INVENTORY CONTROL MODELS APPLICABLE TO AGRICULTURAL INVENTORIES

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ABSTRACT:

Inventory control has become an extensive field of the operations Research investigations specially after the wide use of computers in this field. Although most of the literature was devoted to inventory control problems in industry and commerce, neglecting its application to agricultural inventories. Therefore, the undertaken paper deals with the application of inventory control models to agricultural inventories.

The paper considers the application of inventory control models to the stocking of insecticides, wheat, flour (extera and normal), and animal foods. In addition, the various developed inventory models are reviewed, laying emphasis on the models that suit the situations of the prementioned agricultural inventories.

Objective and Scope of work:

The undertaken research, aims at finding out the inventory control models applicable to some agricultural inventories. This necessitates studying the different inventory models very well, the assumptions on which each is based, the limitations of application and the methods of treatment.

It is essential in addition, to study very well the circumstances and the nature belonging to each inventory item of the agricultural inventories under consideration.

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The Study May Include:

1. The method of stocking the item.
2. The system of providence and consumption.
3. The importance of the item and the effect of its shortage.
4. The probabilities of spoilage and deterioration according to the time of stocking.
5. The statistical distribution of the demand.

It remains then to fit the adequate model for each item according to the prementioned studies.

One of the most important objectives of the paper is to overcome the difficulty of computation associated with the inventory models specially those including the substitution of complicated statistical distributions and the corresponding mathematical treatment. This is performed through developing computer programs ready to be used for different situations of applications. Thus enabling attaining the effect of changing different factors i.e., for sensitivity of the program with respect to different factors involved in each case of application.

Needless to say, the main objective of the paper is to emphasize that the techniques of Operations Research in general and inventory control techniques in particular are applicable to the field of agriculture exactly as well as in industry and commerce (1).

INTRODUCTION:

Inventory control models can be classified generally into two categories; the first is the category of deterministic models, in which the demand of certain item is specifically known with certainity. It may be constant allover the periods, therefore, the problem is to determine the optimal reorder quantity (EOQ). On the other hand it may vary from period to another (known in each period), in this case, by the use of dynamic programming technique, the optimum reorder point and optimum reorder quantity can be determined. The second is the category of probabilistic inventory models, in which the demand is a random variable with known or unknown probability distribution function. Although, the input and lead time may be random variable with probability distribution or it may be deterministic variables.

The determinstic models are mostly familiar and widely handled in several references. Thus the undertaken paper lays emphasis on the probabilistic inventory models.

The probabilistic inventory models are classified mainly to:

1 - Single period model, in which the problem is to determine how much of a single item to have on hand at the begining of the period to minimize the total purchase cost, ending inventory holding cost, and stock out cost. This model can be applied to the stocking of seasonal products and stocking of short-lived perishable items.

2 - Multi-period models, dealing with inventory situations where the item must be recorded periodically. There are basically two types of multi-period models:
    a) Continuous review models.
    b) Periodic review models.

    In the continuous review models, time is treated as a continuous variable and it is assumed that a replenishment order occurs whenever the inventory level reaches the recorder point. While in the periodic review models, a reorder decision is permitted to occur only at a fixed intervals of time.

Many authors, handled the different inventory models with different assumptions. Archibaled and Silver*2 studied the (s, S) policies for continious review and discrete compound Poisson demand. Gross and Harris*4 described the development of continious review (s, S) inventory model with complete backordering and state-dependent, stochastic lead times. Hadley and Whithin[3] analyzed the complete backlogging and lost sales for Poisson demand and any distribution of replenishment time. Galliher, Morse, and Simond[5] studied the complete backlogging case under stuttering Poission demand. Gross and Harris[4] analysed the backlogging case for Poisson demand but allow replenishment time to depend on the level of unfilled demands.

The application of continuous review models depends on the inventory system situation. Continuous review models can be applied in inventory systems in which the stocking level is of great importance, so the inventory level can not be checked at constant intervals as the demand may exceeds the inventory level at any period. Continuous review models will be applied to control the inventory level required for the stocking of strategic goods such as wheat and flour, where it is impossible to permit shortage in inventory. In addition the models are applicable for situations of high probabilistic demand.

