BOUNDARY ELEMENT ANALYSIS OF TRANSMISSION LINES FOR TRANSVERSE ELECTROMAGNETIC WAVES

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Abstract- The boundary element method is used to compute the characteristic impedance and capacitance matrix of various types of transmission lines supporting transverse electromagnetic waves. Multiconductor lines between infinite ground planes are treated by combining the numerical solution with appropriate analytical representation of the potential in the conductor-free region. Comparison is made with available published results.

I INTRODUCTION

Various types of transmission lines supporting transverse electromagnetic (TEM) waves have been developed and are widely used in high frequency communication systems. However, only few types with simple or highly symmetric structure do have exact formulae expressing the electric characteristics of the line in terms of its geometrical parameters. Considerable effort has therefore been spent on approximate and numerical techniques including, for example, approximate conformal mapping methods [1-6], variational methods [7], Fourier transform methods [8], finite differences [9], numerical methods based on Green's function integral equation approach [10-15], ...etc. A considerable body of literature has accumulated in the last few decades and a literature review may be found, for instance, in references [1-15].

Recently, the boundary element method (BEM) has been introduced and efficiently applied to many potential problems involving Laplace's equation [16]. Beside its ability as a numerical method to deal with quite arbitrary geometric structures, it is often more memory-economic as compared with its rival finite differences and finite elements methods. This advantage is due to the fact that the BEM confines the analysis of a homogeneous domain to the boundaries of the domain, and so the problem is in effect treated with one less dimension. Another advantage of the BEM is that, unlike other techniques based on integral equation formulation, the BEM does not require a knowledge of the Green's function of the particular problem under consideration; the BEM formulation involves only the free space (unbounded) Green's function, which for Laplace's equation is the well known logarithmic potential.

The present work has, therefore, been set to make use of the BEM as a basis for computer code (s) suitable for use on a relatively small computer system and applicable to a wide class of TEM-tranmission lines. These include

1. Multiconductor lines in free space or inside a tubular conducting shield (boxed lines). The familiar parallel-wires and coaxial lines are typical examples of this class.
2. Strip lines consisting of one or more strips arbitrarily located between two parallel ground planes.
3. Shielded microstrip lines (either boxed-line or a microstrip over a substrate of finite width between two infinite ground planes).

In all cases, the conductors are assumed to extend infinitely, parallel to the z-axis, but their cross-sections in the xy-plane can be of quite arbitrary shape.

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II PARAMETERS OF MULTICONDUCTOR TEM-TRANSMISSION LINES

This work is concerned with the computation of the distributed parameters (capacitance and inductance per unit length) as well as the characteristic admittance and coupling coefficients for TEM-transmission lines. For the sake of generality, we shall assume a multiconductor line. It is well known that the essential parameters of such a line are the elements of the Maxwellian capacitance matrix C, from which the characteristic admittances for the various operating TEM modes and the coupling coefficients can be easily obtained [11-15]. The capacitance matrix is defined as follows. Consider a system of n-conductors (plus ground) with potentials $V_1, V_2, ..., V_n$, and let $Q_1, Q_2, ..., Q_n$ be the corresponding total charges per unit length on the conductors. The potentials and charges are then related by

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_n \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \cdots C_{1n} \\ C_{21} & C_{22} \cdots C_{2n} \\ \cdots & \cdots\cdots \\ C_{n1} & C_{n2} \cdots C_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdots \\ V_n \end{bmatrix} \qquad \ldots (1)$$

Reciprocity and conservation of charge imply the following known properties of the elements of the C-matrix [14,15].

$$C_{ij} = C_{ji}$$
$$C_{ii} > 0, \quad C_{ij} < 0 \quad i \neq j \qquad \ldots (2)$$
$$\Sigma C_{ij} \geq 0$$

The equality sign in the last equation holds for a 'closed system', where the summation runs over the grounded conductor. This is the case, for instance, of a boxed line.

Similarly one can define the inductance matrix **L** which relates the fluxes and currents in the various conductors. It can be shown that, for a system of n conductors in a homogeneous medium

$$LC = \mu\epsilon\, I \qquad \ldots (3)$$

where I is the unit matrix of order n and u and ϵ are the permeability and permitivity of the medium, respectively.

