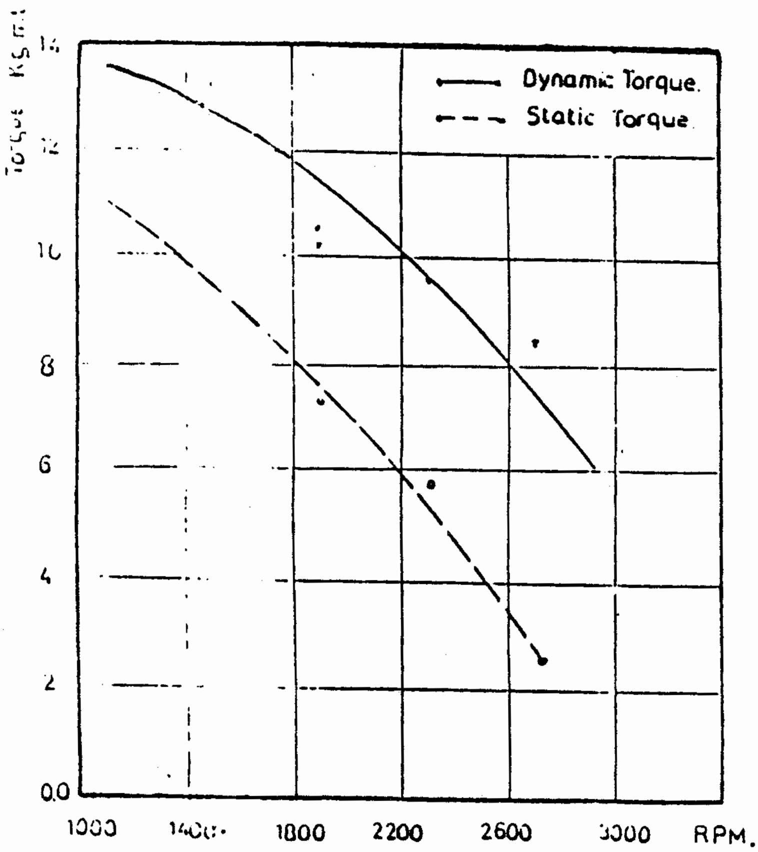


FIG (9) Dynamic & Static Torques.

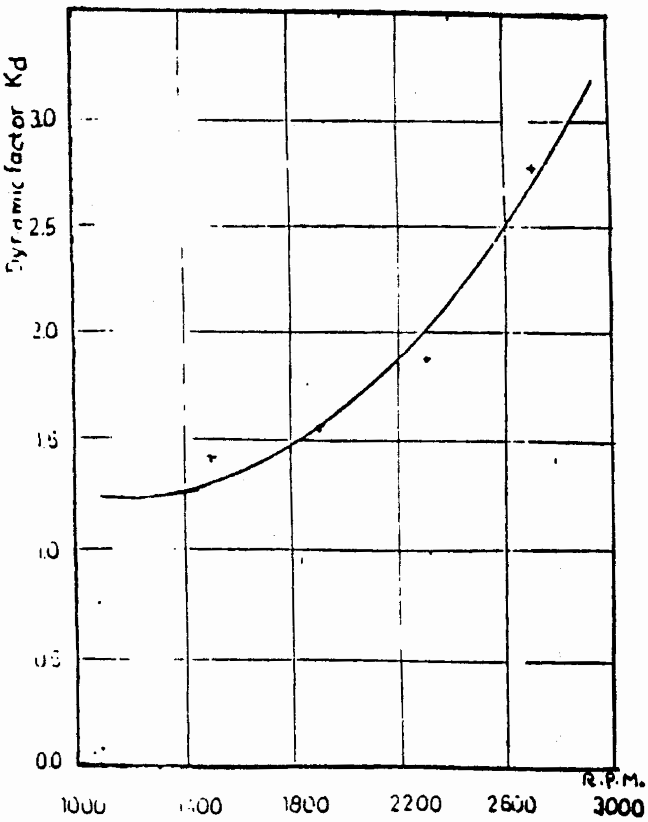
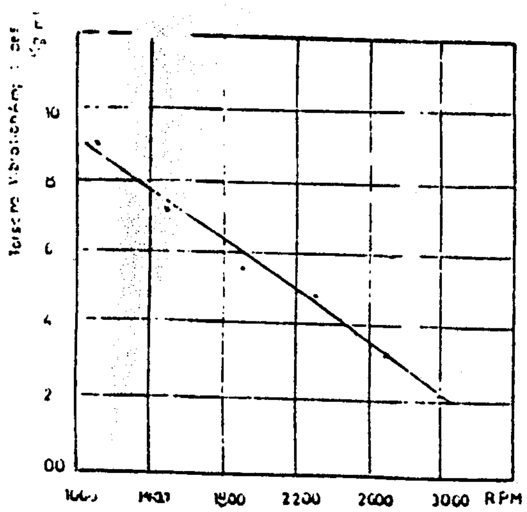
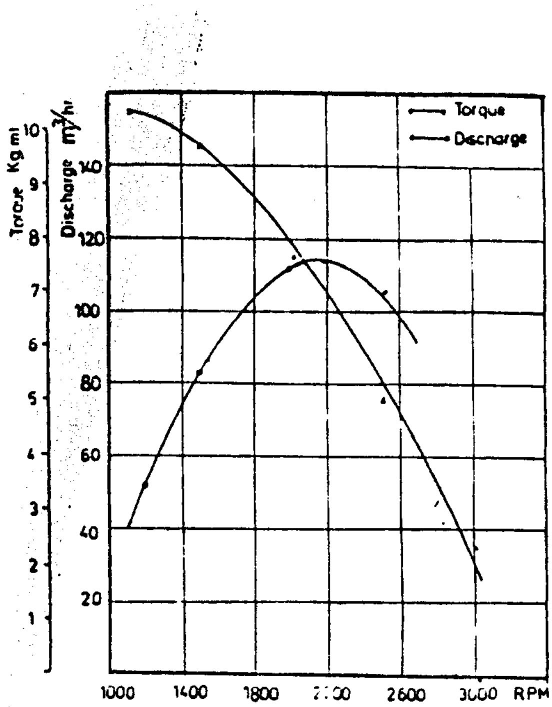


