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RELIABILITY MEASURES FOR UNCORRELATED AND CORRELATED OBSERVATIONS

معايير الثقة للأرصاد المرتبطة وغير المرتبطة

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خلاصة :

في هذا البحث تم إستعراض أهم معايير الثقة الداخلية والخارجية في حالة توافر الأرصاد المرتبطة والغير مرتبطة . وقد كان ما يطلق عليه الرقم الوفير (redundancy number) هو من أفضل وأشهر معايير الثقة الداخلية للشبكات الجيوديسية ، وكانت قيمته العددية تتراوح ما بين الصفر والواحد الصحيح . وقد لوحظ في حاله ما إذا كانت الأرصاد مرتبطه مع بعضها أن القيمة العددية للمعيار تتعدى هذه الحدود ، وعليه فإن هذا المعيار لا يكون مناسبا لبيان مقدار و درجة الثقة الداخلية والخارجية للشبكات الجيوديسية . ويهدف هذا البحث إلى إيجاد معيار جديد لبيان وصف درجة الثقة الداخلية والخارجية للشبكات الجيوديسية التي تكون أرصادها من النوع المرتبط ، وقد تمت المقارنه بين المعيار الجديد المقترح والمعايير السابقة على ثلاثة مسائل مساحية مختلفة (شبكة ميزانيات دقيقة وشبكة مقيسة الأضلاع ومسأله رصد زوايا أفقية بطريقة كل الأتجاهات) لتوضيح مدى كفاءه وشمولية وملائمه المعيار الجديد للتطبيقات المساحية المختلفة .

ABSTRACT

The redundancy number is usually used as the internal reliability measure. It is commonly stated in literature that the value of redundancy number ranges between zero and one. This study, however, indicates that the redundancy number may be beyond that range for correlated observations. Therefore, the internal reliability can not correctly be indicated by the redundancy number. A new suitable internal reliability measure for correlated observations is proposed . In this research, the relationship between the redundancy number and the proposed internal reliability measure is discussed thoroughly.

Finally, three numerical examples representing different surveying problems such as, vertical and horizontal geodetic networks as well as theodolite angle observations were given in order to illustrate the application and the efficiency of the proposed reliability measure.

KEYWORDS: **Correlated observations ; Internal reliability , Redundancy number , Marginal detectable error ; External reliability ; Redundancy matrix , Reliability matrix ; Reliability number ; Internal reliability factor ; Absorption factor.**

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1. INTRODUCTION:

The reliability theory developed by BAARDA (1968) has been successfully used to evaluate the ability to control observations called the internal reliability, and the influence of non detectable outliers on the unknowns of an adjusted system, called the external reliability. In order to interpret the concept of reliability, several studies on the relationship between the reliability and geometry of a network have been made, summarized as the theory of residuals [1,3] . According to this theory, the larger the redundancy number of an observation is, the better the outlier in the corresponding observation will be revealed in its residual, and thus, easier to be detected. Therefore, the redundancy number is widely used as a reliability measure. However, this is appropriate only if the observations are not correlated. For correlated observations the situation turns out to be completely different as pointed out by WANG and CHEN (1994) . The redundancy number gave not only negative values but also values bigger than one.

In this paper, firstly the theory of residuals is reviewed. Then internal and external reliability measures for uncorrelated observations are summarized. In addition, a formula of the new reliability number for correlated observations is derived as a function of the multiple correlation coefficient, from which the range of the new reliability number is discussed. Finally, the internal and external reliability measures for correlated observations are investigated and discussed.