The characteristic admittances are also readily obtained from C [11]. If but a single conductor is involved (n = 1), the admittance is given by

$$Y = v\,C \qquad \ldots (4)$$

where v is the phase velocity of TEM-waves in the surrounding medium. In the case of a microstrip over a dielectric substrate

$$Y = \sqrt{C_d / C_o}\; Y_o \qquad \ldots (5)$$

where C_d is the capacitance per unit length of the actual microstrip structure and C_o and Y_o are the capacitance and characteristic admittance of the structure with air as dielectric.

When two conductors are involved (n = 2), there are two basic modes of operation: an odd mode ($V_1 = 1$, $V_2 = -1$), and an even one ($V_1 = V_2 = 1$). From (1), the corresponding total charges are

$$Q_{iodd} = C_{i1} - C_{i2} \quad i = 1,2$$
$$Q_{ieven} = C_{i1} + C_{i2} \qquad \ldots (6)$$

The line admittances for these modes are, therefore,

$$Y_{iodd} = v\,(C_{i1} - C_{i2}) / 2$$
$$Y_{ieven} = v\,(C_{i1} + C_{i2}) \qquad \ldots (7)$$

For $n > 2$, the various line admittances for a particular mode of operation may be obtained in a similar way.

III BOUNDARY ELEMENT ANALYSIS

Following Collin [17], the elements of the capacitance matrix for a multiconductor line are obtained as follows. Let u (x, y) be the potential function satisfying the following boundary value problem

$$\nabla^2 u_j = 0 \quad \text{outside the conductor}$$
and
$$u_j = 1 \quad \text{on the conductor } j$$
$$= 0 \quad \text{on all other conductors} \qquad \ldots (8)$$

It then follows from (1) and Gauss's theorem that

$$C_{ij} = Q_i = -\epsilon \int_{Bi} q_i \, dc \qquad i, j = 1, 2, \ldots, n \qquad \ldots (9)$$

where the integration is carried out along the boundary Bi of the i th conductor and q_j is the outward normal derivative of u .

The determination of the capacitance matrix, therefore, requires finding n potential functions of the type given by equation (8). This repeated solution of the same potential problem for different boundary conditions is considerably simplified in the boundary element method, which reduces the boundary value problem to one of solving a system of linear equations with the source term being the known values of the potential and its normal derivative at the boundary nodes. If the system matrix is decomposed, for instance, using a Gaussian elimination technique and stored in triangular form then, getting a new solution for a new set of boundary values amounts only to a simple back-substitution process.

The BEM solves the boundary value problem (8) as follows. By means of the weighted residual method or Green's identity, the differential equation (8) can be transformed into the following integration

$$u_p + \int_B hu \; dc = \int_B gq \; dc \qquad \ldots (10)$$

where g is the free-space (unbounded) Green's function and h is the corresponding normal derivative. It is well known that $g = 1/2\pi \ln (1/r)$. The subscript p in equation (10) means an arbitrary point inside a homogeneous region R where u satisfies Laplace's equation and B is the boundary of this region. In the special case when p lies on B, equation (10) reduces to

$$cu + \fint_B hu \; dc = \fint_B gq \; dc \qquad \ldots (11)$$

where $\fint$ denotes the Cauchy's principal value of the integral, $c = a/2\pi$, and a is the inner domain angle at p.

Equations (10) and (11) are the basic equations in the BEM analysis. The first of them is used to compute the fields at any internal point once the potential and its normal derivative are known on the boundary, These boundary values are in turn obtained by solving the boundary integral equation (11). To this end, the boundary B is discretized in the usual isoparametric finite element manner into, say, N-elements and the unknown functions u and q are approximated by suitable polynomials over each element. In the present work we confine ourselves to linear polynomials. Accordingly, equation (11) is reduced to the discrete form

$$c_i u_i + \Sigma h_{ij} u_j = \Sigma G_{ij} q_j \qquad i = 1, 2, \ldots, N \qquad \ldots (12)$$

where h_{ij} and g_{ij} are made up of integrals over the two elements through node i of appropriate products of Green's functions and the interpolation polynomials. Detailed expressions for the various quantities and method of computation are given, for instance, in refernce [16]. In matrix form equation (12) becomes

