

Many experimental studies have been carried out to study the efficiency of submerged breakwater on wave transmission Dick et al. (1968), Abu-Zied (1970), Abdul Kader and Rai (1980) and Heikal et al. (1997). Theoretical studies have been obtained to study the performance of thin vertical submerged breakwaters sited on horizontal beaches by Dean (1945), Johnson, Fuchs and Morison (1951), and Abul-Azm (1993).

The objective of this research is to obtain a numerical model of simulating the wave transformation along and shore side of submerged breakwaters. The geometric parameters of submerged breakwaters are investigated and summarized as; submergence ratio ( $d_1/D$ ), relative breakwater tip width ( $B/D$ ) and side slopes,  $S_1$  for sea side and  $S_2$  for shore side, Fig. (1). The submergence ratio have been experimentally studied by Abu-Zied (1970).

A computer numerical model was designed to study the effect of breakwater parameters on wave transformation and to find the most efficient submergence ratio. The model construction depends on the solution of mild-slope equation, Tanaka (1990), using the implicit finite difference technique. The climatological conditions were taken into consideration such as wave characteristics, (wave height  $H$ , wave period  $T$ , wave direction  $\theta$ ), wave steepness ( $H/L$ ). Also, breaking and non breaking of waves were taken into account. The model was calibrated and compared with the hydraulic experimental results by Abu-Zied (1970) along horizontal bed profile. The model was verified and applied on horizontal, uniform sloped bed and natural bed profiles. Dimensionless relations between different variables have been obtained to indicate the effect of each one on the others.

#### MATHEMATICAL FORMULATIONS

The combined refraction-diffraction equation, derived by Berkhoff (1972, 1976), Smith and Sprinks (1975) and Tanaka (1990), to describe the propagation of periodic, small-amplitude, surface gravity waves over an arbitrarily varying, mild-sloped sea-bed is :

$$\frac{\partial}{\partial x} (C C_g \cdot \frac{\partial \Phi}{\partial x}) + \frac{\partial}{\partial y} (C C_g \cdot \frac{\partial \Phi}{\partial y}) + \omega^2 \cdot C_g / C \cdot \Phi = 0 \quad (1)$$

where :

$\Phi$  = two dimensional complex wave potential function,

$\omega$  = angle frequency,

$C$  = phase velocity ( $= \omega/k$ ),

$C_g$  = group velocity ( $= nC$ ),

$n$  = shoaling factor,  $n = 1/2 [ 1 + 2kD / (\sinh 2kD) ]$  (2)

$k$  = wave number defined by the relation,  $\omega^2 = gk \tanh (kD)$  (3)

$g$  = acceleration due to gravity,

$D$  = water depth,

$x, y$  = horizontal coordinates of place.

In general, the water depth is a given function of place, so also the quantities  $k$ ,  $C$  and  $C_g$  are functions of place. Equation (1) is expanded to yield,

$$CC_g \cdot \partial^2 \Phi / \partial x^2 + CC_g \cdot \partial^2 \Phi / \partial y^2 + \partial \Phi / \partial x \cdot \partial CC_g / \partial x + \partial \Phi / \partial y \cdot \partial CC_g / \partial y + \omega^2 \cdot C_g / C \cdot \Phi = 0 \quad (4)$$

### FINITE DIFFERENCE MODEL

The domain of interest is covered with a rectangular mesh consisting of M rows and N columns of nodes giving in total MN nodes, Fig.(2). The distance between each node is  $\Delta$ . At each node the depth is specified. By knowing the period and hence the frequency of the waves under consideration, it is possible, in the absence of a current, to calculate the wave number k which corresponds to each node by using the dispersion relationship (3). Once the wave number field is known, the phase velocity, the group velocity and the product of the phase and group velocity can be found at each node.

