Faculty of Engineering
Math. &Eng. Physics Dept.

Mathematics (3)
final Exam.

First Semester 2013/2014
Time 3hours.

Solve the following questions

Question 1 (27 marks)

(a) Solve the initial value problem

$$\left(y e^{-y} \cos x\right)dx + \left(1 + e^{-y} \sin x\right)dy = 0, \qquad y(0) = 1.$$

(b) Solve the differential equation $\cos x \; y''' + \cos x \; y' = \sec x$.

(c) A body of mass *2 kg* is thrown vertically upward with initial velocity of $v_0 = 100\ m/sec$. Assume that the air resistance is twice the velocity of the body.
Find **(i)** the equation of motion,
(ii) the velocity of the body at any time t,
(iii) the time at which the body reach its maximum height,
(iv) the maximum height.

Question 2 (28 marks)

(a) Solve the integro-differential equations

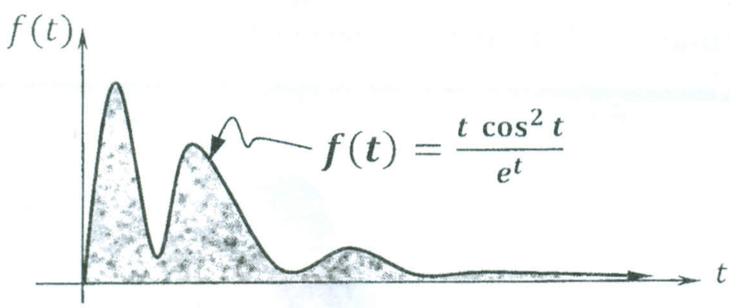
$$y'(t) = e^t - \int_0^t y(x)\cosh(t-x)\,dx, \quad y(0) = 1.$$

(b) Find $L^{-1}\left\{ e^{-\pi s} \times \ln\left(\dfrac{e^{\tan^{-1} s}}{\sqrt{s^2+1}} \right) \right\}$

(c) Find Laplace transform of $f(t)$, where

$$f(t) = \begin{cases} e^t, & 0 \le t \le \pi \\ \sin t, & t > \pi \end{cases}$$

(d) Evaluate the following shaded area



[3] (a) [6 pts] The domain of Ali's garden is describe by the domain of the function

$$f(x,y) = \sqrt{x-|y|} + \sqrt{4x - x^2 - y^2}$$

i) Find the domain of Ali's garden and sketch

ii) Using double integration prove that area of garden $= 2\pi + 4$

(b) [9 pts] If z, u, v are three **positive** real numbers satisfy equation

$z^2u - uvz + u^2 + v^2 = 8v$ where $u = (x+1)e^y$ and $v = (y+1)\cos(x)$

prove that $z_x = -1.6$ and $z_y = 0.2$ at ($x = 0$ and $y = 0$)

(c) [8 pts] Suppose that the elevation z of a hill is given by

$$z = f(x,y) = 39 + 10x - x^2 + 12y - y^2$$

i) If a small stone moves from site (6,8) to (9,12) . Find the rate of change of elevation in that direction

ii) Using second-order approximation, find the elevation z at point (6.01,8.02) use $[x_0 = 6 \; and \; y_0 = 8]$

[4] (a) [7 pts] Find $I = \int_0^1 \int_0^1 \frac{1}{1-(xy)^2} \, dx \, dy$ using transformation

$x = \frac{\sin(u)}{\cos(v)}$ and $y = \frac{\sin(v)}{\cos(u)}$ (note : this transformation transform square $0 \le x \le 1$, $0 \le y \le 1$ into triangle $0 \le u \le \frac{\pi}{2} - v$, $0 \le v \le \frac{\pi}{2}$)

(b) [7 pts] Find the center of mass of the lamina for the shape inside curve $r = 2 - 2\sin(\theta)$ shown in figure 1. If the mass density given by $\rho(x,y) = 1$

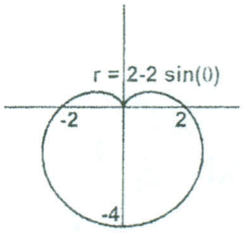


Figure 1

(c) [4 pts] For $\vec{F} = (x^2y)\,\hat{\imath} + (3x - yz)\,\hat{\jmath} + (z^3)\,\hat{k}$. Find Curl $\vec{F}$ and Div $\vec{F}$

(d) [7 pts] Compute the work done by the force field $F(x,y) = (y)\,\hat{\imath} + (-x)\,\hat{\jmath}$ acting on object as it moves along parabola $y = x^2 - 1$ from (1,0) to (−2,3)

(e) [7 pts] Ali has a tent its volume is similar to the volume of the solid bounded by $z = 4 - y^2, x + z = 4, x = 0 \; and \; z = 0$. Find the volume inside tent