$$H\ u = G\ q \qquad \ldots (13)$$

where $u = (u_1, u_2, \ldots, u_n)^T$ and $q = (q_1, q_2, \ldots, q_n)^T$. Matrices G and H depend solely on the geometry of the region R, while vectors u and q comprise the values of the potential and its normal derivative at the boundary nodes of R. Some of these values are given from the boundary conditions or, in some cases , can be deduced from the symmetry of the problem if any. The remaining boundary values are obtained by solving the linear system (13). Next we consider some numerical examples with known analytical or numerical solution to compare with.

IV NUMERICAL EXAMPLES

1- Boxed Lines and Open-wire Lines

The BEM equations in this case are set for the region outside the conductors with the path of integration as shown in Fig. 1. In the case of a boxed-line, the outermost boundary B_0 coinicides with the inner surface of the shield. However, for a system of conductors in free-space, B_0 is taken to be a circle of infinitely large radius. Assuming that the potential is regular (zero) at infinity, the contribution of B_0 to the line integral vanishes in the latter case and the path of integration is confined to the conducting boundaries.

Table 1-3 present the capacitance of a coaxial system utilizing conductors of both circular and square cross-sections. The BEM results are compared with those obtained by Lin using conformal mapping techniques [4]. In case 1 these techniques give an exact result, while in cases 2 and 3 they give only upper and lower limits for the characteristic impedance. The BEM values lie within these limits and satisfactory agree with the exact values in most cases. Also the BEM has been used to compute the characteristic impedance when one of the conductors (or both) is rectangular with its diagonals tilted at an arbitrary angle with respect to the other conductor. Such results are useful, for example, when the structure is used as a coaxial reflection standard [10].

Table 4 gives the capacitance matrix and the odd-mode characteristic impedance of a shielded two-wire line. The accuracy of the BEM results are rather satisfactory as may be seen by checking the properties of the capacitance coefficients (equation (2)) and by comparing the computed values of the characteristic impedance with those calculated from the well-known analytical expression [18].

Table 5 gives the capacitance matrix of a two-coductor boxed strip line computed using the finite difference method and the BEM. In the first case, a mesh size of 100x20 is used and the corresponding system of linear equations is solved using a successive over relaxation technique. With the BEM, however, a total of 100 elements on inner and outer conductors have been found sufficient to give results of comparable accuracy. Such a relatively small grid size can be easily handled on a moderate computer system using, for instance, a simple Gaussian elimination algorithm.

2- Multiconductor Lines Between Parallel Ground Planes

This type of transmission lines is an example of potential problems where the boundaries involved extend to infinity. In boundary elements, as well as in finite elements analyses, these infinite boundaries have been dealt with in various ways:

simple truncation, use of infinite elements, or by combining numerical methods with appropriate analytical ones. Simple truncation, by introducing end walls at far enough distance, reduces the problem to one of the type considered in the previous subsection but eventually increases the mesh size. Also, it has been found that the results depend on whether Dirichlet's or Neumann's condition is imposed on the artificial end wall [19]. On the other hand, the use of an element shape function extending to infinity and including an exponential decay does not involve much increase of the number of elements, but the choice of the rate of exponential decay is somewhat arbitrary and may affect the result [20].

The third approach, the one adopted here, consists in the following. As shown in the figures accompanying Tables 5 and 6, the entire domain between the ground planes is divided into two parts: an interior region I enclosing the conductors and bounded by the rectangular contour B_r, and an outer region II extending to infinity. Inside this latter homogeneous region the potential is represented by

$$u = \Sigma A_m \exp(-m\pi|x| / L) \sin(m \pi y / L)$$
$$q = \Sigma A_m m \pi / L \exp(-m\pi |x| / L) \sin(m \pi y / L) \qquad \ldots (14)$$

In the interior domain, the BEM is applied with the variation of the functions along the boundary elements assumed as linear. This means that

$$u(y) = \Sigma U_n \phi_n(y)$$
$$q(y) = \Sigma q_n \phi_n(y) \qquad \text{on AB} \qquad \ldots (15)$$

where ϕ_n is a triangular pulse function with a unit peak at node n and zeros at nodes n+1 and n-1, respectively. The BEM equations are then formed in the usual way. However, since neither the potential nor its normal derivative is specified along the interface AB between regions I and II, the number of unknowns would exceed the number of BEM equations unless further enough relations between the variables are provided. This is done by 'matching' the solutions in the exterior and interior domains along AB.