Probabilistic Inventory Models:

In these models, the demand is a random variable, discrete or continuous, with known or unknown probability distribution function. The most familiar policy of these models is the(s,S) policy, where s is the optimum reorder point and S is the optimum inventory level, hence, the optimum amount to order is

$$Q = S - s$$

1. Single-Period Model:

This model considers the inventory problem of stocking an item of one period (Long or short). The model therefore suits the case of stocking insecticides, where the insecticides are used for the protection and to remedy crops against insects. To avoid the immunity generated against a certain insecticide, the insecticide must be changed every season, otherwise it looses its effectiveness.

The single period model is based on the following assumptions:

1. The demand for the product is a random variable.
2. No backordering is permitted (unfilled demands are lost sales).
3. The delivery rate is infinite.
4. Lead time is zero.
5. Costs are associated with placing an order, inventory holding cost, and shortage cost.

The expected total inventory cost can be expressed by :

$$E(TIC) = Co + C_3\,(DIL - IOH) + C_1 \int_{-\infty}^{DIL} (DIL - x)\, f(x)\; dx + C_4 \int_{DIL}^{\infty} (x - DIL)\, f(x)\, dx \qquad \text{.............(1)}$$

Where:

x = Demand in the given period.
$f(x)$= Probability distribution function of demand.
$F(x)$= Commulative distribution function of demand.
DIL = Desired inventory level at the start of the period.
IOH = Inventory on hand before placing an order.
C_o = Ordering cost per order or setup cost/setup .
C_1 = Inventory holding cost/unit per unit time.
C_3 = Purchasing cost/unit or production cost/unit.
C_4 = Shortage cost/unit out of stock.

The inventory level that will minimize the expected total inventory cost is the value of the desired inventory level (DIL) such that:

$$P\,(x \leqslant DIL) = F(DIL) = \frac{C_4 - C_3}{C_1 + C_4} \qquad \text{................(2)}$$

The value $(C_4 - C_3)/(C_1 + C_4)$ in equation (2) represents the probability of no stockout when the given item is stocked at the optimal DIL, Otherwise;

$$P(x > DIL) = 1 - F\ (DIL) = \frac{C_1 - C_3}{C_1 - C_4} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

represents the probability of at least one stockout.

If the demand is a discrete random variable, the summation sign is used instead of integration sign and then the optimal desired inventory level is the smallest value of DIL such that:

$$P(x \leqslant DIL) = F(DIL) \geqslant \frac{C_4 - C_3}{C_1 - C_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The data representing the demand of a certain kind of insecticides (Dymitwit) which was obtained from the Bank of Development and Agricultural credit of Shebin EL-Kom Governorate, reveals that the demand follows the normal distribution. This means that f(x) in equation (1) should be substituted by the probability density function of the normal distribution. A computer program is developed to over come the complexity of the computation giving the optimum stock level at the begining of the season which minimize the expected total inventory cost. The program facilitates the computations at different levels of ordering cost, holding cost, and shortage cost.

2. Multi-Period Models:

These models deal with inventory situations which involve a product that must be reordered periodically. The reordering may occur at any interval of time within the cycle time, the models in this case are called continuous review models. On the other hand, the reordering may occur at fixed intervals of time, which is called periodic review models.

1. Continuous Review Models:

These models can be applied to the stocking of all the strategic goods (wheat and flour in our study) in which the stocking level can not be checked periodically as the demand may exceed the reorder point resulting in shortage which is unpermissable in the strategic goods. In addition the models are suitable for the situations of high probabilistic demands. The continuous review models are based on the following assumptions:

1. Demand is a random variable (discrete or continuous) with known or unknown probability distribution function.
2. Lead time may be a random variable (discrete or continuous) with known or unknown probability distribution or it may be determinstic.
3. The distributions of deamnd and lead time do not change from cycle to cycle.
4. Delivery rate may be infinite (a complete order is received at one time) or it may be finite delivery rate.
5. Cycle time is the number of units of time between two successive orders.
6. The planning period is one year.
7. Backordering is probable, but in the application of this model there will be a high safty stock to overcome shortages.
8. An annual expected demand is given.
9. Costs are associated with placing an order, holding inventory, and stock out costs.