2. GAUSS MARKOV MODEL AND RESIDUALS THEORY

The Gauss Markov model of an adjusted system are determined based on the following functional model , which is given in linear form by

$$v = A\,x - l \tag{1}$$

And the stochastic model

$$P = Q_l^{-1} = \sigma_0^2\ C_l^{-1} \tag{2}$$

Where v is the nx1 vector of residuals, A is the nxu coefficient matrix, (which is also commonly known as the configuration matrix or design matrix) , x is the ux1 vector of unknown corrections to the approximate coordinates; L is the nx1 vector of the absolute terms; P is the nxn weight matrix of the observations; Q_l is the nxn cofactor matrix of the observations, C_l is the nxn covariance matrix of the observations, and σ_0^2 is the a priori variance factor. The best estimate for the least squares solution vector x and the corresponding cofactor matrix of the estimated coordinates can be computed as follows [7,10] :

$$x = (A^T PA)^{-1} A^T P\,l = N^{-1} A^T P\,l \tag{3}$$

and

$$C_x = \sigma_0^2\ Q_x = \sigma_0^2 (A^T PA)^{-1} \tag{4}$$

Where Q_x is the weight coefficient matrix of the estimated coordinates and

σ_0^2 is the a posteriori variance factor.

The residual vector of observations estimated from a least squares adjustment and the corresponding cofactor matrix can be expressed as :

$$v = (A Q_X A^T P - I)\, l = R\, l \tag{5}$$

and

$$Q_V = P^{-1} - A Q_X A^T \tag{6}$$

In which $R = A Q_X A^T P - I$ is called , in the statistical literature, the redundancy matrix [1,3] . Equation (5) describes a relationship between an observation l_i and its residual v_i . Let v^*_i be the additional residual due to existence of an outlier Δl_i in the observation . The impact on the residual of observation l_i can be given as follows :

$$v^*_i = -(Q_V P)_{ii}\, \Delta l_i \tag{7}$$

3. RELIABILITY MEASURES FOR UNCORRELATED OBSERVATIONS

3.1 Internal Reliability Measures

The internal reliability of an adjusted system is referred to as the controllability of the observations described by the lower bounds for outliers, which can just be detected at the given significance level α_0 and power of the test β_0.

BAARDA (1968) and ALBERDA (1980) show that an equivalent one-dimensional test for internal reliability is based on the statistic

$$w_i = e_i^T P v \;/\; \sqrt{e_i^T P Q_V P e_i} = v_i / \sigma_{Vi} \tag{8}$$

where $e_i^T = (0\ 0\ 0\ \ldots\ 1\ \ldots\ 0\ 0\ 0)$ denotes an nx1 vector containing one in the ith position and zero elsewhere [5].

PELZER (1980) defines the quantity Z_i which becomes, in the case of independent observations with diagonal cofactor matrix Q_l as follows:

$$Z_i = \sqrt{(e_i^T Q_l e_i)/(e_i^T P Q_V P e_i)} = \sigma_{li} / \sigma v_i \tag{9}$$

This quantity is also useful in design for reliability because it can be evaluated a priori for each observation l_i . It is the ratio of the standard error of an observation l_i , which is obtained from the proposed covariance matrix C_l of observations, to the standard error of its least squares residual, which is a function of C_l and the proposed design matrix A [8,9].

ASHKENAZI (1980) proposes another quantity as an local internal reliability measure, which can be given as:

$$\tau_l = \sigma_{\hat{l}l} / \sigma_{ll} \quad (10)$$

This expresses the ratio of the standard error of a corrected observation to the standard error of the observation itself. The usefulness of this measure of local internal reliability is that it represents the way in which the network increases precision of the measured elements. For a completely internally reliable observation $\sigma_{\hat{l}l} = 0$ so $\tau_l = 0$ and for a completely unreliable observation $\tau_l = 1$.

The most popular local measure of the internal reliability is known as the redundancy number r_i, which can be calculated for each observation from the redundancy matrix using the following relation :

$$r_i = (Q_V P)_{ii} \quad (11)$$

The redundancy number is a contribution of the observation l_i to the total redundancy (n - u) , i.e.

$$\Sigma r_i = \mathrm{Trace}(Q_V P) = n - u \quad (12)$$

This means that, the sum of the redundancy numbers equals the degree of freedom of the adjustment, whereas for individual redundancy numbers it is widely accepted that the redundancy number ranges from zero to one [1,3] , i.e.