One method is to set the potentials and their normal derivatives at the nodal points (i.e. the set of u_n and q_n values in equations (15)) equal to the corresponding values obtained from equations (14). Although this process ensures the coincidence of the exterior and interior solutions at the nodal points, the two solutions may vary considerably over the whole element unless the element lentgth is too small, which means an excessively large number of nodes and an increased memory requirement. The difference, or mismatch, between the two solutions results in a discontinuity in potential across the element and is therefore physically equivalent to the introduction at the interface of a double layer which may result in an unacceptable error in potential and charge distributions. It is therefore necessary that, for a given number of nodes, the difference between the interior and exterior solutions along the common interface be kept a minimum, for instance, in the least square sense. This can be effected by minimizing the error function

$$F = \int (u_e - u_i)^2 + w(q_e - q_i)^2 \, dc \qquad \ldots (16)$$

where w is some weighting function and the integration is along the interface AB. Substituting from equations (14) and (15) into (16) and equating the partial derivatives with respect to the nodal values $[u_n]$ and $[q_n]$ to zero, we get after eliminating the expansion coefficients $[A_n]$

$$(q_1, q_2, \ldots, q_{11})^T = C(u_1, u_2, \ldots, q_n) \qquad \ldots (17)$$

where C a square matrix of order n.

The linear relation (17) between the nodal values on the interface AB and the vanishing of the potential on the upper and lower ground planes provide a sufficient set of boundary conditions for solving the BEM equations in the interior region II.

For the purpose of comparison, we have applied the above approach to compute the capacitance matrix for the structure shown with Table 5, but with the end walls removed. The results are given in the same table together with the values obtained by Kammler using a Green's function integral equation technique [11]. The BEM gives slighgtly higher values, presumably because we have assigned a very small (but non-zero) thickness to the strips. This has been found computionally more convenient as it allows the use of the routines developed for the boxed line with slight modifications and avoids troubles caused by singularities at the ends of an infitismally thin strip (although in principle these singularities can be dealt with in the manner discussed below).

Table 6 presents another example taken from reference [13] which uses a Green's fuction integral equation approach similar to the one developed earlier by Kammler [11]. The same example has been solved in a recent work using a moment. method [15]. The BEM results are closer to the results of refrence [13] than to those of the moment method in which the upper ground plane had to be truneated at a finite width.

Finally, we present in fig. 2 an example of an infinitesmally thin microstrip over a dielectric substrate of finite width; a practical case whose analysis has received much less attention than the infinite-width substrate [21]. The BEM analysis of this problem is carried out in the manner outlined above with the integration path taken as shown in Fig. 2. The singularity due to the sudden change in boundary condition at the edge of the microstrip has been treated by refining the mesh near this edge and by using a shape interpolation function proportional to the square root of the distance from the edge, so as to satisfy the edge condition. This treatment of singularities in BEM is discussed in detail in reference [22]. Examination of the results presented in Fig. 2 shows that, unlike the narrow substrate, the BEM results for the relatively wide substrate do not differ appreciably from the corresponding values for a microstrip over an infinite substrate reported in earlier work [3, 8]. The present method can therefore be used for the analysis of the infinite-width case, provided the substrate is truncated at an appropriate width.

V CONCLUSION

Beside its simplicity and memory-economic character, the boundary element method is capable of treating a wide class of transverse electromagnetic transmission lines with an accuracy comparable with other numerical and analytical techniques. Problems of shielded strip and microstrip lines involving infinite boundaries can be treated by combining BEM analysis with an appropriate expansion of the potential in the conductor free region without resort to infinite elements, the accuracy of which may depend on arbitrary chosen parameter. In principle, the present techniques can be applied to problems involving multilayered dielectric media. Assessment of the method in such applications and a comparison with other techniques from the point of view of accuracy and memory requirements will be considered in a future study.