The models are formulated and mathematically treated in such away that one can determine the optimal reorder point (s) and the optimal reorder quantity (Q) that minimize the total cost. In what follows the models are explained in some detail.

Model 1 Fixed Reorder Point-Fixed Reorder Quantity Model.

This model can be one of the two models illustrated in Fig. (1) where Fig. (1-a) represents fixed reorder point-fixed reorder quantity model with infinite delivery rate (complete order is received at one time). Therefore by substituting the ratio $\gamma = \frac{R_c}{R_d}$ (in the formula of reorder quantity) equal 0 gives the optimum reorder aquantity for model (1-a). While Fig. (1-b) represents the fixed reorder point-fixed reorder quantity model with finite delivery rate (delivery occurs during a certain period of time t_d).

The expected total inventory cost can be expressed as follows:

$$ETAIC\ (S,\ Q) = AOC + EAIHC + EASC$$
$$= C_o\ \frac{M}{Q} + C_1\left[S + \frac{Q}{2}(1 - \gamma) - M_L\right]$$
$$+ C_4\ \frac{M}{Q}\left[\sum_{y=s}^{\infty} (y-s)\ h(y)\right] \qquad \text{.......(5)}$$

Where:

- M - Expected demand per unit of time.
- M_L - Expected demand during lead time. ($M_L = M.L$). in case of constant lead time.
- L - Lead time.
- Y - demand during lead time.
- $h(y)$- Distribution function of demand during lead time.
- S - Reorder point.
- $\frac{M}{Q}$ - Number of cycles.
- Q - Order quantity.
- γ - $\frac{R_c}{R_d}$ (where R_c is the consumption rate, R_d is the delivery rate).
- C_o - Ordering cost per order.
- C_1 - Annual inventory holding cost.

C_3 - Purchase cost per item.

C_4 - Shortage cost/unit out of stock.

AOC- Average ordering cost.

EAIHC- Expected annual inventory holding cost.

EASC - Expected annual shortage cost.

ETAIC- Expected total annual inventory cost.

$$\frac{\partial [ETAIC\ (s,\ Q)]}{\partial s} = 0 \quad \text{gives}$$

$$P(Y > S) = 1 - H(s) = \frac{C_1 \quad Q}{C_4 \quad M} \quad ....(6)$$

Where H(Y) - Cumulative demand distribution during lead time.

The optimal S and Q values must be such that the probability that the demand during lead time is greater than s is $\left(\frac{C_1 Q}{C_4 M}\right)$

$$\frac{\partial [ETAIC\ (S\ ,\ Q)]}{\partial Q} = 0 \quad \text{gives}$$

$$\text{Optimal } Q = \sqrt{\frac{2\ M}{C_1(1-\gamma)} \left[C_o + C_4 \left[ENS\ (s)\right]\right]} .....(7)$$

Where:

ENS(s) = Expected number of stockouts per cycle

$$= \sum_{y=s}^{\infty} (y-s)\ h(y) \qquad \text{discrete}$$

$$= \int_{s}^{\infty} (y-s)\ h(y)\ dy \qquad \text{continious}$$

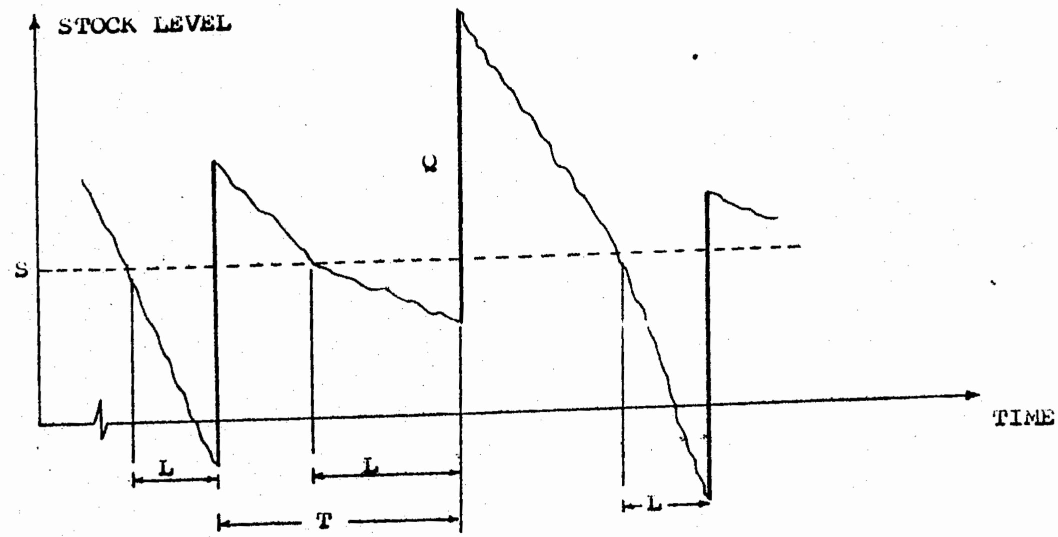
The optimal values of s and Q must satisfy equations (6) and (7) simultaneously.

Model 2 Continuous Review Model With Fixed and Known Lead Time.

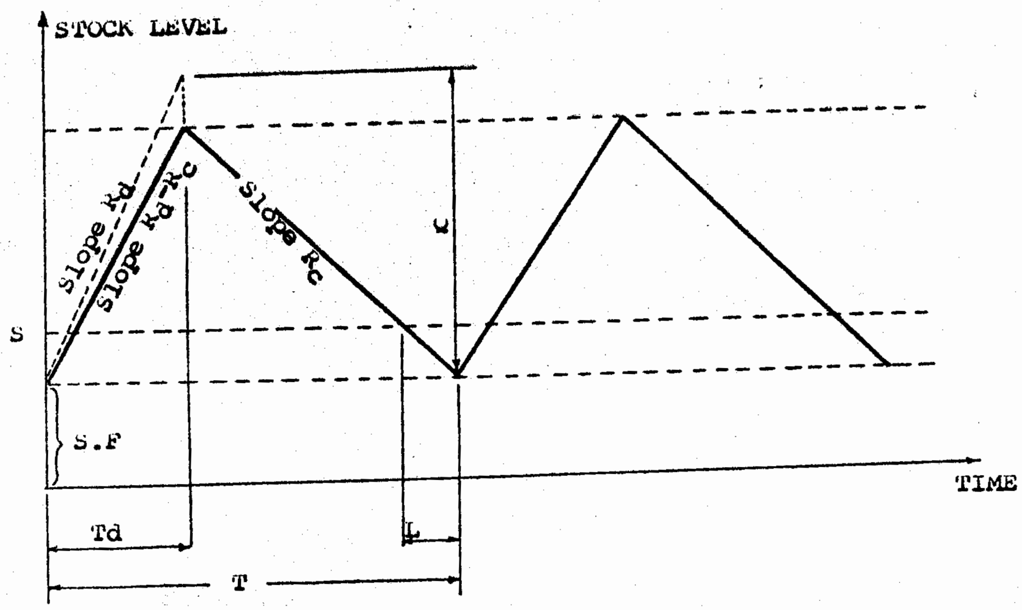
The expected total annual inventory cost per unit of time can be expressed as follows:

$$ETAIC = C_o \frac{M}{Q} + C_3 M + C_1\left(\frac{Q}{2} - ML + s\right) + \frac{M}{2Q}(C_1 L + 2C_4) \sum_{y>s}^{\infty} (y-s)h(y)$$

$$.........(8)$$



a- Infinite delivery rate (R_d = oo, γ = 0)



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b- Finite delivery rate ($\gamma \neq 0$)

FIG. 1 STOCK TIME CHART

s = Optimum reoreder point
L = Lead time
T = Cycle time
S.F = Safty stock

Q = Optimum order quantity
T_d = delivery time
R_c = Average consumption rate
R_d = Average delivery rate.

$$\frac{d(ETAIC)}{dQ} = 0 \text{ gives.}$$

$$\text{Optimal } Q = \sqrt{\frac{2\,C_o\,M}{C_1} + M\left(L + \frac{2C_4}{C_1}\right) \sum_{y>s}^{\infty} (y-s)\; h(y)} \quad \ldots\ldots(9)$$

Define the cumulative demand distribution as

$$H(Y) = \sum_{y=0}^{Y} h(y) \quad \ldots\ldots(10)$$

Then the optimal s is the smallest integer such that

$$H(S) \geqslant K \quad \ldots\ldots(11)$$

$$\text{Where} \quad K = 1 - \frac{2\,C_1\,Q}{C_1ML + 2C_4M} \quad \ldots\ldots(12)$$

The order quantity Q and the reorder point s are optimal if they simultaneously satisfy equations (9) and (10).