$$\begin{aligned} & (CC_g)_{ij} [(\Phi_{i,j-1} + \Phi_{i,j+1} + \Phi_{i-1,j} + \Phi_{i+1,j} - 4\Phi_{i,j}) / \Delta^2] + \\ & \{[(CC_g)_{i+1,j} - (CC_g)_{i-1,j}] / 2\Delta\} \cdot \{[\Phi_{i+1,j} - \Phi_{i-1,j}] / 2\Delta\} + \\ & \{[(CC_g)_{i,j+1} - (CC_g)_{i,j-1}] / 2\Delta\} \cdot \{[\Phi_{i,j+1} - \Phi_{i,j-1}] / 2\Delta\} + \omega^2 \cdot (C_g/C)_{ij} \cdot \Phi_{i,j} = 0 \end{aligned} \quad (5)$$

After manipulation this becomes ;

$$\begin{aligned} & \Phi_{i,j-1} \{1 - [(CC_g)_{i,j+1} - (CC_g)_{i,j-1}] / 4(CC_g)_{ij}\} + \Phi_{i,j+1} \{1 + [(CC_g)_{i,j+1} - (CC_g)_{i,j-1}] / 4(CC_g)_{ij}\} \\ & + \Phi_{i-1,j} \{1 - [(CC_g)_{i+1,j} - (CC_g)_{i-1,j}] / 4(CC_g)_{ij}\} \\ & + \Phi_{i+1,j} \{1 + [(CC_g)_{i+1,j} - (CC_g)_{i-1,j}] / 4(CC_g)_{ij}\} \\ & + \Phi_{i,j} \{-4 + (\Delta^2 \cdot \omega^2) / C_{ij}^2\} = 0 \end{aligned} \quad (6)$$

From the dynamic free surface boundary condition

$$\eta + \Phi_1 / g = 0 \quad (7)$$

and as the potential is assumed to be harmonic in time, it can be shown that ;

$$H = 2 \omega \Phi_{\max} / g \quad (8)$$

where  $\Phi_{\max}$  is the maximum value of potential attained at a node.

In order to solve equation (6) over a domain of interest, boundary conditions must be specified. On the sides parallel to the direction of the incoming waves, the condition of no flow normal to the boundary is imposed, i.e.

$$\partial \Phi / \partial n = 0 \quad (9)$$

On the boundary facing the incoming waves the value of potential and its gradient are specified. Included in these boundary conditions is the information about the phase  $\theta$  at which the wave meets this boundary.

Applying equation (6) at each node and imposing the boundary conditions in simultaneous linear equations are defined. This system of equations is then solved by using the conjugate-

gradient method. The solution is a field of velocity potentials. A phase difference of  $\pi/2$  is then added to the original phase on the boundary facing the incoming waves and the corresponding new system of equations is solved in the same way. The new field of velocity potentials is  $\pi/2$  out of phase with the previous set and the maximum value of potential  $\Phi_{\max}$  attained at each node can be calculated by using the relation;

$$\Phi_{\max}^2 = \Phi_{\max}^2 \cdot \cos^2\theta + \Phi_{\max}^2 \cdot \cos^2(\theta + \pi/2) \quad (10)$$

Then by using equation (8) H and h/H can be calculated.

#### Model Construction

The model consists of one main program and six subprograms. The subprograms are used to calculate the different wave parameters and to minimize the size of the main program.

#### Boundary Conditions

For lateral boundary condition, The value of all variables at cell  $i = N$  and  $i = N-1$  are equals and at cell  $i=1$  and  $i=2$  are equals too. And offshore boundary condition, is that, the deep water wave characteristics are transformed from the deep water into the simulated area under the use of Snell's Law, assuming that the sea bed contour lines are parallel. The wave height, at the outer edge of the simulated area, are calculated with the aid of Snell's Law to find the wave direction. The wave parameters at cell  $j = M$  is used as initial values to the model. But the onshore boundary condition, there are two conditions, one at the beginning of the surf zone which dependent on the breaking limit, at which the wave height is calculated as the breaking wave height. The other one is close to the shoreline which is the limit of the water depth called the minimum water depth at which the wave height is calculated using Weggel formula (1972). This condition is required for the stability of the model.

#### Model Calibration

The model was calibrated and compared with the experimental results of the hydraulic experimental model of Abu-Zied (1970). Figure (3), indicates the result of the comparison. It shows good agreement between the two results, and it is concluded that the model is reliable and could be applied on similar cases.