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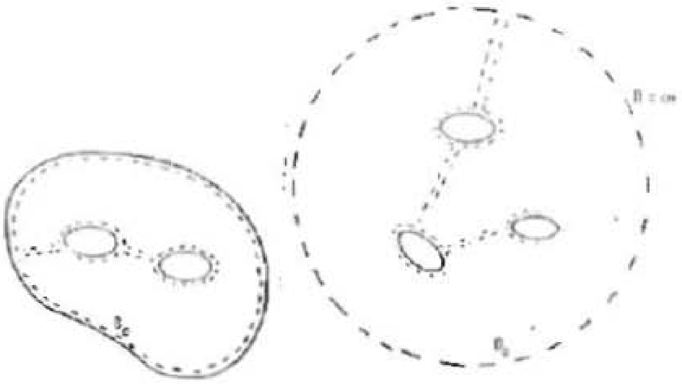


Fig. 1 Shielded and open-wire TEM transmission lines. The dashed line is the BEM path of integration.

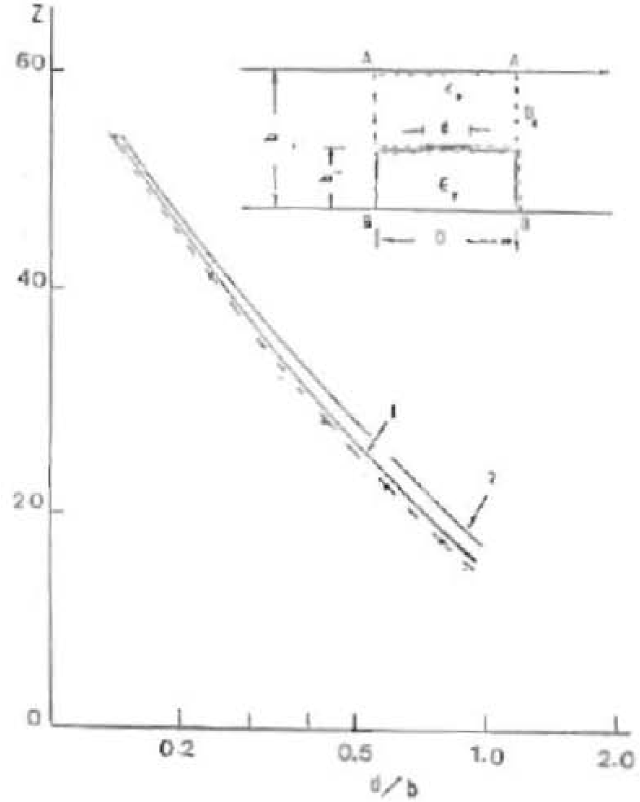
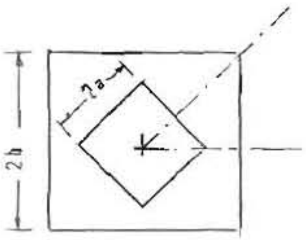
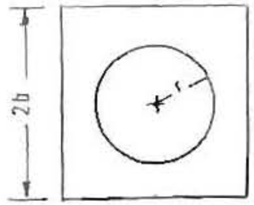


Fig. 2 A microstrip line problem. = 9.9, b / b-1/6. Curves 1 and 2 are BEM values for d/D=0.2 and 0.8, respectively. The dots and the dashed line are values for an infinite-width substrate (refrences [3,8]).



a / b	z_o (exact)	z_o (BEM)
0.638732	20.0	18.519
0.547498	30.0	29.244
0.464000	40.0	39.853
0.388874	50.0	50.709
0.323103	60.0	61.971
0.232320	80.0	81.846
0.168230	100.0	101.226
0.121257	120.0	120.864

Table 1. Exact (reference [4]) and BEM characteristic impedance of a coaxial line of square inner and outer conductors.
Number of boundary elements N = 60.



r / b	z_{min}	z_{max}	z (BEM)
0.239085	90.315	90.372	0.401
0.399780	69.199	69.319	9.291
0.445150	52.840	53.217	3.129
0.557536	39.139	39.999	9.569
0.676602	27.031	28.912	7.970
0.	15.878	19.563	7.552
0.933520	05.238	11.981	7.650

Table 2. Lower (Z_{min}) and upper (Z_{max}) limits (reference [4])and boundary element value of the characteristic impedance (ohms) of a coaxial line of square outer conductor and circular inner conductor. N = 60.

