Answer the following questions

1.

a) For the function $f(x)=\begin{cases}\cosh x, & x\le 0\\ \ln x, & x>0\end{cases}$ complete the following

i- The figure representing $f(x)$ is .....

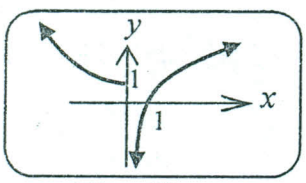

Fig. (1)

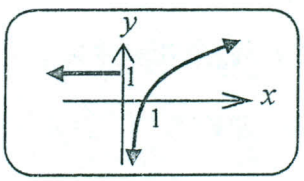

Fig. (2)

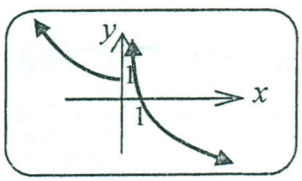

Fig. (3)

ii- The equation of the asymptote is ..... and its type is.....

iii- The zero (or zeros) of the function is (or are) .....

iv- $\lim_{x\to 0^-} f(x)=.....$

v- The function is non-differentiable at $x=.....$

b) For the function $y=e^{\sinh^{-1}x}$ *prove that* $(1+x^2)y''+x\,y'-y=0$,

hence prove that $(1+x^2)y^{(n+2)}+(2n+1)x\,y^{(n+1)}+(n^2-1)y^{(n)}=0$.

c) Find $\frac{dy}{dx}$ *if* $(\cos^{-1}x)^y=\frac{\tan^{-1}x}{\sec x}$.

2.

a) Find $\frac{dy}{dx}$ of the parametric function $x=3^{\sinh t}$, $y=\sin^{-1}(\tan t)$ *at t=0*.

b) Using L'Hopital's rule, what is the value should m assigned to make the function

$f(x)$ continuous at $x=\pi$, where $f(x)=\begin{cases}\frac{\sin x}{x-\pi} & ,x\neq\pi\\ -\cos m & ,x=\pi\end{cases}$

c) Prove that $\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$, $|x|\le 1$.

d) Find $\lim_{x\to\infty} x^2e^{-x}$

3. **(a) [5 points]** Use the polar form to write $\dfrac{\left(1+i\sqrt{3}\right)^3\left(1-i\right)^4}{\left(-\sqrt{3}-i\right)^6}$ in the form $a+ib$.

(b) [5 points] Use the binomial theorem to find the coefficient of x^n in the expansion $\left(\dfrac{4+x}{1-x}\right)^2$.

(c) Given the polynomial $f(x)=x^5-5x^4+9x^3-2x^2+1$.

(i) [5 points] Find the quotient when dividing the polynomial $f(x)$ by x^2-x-2.

(ii) [5 points] What is the quotient and remainder when dividing this polynomial by $\left(x^2-1\right)(x-2)$?.

(iii) [5 points] Solve the equation $f(x)=x^2+10x-7$.

(d) [10 points] Find the partial fractions decomposition of the function

$$\frac{x^5-5x^4+9x^3-2x^2+1}{\left(x^2-1\right)(x-2)}.$$

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4. **(a) [5 points]** Solve the equation $x^3-81x^2+5x-405=0$, given that one of its root is the negative of the other.

(b) [5 points] If A is a square matrix prove that $A-A^t$ is skew-symmetric.

(c) [15 points] Given the matrices

$$A=\begin{bmatrix}1 & 2 & -2\\ 2 & 5 & -1\\ 4 & 5 & -3\end{bmatrix},\quad B=\begin{bmatrix}1 & 0 & 1\\ 2 & -2 & -1\\ 3 & 0 & 0\end{bmatrix},\quad C=\begin{bmatrix}0\\ 5\\ -6\end{bmatrix}$$

Find $|A|$, $\left(4A-2B^t+3I\right)$, CB, A^{-1}, and $\left(A^tB^{-1}\right)^{-1}$.

(d) [10 points] Solve the linear system of equations

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 8\\ 3x_1 + 2x_2 - 5x_3 &= 10\\ 6x_1 - 4x_2 + 7x_3 &= 25 \end{aligned}$$

With all best Wishes **Thursday, 12/1/2012**