FIG.(10) Dynamic Factor of Torques.



FIG(11) The measured Torsional Vibration Amplitudes.



FIG(12) Torque & Discharge Measurements

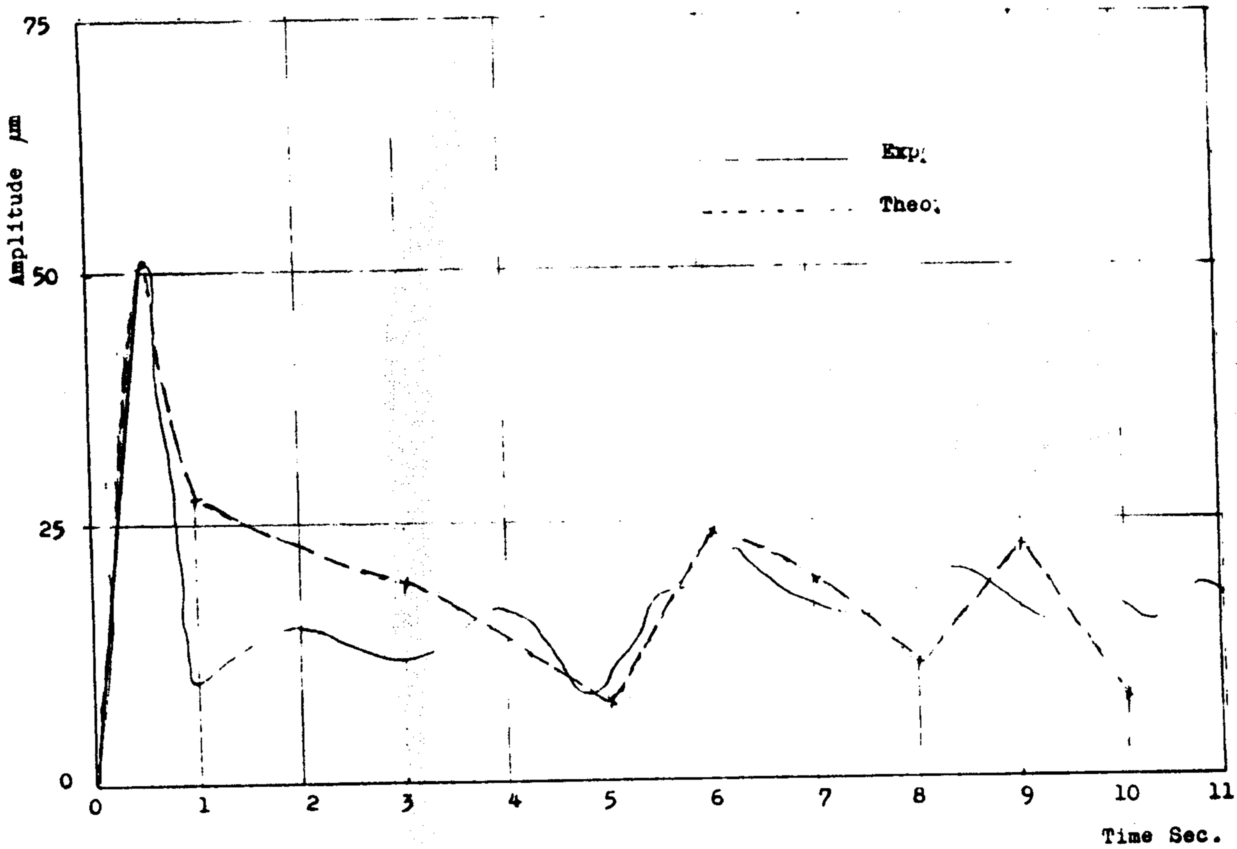
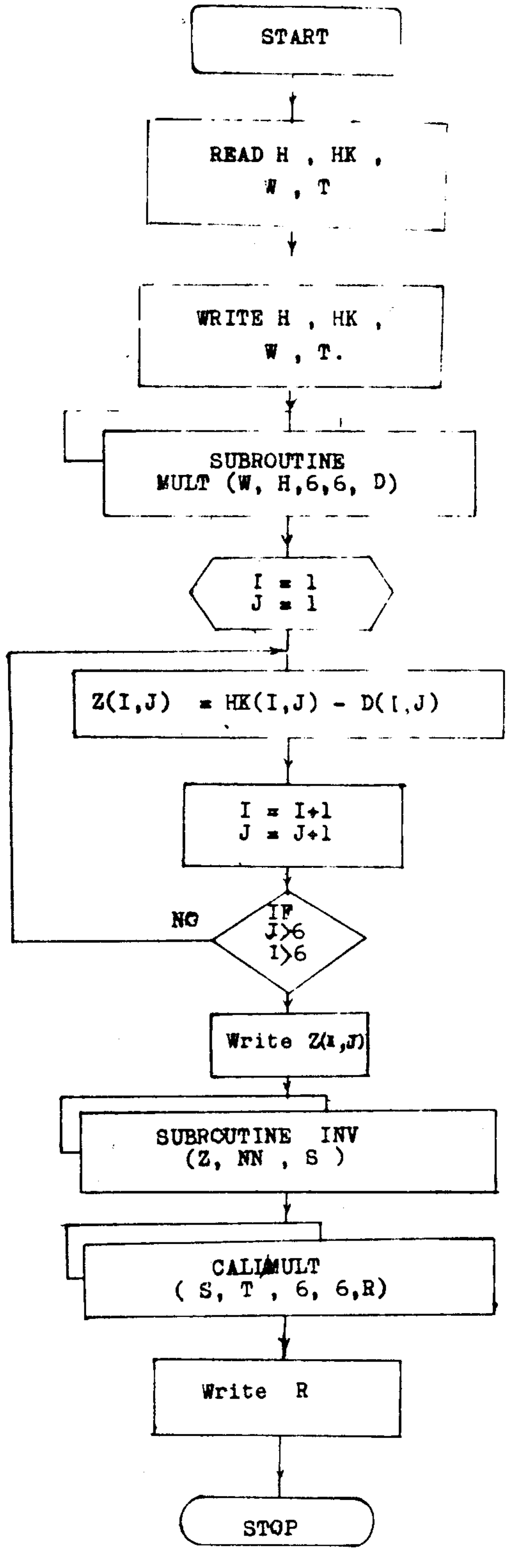


Fig.(13): Theoretical and Expermintal Response Results.



SUBROUTINE MULT
TO Multiply two Matrices

SUBROUTINE INV
To calculate the inverse of matrix .

Fig.(14) Flowchart of Computer Programe.