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INVESTIGATION ON INCOMPRESSIBLE TURBULENT BOUNDARY LAYER WITH MAXIMUM DECELERATION

بحث عن الطبقة الجدارية اللاانضغاطية المضطربة عند اقصى عجلة تقصيرية

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خلاصة : مشاكل الطبقة الجدارية المضطربة المنضغطة تعامل على اساس فرض احوال ابتدائية محددة وتكامل معادلة "براندتل التكاملية"، بتمثيل معاملات انحدار السرعة كدالة من ارقام تشابه لابعدية للطبقة الجدارية المضطربة ، فانه يمكن الحصول على حد انفصالها مما يساعد في اجراء حسابات لطبقة الجدارية الغير منفصلة ذات اقصى عجلة تقصيرية .

النتائج النظرية التي حصل عليها قورنت مع النتائج المعملية المتاحة والتي اجريت على ناشر قطري. بالاضافة الى ذلك يحتوي البحث على علاقة بين انحدار السرعة المضطربة وانحدار اجهاد القص كعلاقة معاملات للطبقة الجدارية المتزنة والتي وجد انها تتفق مع التي حصل عليها من نظرية "ميلور - جيبسون" .

ABSTRACT- Incompressible turbulent boundary layer problems are solved based on certain assumed boundary conditions, by integrating Prandtl integral equation. Representing the velocity profile parameter as a function of dimensionless similarity number of turbulent boundary layers, a separation criteria can be achieved. This helps in solving the attached boundary layers with maximum deceleration.

The obtained theoretical results are compared with previous available experimental data obtained for radial diffuser.

Moreover, this investigation contains relation between the turbulent velocity profile and shear stress profile in a parametric form for equilibrium boundary layer and agree well with that obtained from Mellor-Gibson theory.

NOMENCLATURE

b	diffuser width	(mm)
$\bar{b}$	effective width	(mm)
$\bar{c}$	velocity at the outer edge of boundary layer,	(m/s)
c_f	local skin friction coefficient, $\tau_w / (1/2 . \rho \bar{c}^2)$	
c_τ	shear stress velocity, $\sqrt{\tau_w / \rho}$,	(m/s)
c_∞	free stream velocity,	(m/s)
c_x	velocity of the fluid inside the boundary layer in x-direction (m/s)	(m/s)
c_y	velocity component inside the boundary layer in y-direction, (m/s)	(m/s)
$H_{1,2}$	convential shape factor , δ^* / δ^{**}	
I	defect-shape factor	
P	pressure ,	(N/m^2)
$Re_{\delta^{**}}$	momentum thickness Reynolds number, $\bar{c} . \delta^{**} / \nu$	

x	coordinate in the direction of the wall,	(m)
y	coordinate normal to the direction of the wall,	(m)
δ	boundary layer thickness,	(m)
δ^*	boundary layer displacement thickness, $\int_0^{\infty} (1-c_x/\bar{c})\, dy$,	(m)
δ^{**}	boundary layer momentum thickness, $\int_0^{\infty} c_x/\bar{c}\,(1-c_x/\bar{c})\, dy$,	(m)
Λ	Euler number, $1/\bar{c} \cdot d\bar{c}/dx \cdot \delta^{**}$	
ν	kinematic viscosity of fluid,	(m^2/s)
ρ	density of fluid,	(kg/m^3)
τ	shear stress,	(N/m^2)
τ_w	wall shear stress,	(N/m^2)

1- INTRODUCTION

The prediction of boundary layer development under various flow conditions playes an important role in the design of a wide variety of fluid machinery components. For example, boundary layers in the presence of unfavourable pressure gradient play a significant role in the case of radial diffusers. This causes the viscous boundary layers to break away from the walls and greatly affects the diffuser performance.

There have been several studies applied to the problem of predicting diffuser efficiency and design parameters. In particular, Ackeret and Albring [1,2], Leibe und Jahn [3], and Ruchti [4] and Sörgel [5] have all presented experimental work to deal with this phenomenon. In [6] an investigation about turbulent boundary layer by different roughnesses was given experimentally.

This paper is concerned with developing of a mathematical model to obtain a criteria for maximum deceleration in radial diffusers. This is based on the integration of the momenium integral equation under certain assumed boundary conditions. Assuming that the velocity profile parameter has an optimum value, Λ_{opt} of a magnitude about 0.00094, and the form parameter, $H_{1,2}$ ranges between 1.4 and 1.6 for maximum deceleration.

2- GOVERNING EQUATIONS

The governing equation of mean flow for incompressible turbulent flow is the momentum integral equation [7] given as :

$$\frac{d\delta^{**}}{dx} + \left(2 + \frac{\delta^*}{\delta^{**}}\right) \frac{\delta^{**}}{\bar{c}} \frac{d\bar{c}}{dx} = \frac{\tau_w}{\rho \bar{c}^2} = \frac{c_f}{2} \qquad \ldots (1)$$

Equation (1) is normalized by using ,

$$x^* = \frac{x}{x_o} \; ; \; \bar{c}^* = \frac{\bar{c}}{\bar{c}_o} \; ; \; \Delta = \frac{\delta^{**}}{x_o} ,$$

x_o and $\bar{c}_o$ being appropriate reference quantities. Thus, x_o will be the length from the center of the radial diffuser to the beginning of the parallel walls ; $\bar{c}_o$ the component of velocity at the inlet by x_o at the edge of boundary layer. Fig. (1) illustrates the coordinate system used for the radial diffuser.

