Mansoura University
Faculty of Engineering
Eng. Math. & Phy. Dept.

MATHEMATICS (2) - Preparatory Year
Final Term Exam.

Total Marks: 130

(تخلفات)

Time: 3 hours

Part (I)—Integral Calculus

Question (1): [38 Marks]

(a) Evaluate each of the following integrals:

(i) $I = \int x^2 \cos 3x \, dx$

(ii) $I = \int \frac{\csc^{-1}(1/x)}{\sqrt{1-x^2}} \, dx$

(iii) $I = \int \frac{5}{3\cos x + 2\sin x + 2} \, dx$

(iv) $I = \int_{-2}^{2} \frac{1}{(4+x^2)^2} \, dx$

(b) Drive a reduction formula for the integral $I_n = \int \cos^n x \, dx$. Hence, find $\int (\sec x)^{-4} \, dx$.

(c) Determine if the following integrals are convergent or divergent:

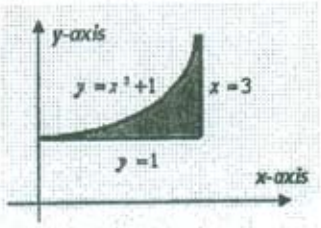
(i) $I = \int_0^{\infty} \frac{x}{x^2 + 3x + 2} \, dx$

(ii) $I = \int_{-\infty}^{3} \frac{1}{\sqrt{3-x}} \, dx$

Question (2): [32 Marks]

(a) Compute the area bounded by $y = \sin x$, $y = \cos x$ and $y - axis$.

(b) Find the volume of the solid generated by revolving of the area shown in figure:



(i) about $x = 3$.

(ii) about $y = -1$.

(c) Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2}$, $-2 \le x \le 2$ about x-axis.

(d) Use Simpson's rule with $h = 0.1$ to approximate $I = \int_0^1 e^{x^2} \, dx$.

3) a) Find the equation of the line through **(3,6)** which makes an angle $tan^{-1}3$ with x-axis. Then find the equations of the lines which parallel to it.

b) Prove that the two lines: $x^2 + 2xysec\alpha + y^2 = 0$ are always real and the angle between them is α. Then find:

(i) The equation of their bisectors.

(ii) The equation of the pair of lines passing through the point **(2, 1)** and parallel to (i).

c) Given the conic section whose equation is: *(35 marks)*

$$4x^2 + 9y^2 - 48x + 72y + 144 = 0$$

Find its center, semi – axes, vertices, foci, the length of LR and graph it.

4) a) Find the equation of the plane passing through the line:

$$x + y + z = 6 \quad \text{and} \quad 2x + 3y + 4z + 5 = 0$$

and perpendicular to the plane $4x + 5y - 3z = 8$

b) Find the equations of the spheres which pass through the circle:

$$x^2 + y^2 + z^2 = 5 \quad , \quad x + 2y + 3z = 3$$

and touch the plane $4x + 3y - 15 = 0$.

c) Discuss the nature of the surface whose equation is : *(35 marks)*

$$x^2 + 2y^2 - 3z^2 + 4x - 4y - 6z - 9 = 0.$$

Find the equation, center and axes of the conic sections obtained by cutting the given surface by the planes $x = -2$, $y = 1$ and $z = -1$, respectively.