#### MODEL APPLICATIONS

The model was applied on natural bathymetry of a coastal area with uniform mild slope. This area was divided into equal spaced mesh in two directions as shown in figure (2). The submerged breakwater was represented with water depth, according to the side slopes of the breakwater. The breakwater submergence ratios were classified into five groups, ( $d_1/D = 0.1, 0.2, 0.3, 0.4, 0.5$ ) and the relative breakwater top width was divided into five groups also ( $B/D = 1,2,3,4,5$ ). The breakwater side slopes are equal to  $S_1$  (sea side),  $S_2$  (shore side). Put  $S_1 = 1:1 \ \& \ 1.5:1 \ \& \ 2:1$  and  $S_2 = 1:1$ . Wave characteristics were grouped into three values ( $H/L = 0.0384, 0.0250, 0.0160$ ). Wave direction was taken constant.

## RESULTS AND DISCUSSION

The results of the model application were divided into two :-

### Effect of Relative Breakwater Width (B/D)

The effect of breakwater top width on wave height shore side of the breakwater was studied. Figure (4) shows the relationship between the wave height along cross section normal to the shoreline and the distance from the breakwater position. These curves were drawn for breakwater top width  $B= 1.0$  to  $5.0$  water depth at a certain submergence ratio  $d_1/D = 0.30$ , and constant wave steepness  $H/L = 0.0250$ . From this figure, the dissipation of wave energy increased with the increase of top width. But when breakwater top width increases more than  $4.0$  times water depth, there is no significant effect on wave height shoreside. It is concluded that, the breakwater top width equals to  $3.0-4.0$  times water depth, for economic considerations.

### Effect of Breakwater Submergence Ratio ( $d_1/D$ )

The effect of breakwater submergence ratio on transmission of waves shoreward was investigated. Figures 5 to 9, show the analysis of the results. Dimensionless parameters were obtained to indicate the effect of breakwater submergence ratio on the transformed wave height,  $h$ . The figures indicate the relation between the transmission ratio of wave height ( $h/H$ ), and submergence ratio. From these curves, it is clearly that the shoreward wave height decreased with the increase of breakwater submergence ratio for different values of relative breakwater top width and for various values of wave steepness. This means that the transformed wave energy shoreward increased with the increase of submergence ratio. Moreover, the submerged breakwater has remarkable effect on transformed wave height when  $d_1/D=0.0-0.3$ , for different values of wave steepness, (i.e. submerged breakwater height =  $1.0-0.8$  water depth). Also, submerged breakwaters have very good efficiency for moderate wave steepness. It is useful to construct submerged breakwaters with moderate side slope seaside to help in breaking of waves on breakwater body.

## CONCLUSIONS

A reliable computer numerical modeling was designed to simulate the transformation of wave field around and shoreward of submerged breakwaters. The effect of refraction-diffraction and breaking of waves has been taken into consideration. Through this model, the effect of submerged breakwater parameters were investigated and concluded that.

- The breakwater top width equals to  $3.0-4.0$  times water depth, for economic considerations.
- The submerged breakwater has remarkable effect on transformed wave height when  $d_1/D=0.0-0.3$ , for different values of wave steepness, (i.e. submerged breakwater height =  $1.0-0.8$  water depth).
- Submerged breakwaters have very good efficiency for moderate wave steepness.

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#### NOTATION

B	breakwater top width,
C	phase velocity,
$C_g$	group velocity,
D	water depth,
dI	water depth above breakwater,
g	gravational acceleration,
H	wave height sea side,
h	wave height shore side,
i, j	number of grid points along x and y directions respectively,
k	wave number,
L	wave length,
n	shoaling factor,
$S_1, S_2$	breakwater side slopes sea side and shore side respectively,
T	wave period,
x, y	horizontal coordinates,
$\Delta$	grid step length in x & y directions,
$\Phi$	two dimensional velocity potential,
$\omega$	angular frequency,
$\theta$	incident wave direction with y-axis.