$$0 \leq r_i \leq 1 \quad (13)$$

BILL and MÜRLE (1984) proposed the following lower and upper boundaries for the redundancy number as follows [2] :

$$\left.\begin{array}{ll} 0 \leq r_i < 0.01 & \text{(not control ability)} \\ 0.01 \leq r_i < 0.10 & \text{(bad control ability)} \\ 0.10 \leq r_i < 0.30 & \text{(sufficient control ability)} \\ 0.30 \leq r_i \leq 1 & \text{(good control ability)} \end{array}\right\} \quad (14)$$

This mean that, zero redundancy number implies uncontrolled observation in that an outlier enters into the solution with its full size. The larger the redundancy number r_i is , the easier the outlier can be detected by testing the residual. Large redundancy number (close to one) are a sign for good reliability . If the redundancy number is one , then the outlier is completely recovered by the residual . In this case the observations said to be completely controlled by the other observations.

Alternatively, an average value of the redundancy number for the entire network can be referred to as a global measure of internal reliability and seems to be more

practical to utilize. The average redundancy number $(r)_{ave}$ can be calculated as a single representative number as follows [6]:

$$(r)_{ave} = \Sigma r_i / n = (n - u) / n \quad (15)$$

POPE (1976) stated that for a good geodetic network, $(r)_{ave}$ should not be less than (0.50). The higher the degree of freedom, the closer the average redundancy number is to one [3] .

The marginally detectable errors computed for all observations are a measure of the capability of the network to detect blunders with probability (1-β). They constitute the internal reliability of the network because the marginally detectable errors do not depend on the observations or on the residuals. They can be computed as soon as the configuration of the network and the stochastic model are known. This marginally detectable errors are determined by BAARDA (1968) for certain probability levels α and β_0 as follows [6] :

$$|M_i| \geq \delta_0 . \sigma_{li} / \sqrt{r_i} \quad (16)$$

where δ_0 = the non centrality parameter of the normal distribution which depends on the given probability α_0 and β_0 and can be determined as (for $\alpha = 0.10$ and $\beta_0 = 0.20$ then $\delta_0 = 4.13$).
σ_{li} = the standard error of the ith observation.
r_i = the redundancy number of the ith observation.

Equation (16) states that in 100 (1 - β_0) % of the cases, the outliers greater than those given in equation (16) are detected. In 100 β_0 % of the cases, outliers greater than those given in equation (16) remain undetected. The larger the redundancy number of the observation, the smaller the marginally detectable blunder. Because one would like to have small magnitudes for the marginally detectable blunders, the desirability of larger redundancy number follows. If the limits in equation (16) are of about the same size, the observations are equally well checked, and the internal reliability is said to be consistent [6] .

The absorption, which means that the portion of the blunder that propagates into the estimated parameters and falsifies the solution, is

$$A_i = (1 - r_i)\,\Delta_i \quad (17)$$

The factor ($1 - r_i$) is called the absorption number. The larger the redundancy number is, the less absorption of the blunder (less falsification). If $r_i = 1$, the observation is called fully controlled, because the residual completely reflects the blunder. A zero redundancy implies uncontrolled observations in that a blunder enters into the solution with its full size. Observations with small redundancy numbers might have small residuals and instill false security in the analyst. The absorption number can be rewritten as a function of the residuals as follows [6] :

$$A_i = -v_i (1 - r_i) / r_i \quad (18)$$

The residuals can be looked on as the visible parts of errors. The factor in equation (18) is required to compute the invisible part from the residuals.

3.2 External Reliability Measures

The external reliability is used to estimate the influence of undetectable blunders on the estimated unknown parameters (coordinates) as well as any function derived from these parameters in an adjusted system. The impact of blunders on the parameters be minimal. BAARDA suggested the following expression [3,6] :

$$\lambda_{0i} = \delta_0 \sqrt{(1 - r_i) / r_i} \qquad (19)$$

where the values λ_{0i} are the measures of external reliability as a whole. If the λ_{0i} are the same order of magnitude, the network is homogeneous with respect to external reliability. If r_i is small, the external reliability factor becomes large, and the global falsification caused by the blunder can be significant. It follows that small redundancy numbers are not desirable. Low external reliability implies that the absorption of the blunder and subsequent falsification of the solution is minimal . The global measure of external reliability λ_{0i} and the absorption number A_i have the same dependency on the redundancy numbers.