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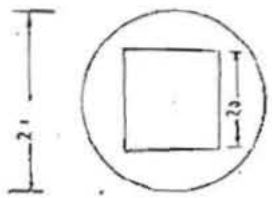
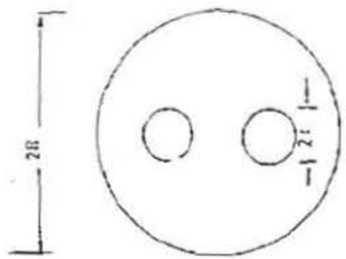


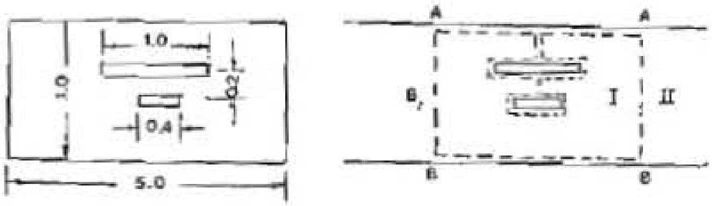
a / r	Z_{min}	Z_{max}	Z (BEM)
0.10	128.105	128.106	127.979
0.20	86.519	86.579	86.498
0.30	62.083	62.396	62.106
0.40	44.430	45.474	44.740
0.50	30.287	32.737	31.268
0.60	17.557	22.851	19.900
0.70	02.619	15.0764	06.010

Table 3. Lower and upper limits (reference [4]) and BEM value of the characteristic impedance of a coaxial line of circular outer conductor and square inner conductor. N = 60.



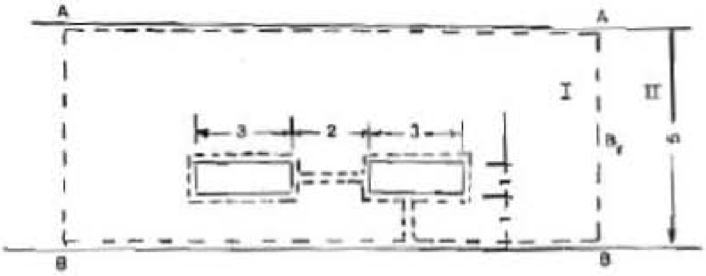
	Capacitance Matrix C / c_o			Z_o (ohms)
R = 4.0	2.0054	-0.7658	-1.2395	
r = 1.0	-0.7658	2.0054	-1.2395	Exact value 271.42
	-1.2399	-1.2399	2.4799	BEM value 271.89
R = 2.0	2.2992	-0.5904	-1.7089	
r = 1.0	-0.5904	2.2992	-1.7089	Exact value 260.38
	-1.7096	-1.7096	3.4192	BEM value 260.75

Table 4. Capacitance matrix and odd mode characteristic impedance Z_o of a shielded two-conductor line. N = 60.



		Capacitance Coefficients C / ϵ_0			
		c_{11}	c_{12}	c_{21}	c_{22}
Boxed-Line	Finite Diff.	8.1688	-3.2416	-3.2399	5.0721
	BEM	8.1568	-3.1686	-3.1750	4.9410
Open-ended Line	Integral Eqn.	7.9039	-3.0640	-3.0640	4.8074
	BEM	7.9636	-3.1024	-3.0978	4.8465

Table 5. Elements of the capacitance matrix of a two-conductor shielded strip line computed by the BEM, finite differences, and integral equation method of reference [11]. N = 100.



	Integral Eqn. [13]	Moment Method [15]	BEM
C_{11}	63.07	62.64	64.21
C_{12}	-05.866	-05.724	-05.923
C_{21}	-05.866	-05.724	-05.923
C_{22}	63.07	62.64	64.21

Table 6. Elements of the capacitance matrix ($\times 10^{12}$ F/m) for a shielded two-conductor line. N = 100.