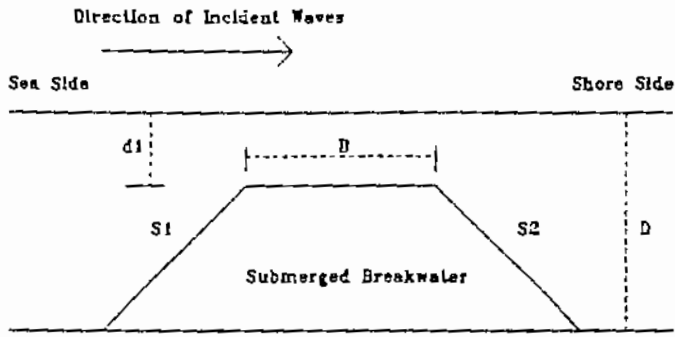


Fig. 1: Definition sketch of submerged breakwater parameters.

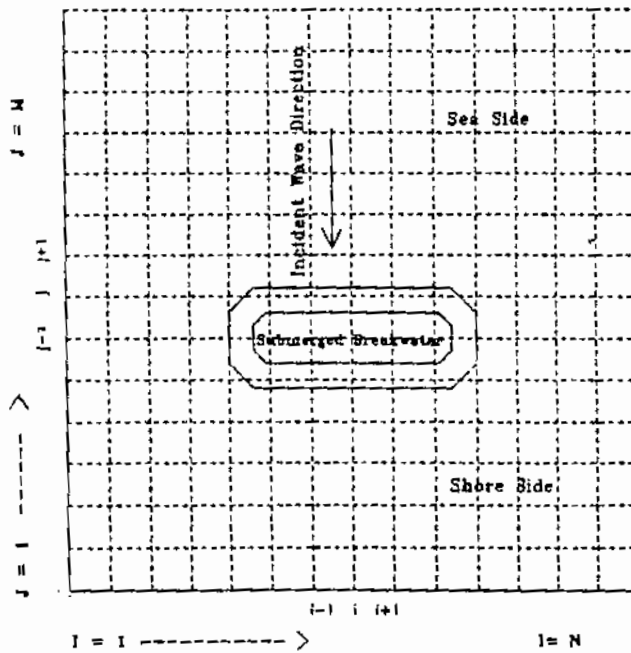


Fig. 2: Definition sketch of finite difference mesh system

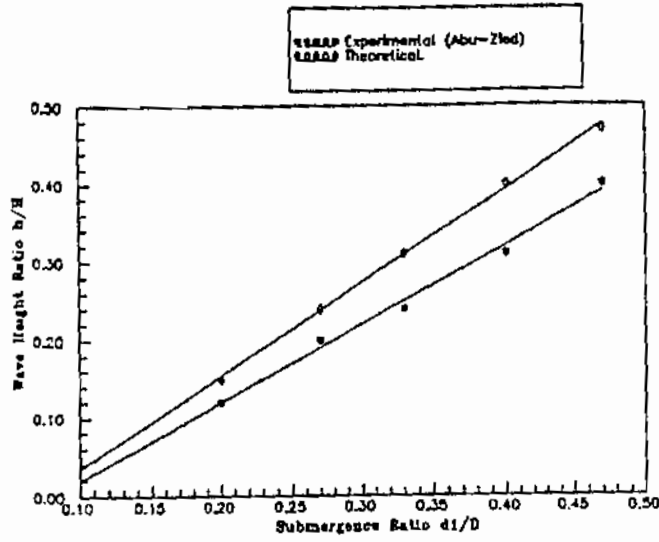


Fig. 3: Comparison between theoretical model and experimental results (Abu-Zied, 1970).