CASPARY (1988) suggested the maximum eigen value Λ_{max} of the reliability matrix ($P Q_V P$) and the trace of the same matrix as a two global measures of external reliability , which can be expressed as follows:

$$\text{Trace}\,(P Q_V P) = \max \quad \text{and} \quad \Lambda_{max}\,(P Q_V P) = \max \qquad (20)$$

This means that for a given error vector the probability of detecting its existence is related to the magnitude of Λ_{max} ($P Q_V P$) . In addition, trace ($P Q_V P$) can be used to define the upper bound of the non centrality parameters. Additional observations increase the number of degree of freedom and hence the magnitude of the trace, which indicates the increased probability of error detection [3] .

4. RELIABILITY MEASURES FOR CORRELATED OBSERVATIONS

4.1 Internal Reliability Measures

The internal reliability of an adjusted system is referred to as the controllability of the observations described by the lower bounds for outliers, which can just be detected at the given significance level α_0 and power of the test β_0.

According to BAARDA and PELZER (1980), the lower bound for the outlier in the observation l_i is expressed as

$$\Delta_0 l_i = \sigma_0 \delta_0 / \sqrt{e_i^T\ P Q_V P\ e_i} = C_{0i} . \sigma_{li} \qquad (21)$$

With

$$C_{0i} = \delta_0 / \sqrt{e_i^T Q_l e_i\ e_i^T\ P Q_V P\ e_i} = \delta_0 / \sqrt{R_i} \qquad (22)$$

in which

$$R_l = e_l^T Q_l e_l \; e_l^T P Q_V P e_l \qquad (23)$$

where δ_0 is the non centrality parameter which depends on the given probability α_0 and β_0, and C_{0l} is called the measure for the controllability of the observation l_i. The measure of the controllability indicates how many times of the corresponding standard error σ_{li} the lower bound $\Delta_0 l_i$ is.

WANG and CHEN (1994) suggested the internal reliability factor R_l, which is identical with the reliability number Z_i, as a local measure of internal reliability in case of correlated observations [11]. However, the range of R_l can be given as follows :

$$0 \leq R_l \leq q_{ii} P_{ii} = 1 / (1 - \rho_l^2) \qquad (24)$$

It should be noted that the internal reliability factor R_l defined in equation (23) is different in essence from the redundancy number r_i for correlated observations. The multiple correlation coefficient ρ_l is defined through the following relation

$$\rho_l^2 = q^T_{(i)} Q_l^{-1}{}_{(ii)} q_{(i)} / q_{ii} \qquad \text{and} \qquad 0 \leq \rho_l^2 \leq 1 \qquad (25)$$

where the $(n-1) \times 1$ vector $q_{(i)}$ corresponding to the ith column of Q_l after eliminating the element q_{ii}, similarly, the $(n-1) \times (n-1)$ matrix $Q_{l(ii)}$ results from Q_l after eliminating the ith row and the ith column.
Consequently, the range for these reliability numbers R_l may well exceed the unit interval, depending on the magnitude of ρ_l, which makes a comparison very difficult whenever the multiple correlation coefficient changes widely.

Based on the previous discussion, the normalized reliability number $\check{R}_l$ can prefer be proposed as a local measure of internal reliability. The proposed measure can be computed using the following relation:

$$\check{R}_l = R_l (1 - \rho_l^2) = (e_l^T Q_l^{-1} e_l)^{-1} (e_l^T P Q_V P e_l) \qquad (26)$$

With the following range

$$0 \leq \check{R}_l \leq 1 \qquad (27)$$

for the detectable outliers in the case of correlated observations.