The minimum expected total annual inventory cost is given by

$$\text{Minimum ETAIC} = C_3M + \sqrt{2C_1M\; C_o + \left(\frac{C_1L}{2} + C_4\right) \sum_{y>s}^{\infty} (y-s)\; h(y)} + C_1(s-ML) \quad \ldots\ldots(13)$$

The continuous review models were applied to control the stock level of wheat and flour. The data obtained from the provisional department in Shebin EL-Kom, representing the demand of wheat and normal flour, reveals that the demand follows the negative exponential distribution and that the demand of extra flour follows the normal distribution. This means that $h(y)$ should be substituted by the probability density function of of negative exponential distribution in case of wheat and normal flour and substituted by the probability density function of the normal distribution in the case of first class flour.

Computer programs were developed for single period model and continious review models (model 1 and model 2). The program of model 1 was provided with a subroutine to determine the distribution of demand during lead time $h(y)$ by simulation if it is unknown. Figures (2) and (3) illustrates the flow charts of the computer programs.

The following notations to be used in the flow chart:

CO	- Ordering cost.
C1	- Inventory holding cost/unit per unit time.
C3	- Purchase cost per unit.
C4	- Shortage cost/unit out of stock.
IOH	- Inventory on Hand before placing an order.
HIGX	- Maximum demand.
LOWX	- Minimum demand.
PDF	- Probability density function of demand x.
CDF	- Cumulative distribution function of demand x.
DIL	- Dersired inventory level at the start of the period.
DILSTAR	- Optimum desired inventory level.
A	- Critical value for determining if an order should be placed.
STARMU	- Expected total cost as a function of demand and DIL, CO.
TESTMU	- Expected total cost as a function of demand and DIL.
M	- Annual expected demand.
$Y = \frac{RC}{RD}$	- Where RC is the consumption rate, RD is the delivery rate.
KNTY	- Number of different values of demand during lead time.
HSMALL(Y)	- Probability distribution function of demand during lead time.
HLARGE(Y)	- Cumulative distribution function of demand during lead time.
Y	- Demand during lead time.
PYGROP	- Probability that the demand Y is greather than reorder point.
ETAIC	- Expected total Annual inventory cost.
ROP	- Reorder point.
OPTROP	- Optimum reorder point.

QSTAR - Maximum order quantity.

OPTQ - Optimal order quantity.

EXPECY - Expected demand during lead time.

KNOWN - 0 if the demand during lead time is unknown.

- 1 if the demand during lead time is known.

DDDLT - A subroutine used for the determination of distribution of demand during lead time in which the data are the demand (x) and lead time (L).