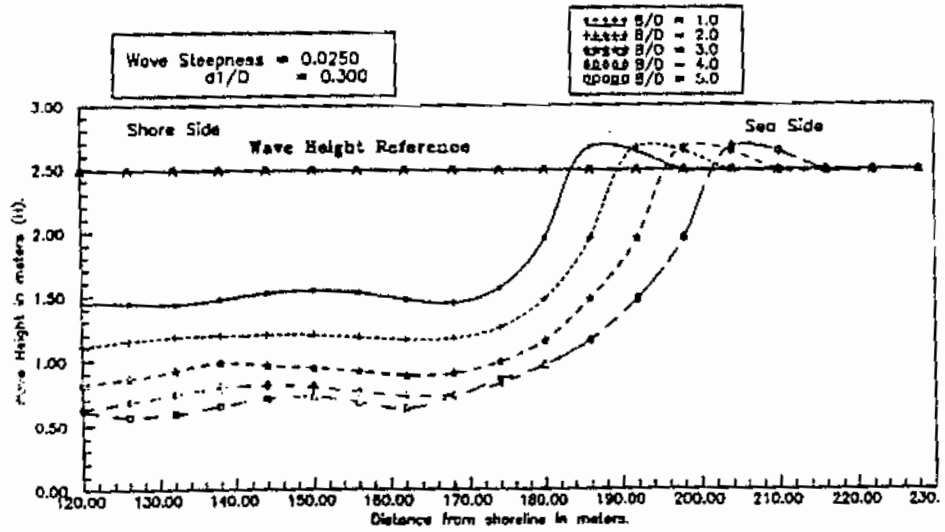


Fig. 4: Effect of relative breakwater top width (B/D) on wave height shoreside of submerged breakwaters.

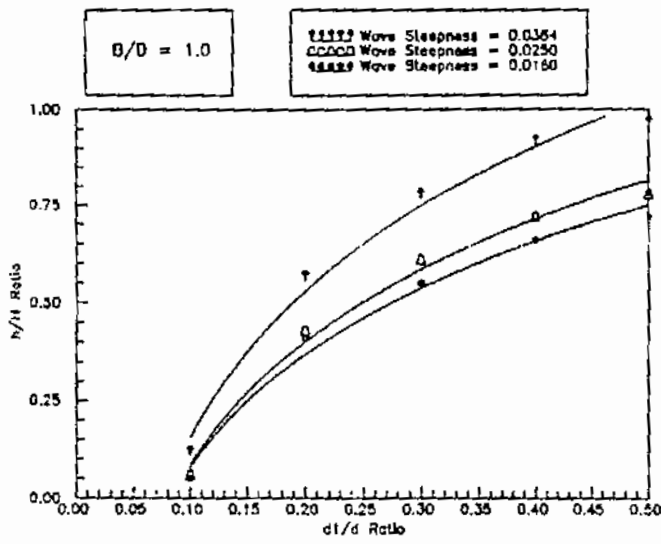


Fig. 5: Effect of breakwater submergence ratio on passing wave height. (for different wave steepness and  $B/D = 1.0$ )

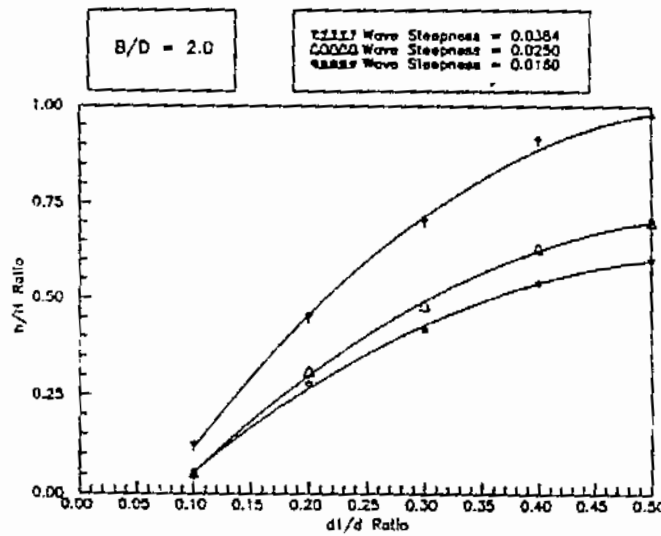


Fig. 6: Effect of breakwater submergence ratio on passing wave height. (for different wave steepness and  $B/D = 2.0$ )