4.2. External Reliability Measure

The external reliability is used to estimate the influence of non detectable outlier on the estimated unknown parameters in an adjusted system. It is generally expressed as BAARDA (1968)

$$\lambda_{0l} = \| \Delta_0 x_l \| = \sqrt{\Delta_0 x_l^T C_X^{-1} \Delta_0 x_l} \qquad (28)$$

where the $\Delta_0 x_i$ is a shift of the unknown parameters vector x caused by the marginally detectable outlier $\Delta_0 l_i$. The λ_{0i} is named as the external reliability measure. Equation (28) can be further simplified as [11] :

$$\lambda_{0i} = \delta_0 \sqrt{ (1 / R_i \ (1 - \rho_i^2)) - 1 } \quad (29)$$

which indicates that the external reliability measure not only depends on the internal reliability factor R_i, but also on the multiple correlation coefficient ρ_i . Finally, the external reliability measure can be expressed using the new reliability number $\check{R}_i$ as follows :

$$\lambda_{0i} = \delta_0 \sqrt{ (1 - \check{R}_i) / \check{R}_i } \quad (30)$$

which nicely corresponding to the usual formula for uncorrelated observations where $\check{R}_i$ is simply replaced by the redundancy number r_i itself.

5. NUMERICAL EXAMPLES

5.1 Simulated leveling Network

To illustrate the efficiency of the proposed reliability number on the different practical surveying applications (problems), let us begin with the example of three vertical control networks in different configurations A, B and C , illustrated in Fig. 1, as discussed WANG and CHEN (1994). The three observations (height differences) were assumed with the non diagonal cofactor matrix of the observations. The internal and external measures as well as the proposed reliability number were computed and listed in table (2).

5.2 Theodolite Angle Observations

The second example for this study was the reduction of theodolite direction observations in connection with subsequent least squares adjustments. These reductions of direction measurements are often called station adjustments. A set of horizontal angles can be measured using the all combinations method, illustrated in Fig. 2. The computation can be performed using the parametric technique. The data being the same as that used by REISSMANN (1980) . The orientation and other nuisance parameters can be removed from the least squares adjustment using the Schreiber technique [10]. The internal and external reliability measures as well as the proposed reliability number were computed and listed in table (3).

5.3 Real Geodetic Network

The third application for this study was a two - dimensional trilateration network comprising six points is established in El – Mansoura city (Sandoub zone). The configuration of the network points is shown in Fig. 3. All distances of the network legs were observed using SOKKIA (SET 5) Total station. The standard errors of the measured distances is ± (5mm + 5PPm) . The approximate coordinates of

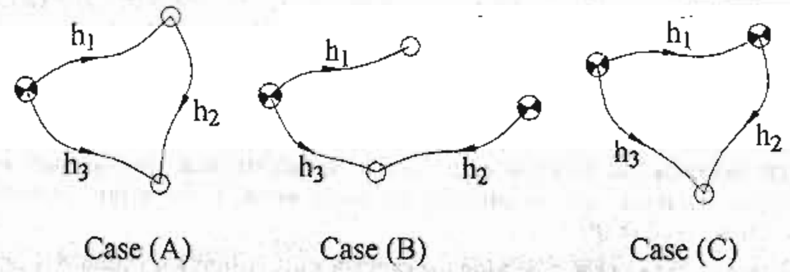

Fig. 1: Simulated leveling networks in different configurations

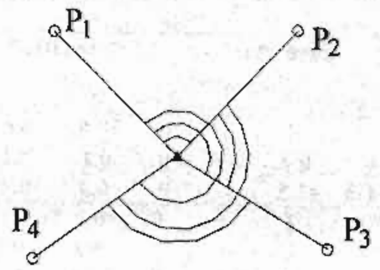

Fig.2: Theodolite angle observations

Table1. The approximate coordinates of the netpoints

Points	X^O (m)	Y^O (m)
P_1	2708.294	2000.235
P_2	2002.848	1999.845
P_3	1920.192	3017.159
P_4	2356.732	4174.899
P_5	3051.645	4003.819
P_6	2750.647	2971.591