Fig.(2) FLOW CHART OF A SINGLE PERIOD MODEL

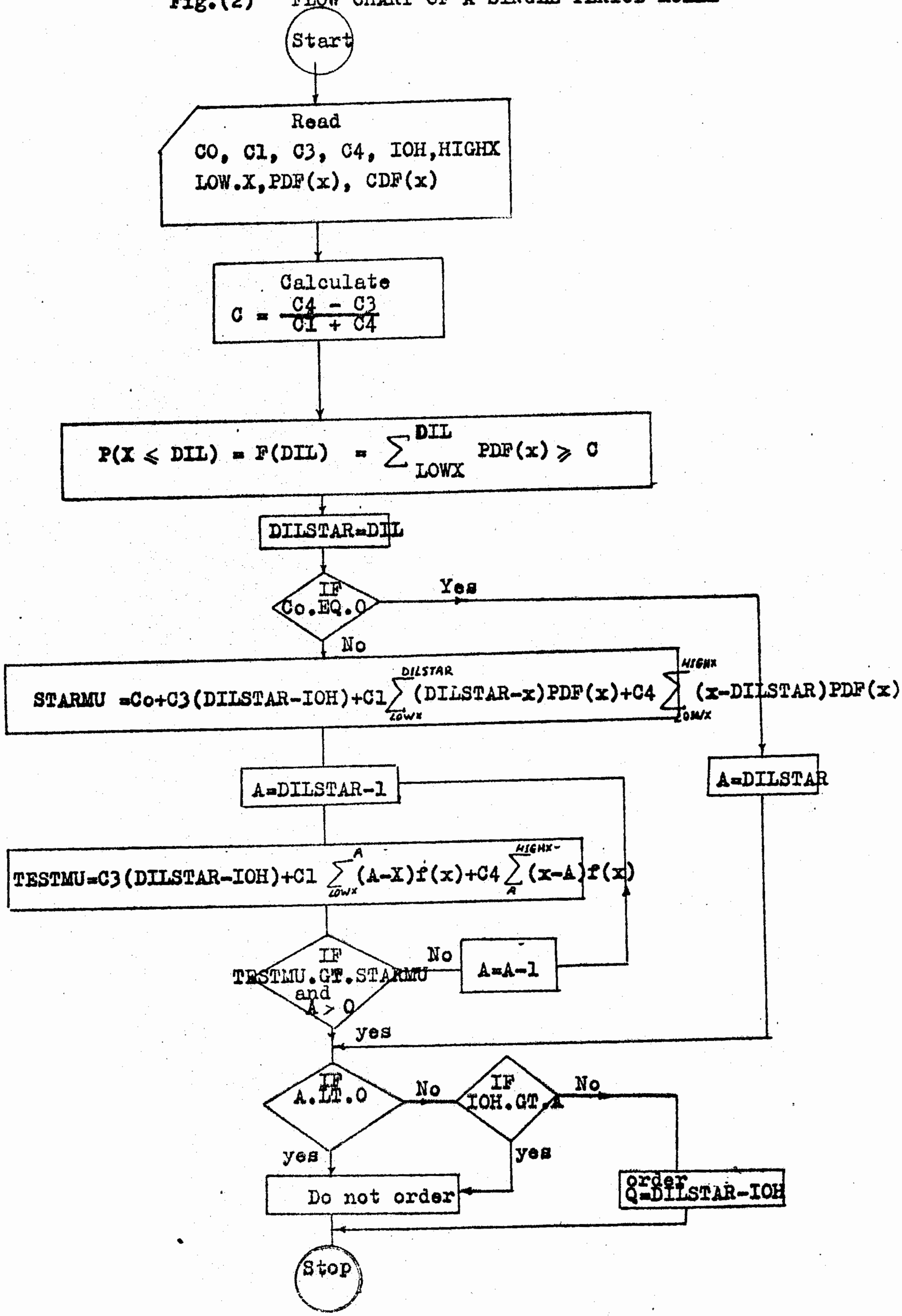
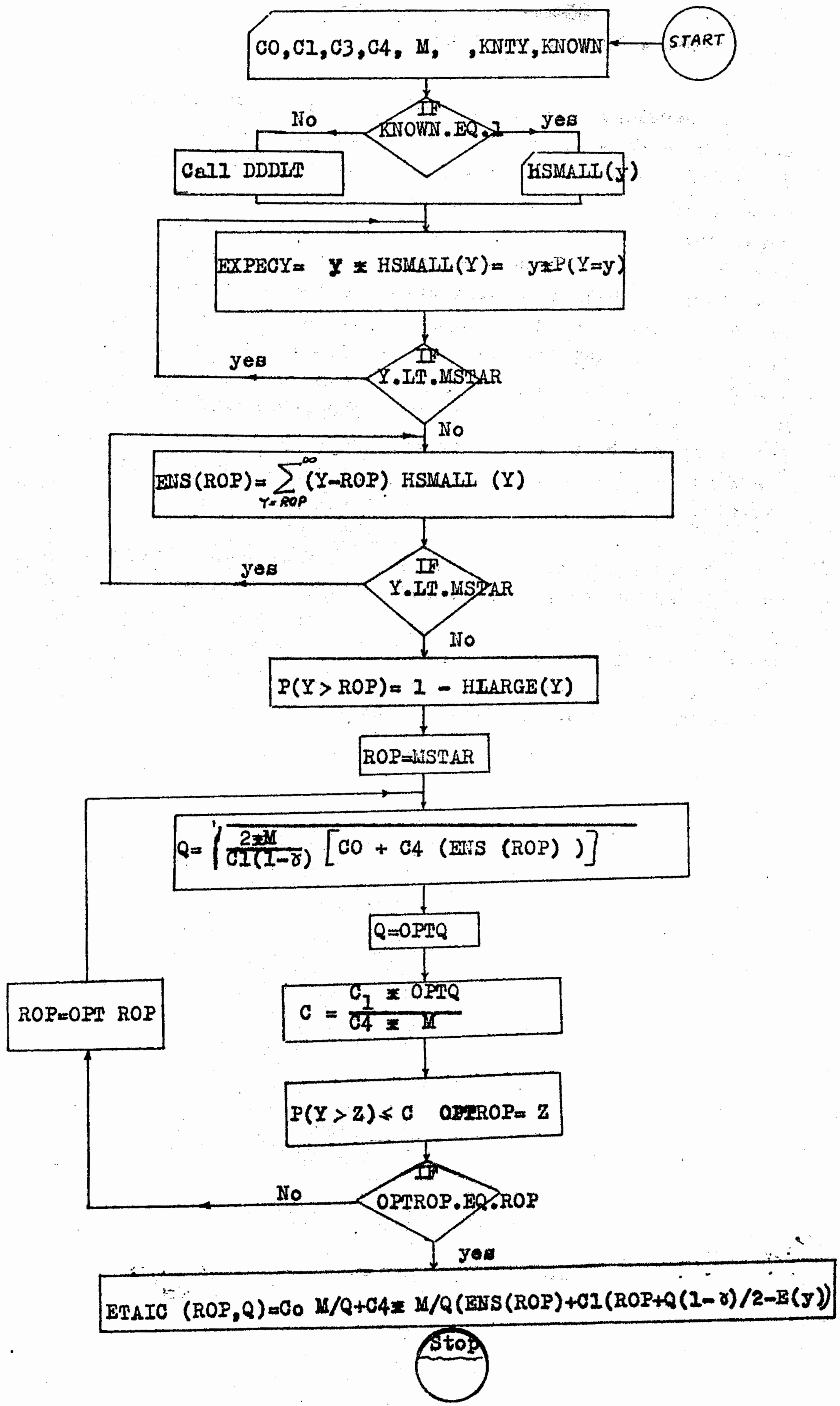


