

Menofia University
 Faculty of Engineering
 Shebien El-kom
 First Semester Examination
 Academic Year : 2017-2018



Department : Electrical Eng.
 Year : 1st
 Subject : Eng. Mathematics
 Time Allowed : 3 hours
 Date : 17 / 1 / 2018

Allowed Tables and Charts : None

الإمتحان في صفحتين

Answer all the following questions: [140 Marks]

Question 1 [70 Marks]

(A) Find the general solution of the differential equations [20 Marks]

(i) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$

(ii) $y \sin 2x dx - (y^2 + \cos^2 x) dy = 0$

(iii) $\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = x \cos x, x > 0$ (iv) $y \frac{d^2 y}{dx^2} + 1 = \left(\frac{dy}{dx}\right)^2$

(B) Find the solution of the ODE problem

$$\left(\frac{dy}{dx}\right)^2 - 2x \left(\frac{dy}{dx}\right) + y = 0 \quad [5 \text{ Marks}]$$

(C) i) Solve the differential equation $(x^2 D^2 - xD + 2)y = x \ln x$ [5 Marks]

ii) Calculate the volume of the body bounded by the surfaces:
 $z = 4 - x, \quad x + y = 2, \quad x = y = z = 0$ [10 Marks]

(D) (i) Solve the following system of simultaneous ordinary differential

equations. $\frac{d^2 x}{dt^2} - 3x - 4y = 0, \quad \frac{d^2 y}{dt^2} + x + y = 0$ [10 Marks]

(ii) Solve the following ODEs [10 Marks]

1. $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$

2. $(D^2 + 9)y = \cos 2x + \sin 2x$

(E) Solve the differential equation $\frac{d^2 x}{dt^2} - 4 \frac{dx}{dt} + 4x = t e^t$ using

Laplace transform with initial conditions: $x(0) = 0$ and $x'(0) = 0$.

[10 Marks]

Question 2 [70 Marks]

(A) (i) By transforming into polar coordinates evaluate the integral (Jacobian)

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 + y^2 dy dx \quad [10 \text{ Marks}]$$

ii) Find the interval of convergence of the series $S_n = \sum_{n=1}^{\infty} \frac{(2x)^n}{n}$. [5 Marks]

ii) Calculate the double integral $\iint_D f(x, y) dx dy$ for $f(x, y) = 3 + x^2 + y^3$

and D is bounded by $0 \leq x \leq 1$, $0 \leq y \leq x$. [5 Marks]

(B) Find the inverse Laplace transform of the functions

$$(i) F(s) = \ln \frac{s+1}{s-1} \quad (ii) F(s) = \frac{1}{(s)(s-2)^2(s^2+1)} \quad [10 \text{ Marks}]$$

(C) Find Laplace transform of the following functions

$$(i) f(t) = \frac{1 - \cos t}{t} \quad (ii) f(t) = \begin{cases} 0 & t < \frac{2\pi}{3} \\ \cos(t - \frac{2\pi}{3}) & t > \frac{2\pi}{3} \end{cases} \quad [10 \text{ Marks}]$$

(D) Test the convergence of the following series:

$$(i) S_n = \sum_{n=1}^{\infty} \frac{2n-1}{2^n} \quad (ii) S_n = \sum_{n=1}^{\infty} \left(\frac{n}{2n+1} \right)^n \quad [10 \text{ Marks}]$$

(E) A periodic function $f(x)$ with half period π is defined as follows:

$$f(x) = x \quad -\pi \leq x \leq 0$$

i) Plot the function. ii) Find the corresponding Fourier cosine series. [10 Marks]

(F) Find the center of the mass of a thin plate bounded by the parabola $y = 6x - x^2$ and the straight line $y = x$, given that it has a mass density $f(x, y) = \rho(x, y) = 1$. [10 Marks]

With my best wishes
Associate Prof. Dr. Islam M. Eldesoky