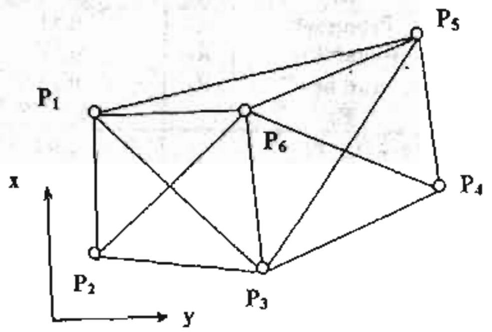

Fig. 3 : A real geodetic network.

the netpoints are listed in table (1) with respect to the selected local horizontal coordinate system. The network was adjusted by the least squares method using the parametric technique. The internal and external reliability measures as well as the proposed reliability number were computed and listed in table (4).

6. RESULTS AND DISCUSSIONS

I- Table (2) contains the internal and external reliability measures as well as the proposed measures for the three cases (A, B and C). From the obtained results, it was found that:

1- In case (A), we can see how wide the range for the redundancy numbers can be found. The value of the redundancy number was found in the first observation negative ($r_1 = -1$) whilst in the third observation was found higher than one ($r_3 = 1.5$).

Table 2: Internal and external reliability measures for correlated observations in leveling networks.

		Case (A)			Case (B)			Case (C)		
Redundancy matrix $Q_v P$		-1	-1	1	0	-0.6	0.6	1	0	0
		0.5	0.5	−0.5	0	0.1	-0.1	1.5	1	-1
		-1.5	-1.5	1.5	0	-0.9	0.9	1.5	0	0
Reliability matrix $P Q_v P$		1	1	-1	0	0	0	5	3	-3
		1	1	-1	0	0.2	-0.2	3	2	-2
		-1	-1	1	0	-0.2	0.2	-3	-2	2
Redundancy number r_i	r_1		-1.00			0			1.00	
	r_2		0.50			0.10			1.00	
	r_3		1.50			0.90			0	
WANG 'S reliability factor R_i	R_1		2.00			0			10.00	
	R_2		1.00			0.20			2.00	
	R_3		5.00			1.00			10.00	
Proposed Reliability number $\check{R}_i$	$\check{R}_1$		0.11			0			0.53	
	$\check{R}_2$		0.50			0.10			1.00	
	$\check{R}_3$		0.25			0.05			0.50	
Trace ($Q_v P$)			1.00			1.00			2.00	
Trace ($P Q_v P$)			3.00			0.40			9.00	
Trace ($P Q_l P$)			12.50			15.10			6.50	
$(r)_{ave}$			0.333			0.333			0.667	
$(R)_{ave}$			2.667			0.400			7.333	
$(\check{R})_{ave}$			0.287			0.050			0.667	

2- No longer are the values for the redundancy number restricted to the unit interval, although they always sum up to the correct number $r = n - u$, which equals one in cases (A) and (B) , and two in case (C).

3- All proposed reliability numbers belong to the unit interval, moreover, the relative magnitude within the internal proposed reliability numbers $\check{R}_i$ may change in comparison to the internal reliability factors R_i . Thus, in case (C) we might conclude that the second observation is poorly controlled, based on the value of $R_2 = 2$, while it turns out to be extremely well controlled as the value of $\check{R}_2 = 1$.

4- Conversely, in case (C) the third observation would appear to be uncontrolled as it is apparent when the redundancy number $r_3 = 0$, whereas it is moderately controlled as $\check{R}_3 = 0.5$. (third observation is not highly controlled as indicated by $R_3 = 10$) .

5- Similar discussions apply to cases (A) and (B) , for instance, in case (B) the first observation will be flagged as fully uncontrolled independent of the chosen criterion $r_1 = R_1 = \check{R}_1 = 0$, and in case (A) the second observation will be relatively best controlled among the three criteria , using the criterion $\check{R}_2 = 0.5 > \check{R}_3 > \check{R}_1$, although the value $R_2 = 1 < R_1 < R_3$ would erroneously indicate otherwise.

