



**Answer the following questions**

**Question 1 ( 25 marks)**

Solve the heat equation:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + x, \quad 0 < x < 1, \quad t > 0$$

Where  $u$  is the temperature,  $k$  is the conductivity of the rod,  $x$  is a dimension and  $t$  time

Subject to: B.C.s  $u(0, t) = 1, u(1, t) + \frac{\partial u(1, t)}{\partial x} = 2$  and I.C.  $u(x, 0) = x$

**Question 2 ( 25 marks)**

Consider waves in a resistant medium that satisfy the problem  $u_{tt} = c^2 u_{xx} - ru_t + 5xe^{-t}$  Where  $0 < x < l, t > 0$  With Boundary conditions,  $u(0, t) = 1, u(l, t) = 2$  and initial conditions,  $u(x, 0) = f(x), u_t(x, 0) = g(x)$  Where  $r$  is a constant,  $0 < r < 2\pi c/l$ . Write down the series expansion of the solution.

**Question 3 ( 25 marks)**

Solve the following 2-dimensional PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{a} \frac{\partial u}{\partial t}, \quad 0 < x < a, \quad 0 < y < b, \quad t > 0$$

Subject to the boundary conditions:

$$\frac{\partial u}{\partial x} = 0 \quad \text{at} \quad x = 0, \quad \frac{\partial u}{\partial x} + H_2 u = 0 \quad \text{at} \quad x = a,$$

$$u = 0 \quad \text{at} \quad y = 0, \quad \frac{\partial u}{\partial y} + H_4 u = 0 \quad \text{at} \quad y = b,$$

$$u = f(x, y) \quad \text{for} \quad t = 0$$

**Question 4 ( 25 marks)**

Solve the following heat equation that describe the heat flow in a non-uniform rod without sources

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( K_0 \frac{\partial u}{\partial x} \right), \text{ where } u \text{ represent the temperature and the thermal properties of the rod } K_0 \text{ poss}$$

non-constant. Also, the boundary and initial conditions are

$$\text{B.C.s } u(0, t) = 0, \frac{\partial u(L, t)}{\partial x} = 0 \text{ and I.C. } u(x, 0) = f(x)$$

**This exam measures the following ILOs**

Question Number	Q1				Q2	Q3		Q4			
Skills	Knowledge & understanding skills				Intellectual Skills				Professional Skills		

*With my best wishes*

*Asst. Prof. Dr. Islam M. Eldesoky*