Fig.(3) FLOW CHART OF A CONTINUOUS REVIEW MODEL 1



CONCLUSIONS & RECOMMENDATIONS:

It develops from the preceding study that the model selected to be applied should take into consideration the conditions, the limitations and the assumptions of the case of application. In other wards, it is not allowed to apply any inventory model to a certain case without studying the validity of the model to the case. It is concluded also that the nearest distribution of demand has to be found by statistical methods in order to calculate to a considerable degree of approximation the economic order quantity and the recorder point. The ordering cost per order, the holding cost per unit per period and the shortage cost per unit per period should be evaluated as accurately as possible since the final result of optimization depends mainly on these evaluations.

It was found that the single period inventory model matchs the case of stocking insecticides and the continuous review inventory models match the case of stocking wheat and flour. However, it is recommended to study the case of stocking vegetables, fruits, seeds and fertilizers in order to find the suitable inventory model.

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تطبيق نظام التخزين فى المخازن الزراعية

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يعتبر التخزين من المجالات الهامة فى مجال بحوث العمليات خاصة بعد التوسع الهائل فى استخدام الحاسب الالكترونى ولقد اهتمت معظم الأبحاث فى هذا المجال فى تطبيق السياسات المثلى فى التخزين فى مجال الصناعة والتجارة وأهملت تطبيق هذه السياسات فى مجال تخزين الحاصلات الزراعية ٠

ان هذا البحث يهدف الى ايجاد نماذج التخزين المناسبة لبعض المخازن الزراعية وهذا يتطلب دراسة النماذج الرياضية المختلفة للتخزين والافتراضات المبنية عليها وقيودها وطريقة حسابها ٠ هذا بالاضافة الى دراسة ظروف وطبيعة المخزون الزراعى موضوع البحث ٠

وتتناول الدراسة الآتى :-

١ - طريقة التخزين ٠
٢ - نظام الامداد والاستهلاك٠
٣ - أهمية المنتج وتأثير العجز فى المخزون على الاستهلاك٠
٤ - احتمالات التقادم والفساد نتيجة لطول فترة التخزين٠
٥ - التوزيع الاحصائى للاستهلاك٠

ان هذا البحث تم تطبيقه على بنك التنمية والائتمان الزراعى بشبين الكوم حيث يقوم البنك بالأنشطة التالية :-

١ - نشاط تموينى ويختص بتخزين السلع التموينية والاستراتيجية مثل القمح والدقيق والفول والذرة ٠
٢ - نشاط زراعى ويختص بتوفير مستلزمات الانتاج الزراعى مثل المبيدات الحشرية والأسمدة والتقاوى ٠
٣ - نشاط تنمية الثروة الحيوانية ويختص بتوفير الكسب والعلف لتغذية الماشية وكذلك توفير الادوات اللازمة للميكنة الزراعية ٠

ولقد تناولت الدراسة تطبيق نماذج تخزين على القمح والدقيق والمبيدات.

١ - القمح - الدقيق :

يعتبر القمح والدقيق من المواد الاستراتيجية الهامة فى مصر والتى يجب حساب الحجم الأمثل لتخزينها وكذلك الحجم الأمثل الذى يتم عنده اعادة الطلبات ولقد تبين من دراسة النماذج المختلفة أن نموذج التحقق من مستوى التخزين الدورى هو أفضل النماذج حيث أن هذا النموذج ينص " عندما يصل مستوى التخزين الى الحجم الأمثل لاعادة الطلب يتم عمل طلب لرفع مستوى التخزين الى الحجم الأمثل للتخزين" وهذا يحقق أقل تكلفة فى التخزين مع الأخذ فى الاعتبار وجود مخزون احتياطى دائم حتى لا يحدث عجز فى المخزون.

ومن البيانات التى تم الحصول عليها للاستهلاك الشهرى من القمح والدقيق " بلدى وفاخر " وبعد تحليلها احصائيا لايجاد أحسن توزيع احصائى يناسب هذه البيانات وجد أن القمح والدقيق البلدى يتبع نظام التوزيع الأسى بينما الدقيق الفاخر يتبع التوزيع الطبيعى.

٢ - المبيدات :

تستخدم المبيدات فى وقاية المحاصيل من الآفات الضارة وفى علاج المحاصيل المصابة بالآفات أو الأمراض. وحيث أن القطن يعتبر من المحاصيل الهامة فى مصر فان البحث يعمل على ايجاد الحجم الأمثل لتخزين أحد المبيدات المستخدمة فى رش القطن " الدايمثويت" . ولأن الآفات يحدث لها مناعة من كثرة استخدام المبيد لذلك يجب أن يتم تغيير المبيد كل موسم زراعى وهذا يتناسب مع المبيدات التى يتم استحداثها كل فترة لذلك فان النموذج الرياضى لنظام التخزين ذو الفترة الواحدة يصلح لتخزين المبيدات حيث أنه يطبق على المخزون الموسمى أو المخزون القابل للتلف - حيث فترة التخزين تكون قصيرة .

ومن البيانات التى تم الحصول عليها عن استهلاك المبيد السابق ذكره وجد أنه يتبع التوزيع الطبيعى.

ولقد تم التغلب على صعوبة الحسابات لايجاد الحجم الأمثل للطلب والحجم الأمثل لاعادة الطلب وأقل تكلفة للتخزين وذلك بعمل برنامج للحاسب الآلى يسهل الحسابات ويتيح الفرصة لدراسة تأثير العوامل المختلفة على التكاليف الاجمالية.