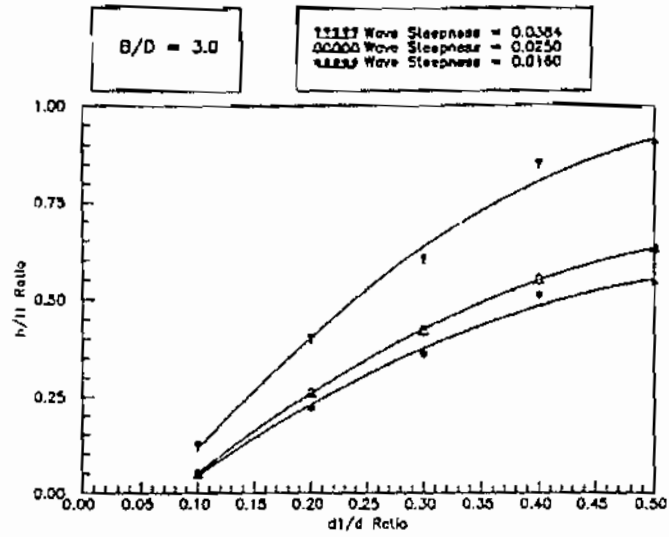


Fig. 7: Effect of breakwater submergence ratio on passing wave height. (for different wave steepness and  $B/D = 3.0$ )

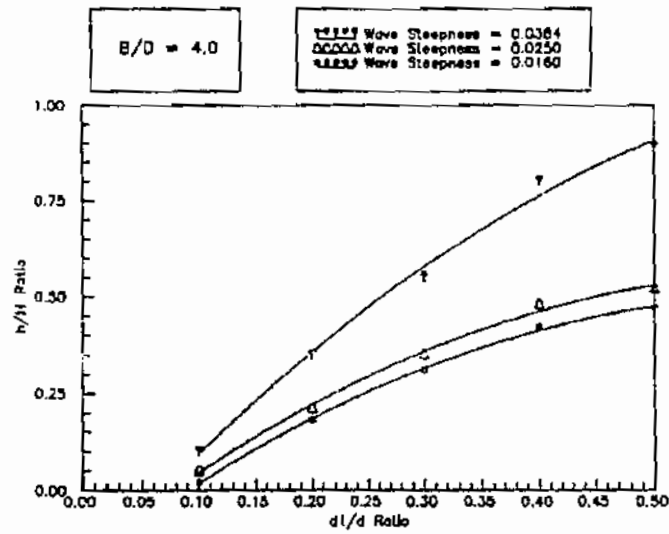


Fig. 8: Effect of breakwater submergence ratio on passing wave height. (for different wave steepness and  $B/D = 4.0$ )

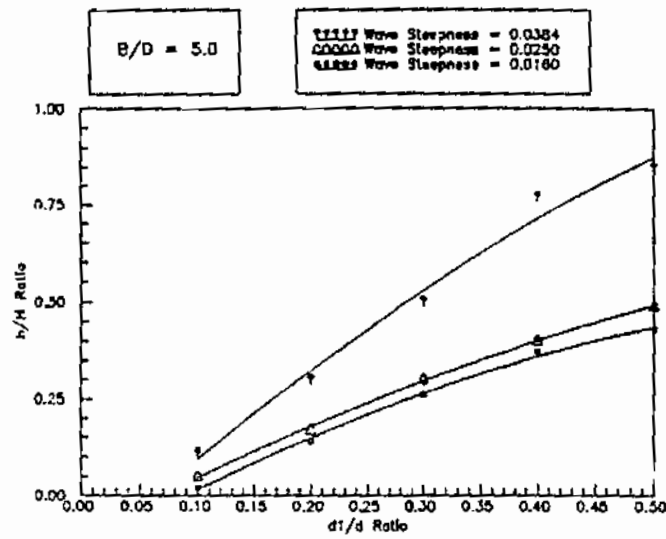


Fig. 9: Effect of breakwater submergence ratio on passing wave height. (for different wave steepness and B/D = 5.0)