II- Table (3) contains the internal and external reliability measures as well as the proposed measures for correlated and uncorrelated theodolite angle observations, while table (4) shows the internal and external reliability measures as well as the proposed measures for correlated and uncorrelated observations in real trilateration network. From these tables, the following results can be stated:

1- In most cases internal reliability factors are higher than one. Therefore, the reliability factor can not be used as a measure of the internal reliability.

Table 3. Internal and external reliability measures for correlated and uncorrelated theodolite angle observations.

	Correlated Observations			Uncorrelated Observations		
	r_i	R_i	$\check{R}_i$	r_i	R_i	$\check{R}_i$
1	0.125	0.786	0.393	0.500	0.500	0.500
2	0.571	1.143	0.571	0.500	0.500	0.500
3	0.518	0.786	0.393	0.500	0.500	0.500
4	0.821	1.143	0.571	0.500	0.500	0.500
5	0.429	1.071	0.536	0.500	0.500	0.500
6	0.536	0.571	0.286	0.500	0.500	0.500
$Tr(Q_vP)$		3.000			3.000	
$Tr(PQ_vP)$		5.500			3.000	
$Tr(PQ_lP)$		6.500			3.000	
$(r)_{ave}$	0.500	0.917	0.458	0.500	0.500	0.500

2-The redundancy numbers r_i are considerably smaller than the corresponding internal reliability factors R_i.

3-The smallest reliability factor R_i does not necessarily correspond to the smallest redundancy number r_i.

4-All values of the proposed internal and external reliability numbers lies between zero and one .

5-In case of uncorrelated observations , the redundancy numbers, the internal reliability factors and the proposed internal reliability numbers gave the same identical numerical values ($r_i = R_i = \check{R}_i$). Therefore, it is not necessary to calculate both the internal reliability factors and the proposed internal reliability numbers for this case.

Table 4. Internal and external reliability measures for correlated and uncorrelated observations in real geodetic network.

	Correlated Observations			Uncorrelated Observations		
	r_i	R_i	$\check{R}_i$	r_i	R_i	$\check{R}_i$
1	0.005	0.173	0.141	0.122	0.122	0.122
2	0.380	0.394	0.197	0.264	0.264	0.264
3	0.354	0.261	0.235	0.334	0.334	0.334
4	0.491	0.461	0.348	0.436	0.436	0.436
5	0.268	0.241	0.196	0.169	0.169	0.169
6	0.121	0.383	0.311	0.268	0.268	0.268
7	0.216	0.588	0.296	0.177	0.177	0.177
8	0.269	1.039	0.520	0.296	0.296	0.296
9	0.165	0.372	0.186	0.142	0.142	0.142
10	0.113	0.320	0.161	0.096	0.096	0.096
11	0.077	0.806	0.403	0.243	0.243	0.243
12	0.542	0.782	0.513	0.454	0.454	0.454
$Tr(Q_vP)$	3.000			3.000		
$Tr(PQ_vP)$	7.275			3.000		
$Tr(PQ_lP)$	22.843			9.000		
$(r)_{ave}$	0.250	0.485	0.292	0.250	0.250	0.250

7. CONCLUSIONS AND RECOMMENDATIONS:

In this paper, the recently introduced internal and external reliability factors for correlated observations have been modified to make them more comparable to the redundancy numbers of BAARDA (1968) , which are only valid for uncorrelated observations. The new proposed internal and external reliability numbers lie between zero and one . They can be used to represent the percentage of controllability of an outlier that occur in particular observations. This makes the interpretation of any problem much easier as far as reliability is concerned.

The proposed internal and external reliability measures, which can be computed using the equations (26) and (30), are suitable for both correlated and

uncorrelated observations. The relationship between the proposed internal reliability number and the internal reliability factor is also given .
This study could be considered as an extension for what WANG and CHEN (1994) have done before. All equations derived with the multiple correlation coefficient can be used for further study of the influence of the correlation between the observations on the reliability of an adjusted geodetic network .

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