Equation (1) becomes :

$$\frac{d\Delta}{dx^*} + (2 + H_{12})\frac{\Delta}{\bar{c}^*}\frac{d\bar{c}^*}{dx^*} = \frac{c_f}{2} \qquad \ldots (2)$$

The optimum value of Euler number, $\Lambda_{opt} = -\frac{\Delta}{\bar{c}^*}\frac{d\bar{c}^*}{dx^*}$, was found to be close to 0.00094 [6] and so one adopts this constant value of, Λ, for the analysis presented here. Moreover, by this value of, Λ, the local skin friction coefficient, c_f, is very small and diminishes, and the form parameter, $H_{12} = \delta^*/\delta^{**}$, has a magnitude of 1.4 to 1.6 [6], so that equation (2) can be integrated with $c_f \to 0$ and Λ_{opt} and H_{12} are constants. Equation (2) therefore yields

$$\frac{d\Delta}{dx^*} + (2 + H_{12})(-\Lambda_{opt}) = 0 \qquad \ldots (3)$$

Separating the variables and integrating, this equation (3) one gets :

$$\Delta = (2 + H_{12})\,\Lambda_{opt}\cdot x^* + C, \qquad \ldots (4)$$

where $C = \Delta_o$, since $\frac{\delta^*}{\delta^{**}}$ = const and $c_f \to 0$ when the first term on the right hand side of equation (4) is equal to zero. Then the solution of Eq. (4) is given as

$$\frac{\delta^{**}}{x_o} = (2 + \frac{\delta^*}{\delta^{**}})(\Lambda_{opt})\frac{x}{x_o} + \frac{\delta_o^{**}}{x_o} \qquad \ldots (5)$$

Recalling the integral definition of ecuation (1), and substituting for $c_f = 0$, this yields

$$\frac{d\delta^{**}/x_o}{dx/x_o} = -(2 + \frac{\delta^*}{\delta^{**}})\frac{d\bar{c}/\bar{c}_o}{dx/x_o}\frac{\delta^{**}}{x_o} \qquad \ldots (6)$$

Rearranging equation (6) by separating the variables and integrating one gets,

$$\ln(\frac{\delta^{**}}{x_o}) = \underbrace{-(2 + \frac{\delta^{**}}{\delta^{**}})}_{=A}\ln\frac{\bar{c}}{\bar{c}_o} + \ln C \qquad \ldots (7)$$

or

$$\ln(\frac{\delta^{**}}{x_o}) = -C\ln(\frac{\bar{c}}{\bar{c}o})^A, \qquad \ldots (7')$$

where $C = \frac{\delta_o^{**}}{x_o}$, since $\bar{c} = \bar{c}_o$ at $x = x_o$

$$\text{Therefore } \frac{\delta^{**}}{x_o} = (\frac{\bar{c}}{\bar{c}_o})^{-A}\cdot(\frac{\delta_o^{**}}{x_o}) \qquad \ldots (8)$$

Equation (5) and (8) are two independent expressions for δ^{**}/x_o. Combining there two equations (5), (8) and one gets a simple equation,

$$(\frac{\bar{c}}{\bar{c}_o})^2 = [1 + (2 + H_{12})\cdot\Lambda_{opt}\cdot\frac{x/xo}{\delta_o^{**}/x_o}]^{-\frac{2}{(2+H_{12})}} \qquad \ldots (9)$$

using a mean value for $H_{12} = 1.5$ and $\Lambda_{opt} = 0.00094$ and substituting in equation (9), the following form for the velocity distribution, or the recovery factor, is

$$\left(\frac{\bar{c}}{\bar{c}_o}\right)^2 = \left[1 + 0.0033 \frac{x/x_o}{\delta_o^{**}/x_o}\right]^{-0.5714} \qquad \ldots (10)$$

Equation (10) is used in the analysis to obtain the effect of the initial velocity on the diffuser performance.

In addition, an equilibrium turbulent boundary layer is one for which the gross properties of the outer region, constituting some 85-95% [8] of the total boundary layer, can be scaled with a single parameter such as the boundary-layer defect thickness, Θ, where

$$\Theta = \int_0^\infty \frac{\bar{c} - c_x}{c_\tau} \, dy \, ,$$

c_τ, the friction velocity, expressed in terms of the local wall shear stress, c_f, by $c_\tau / \bar{c} = (\frac{1}{2} c_f)^{0.5}$. In an equilibrium layer both velocity-defect profiles and shear stress profiles are self similar. This self-similarity of velocity-defect profiles, demonstrated by Clauser [9], that for an equilibrium boundary layer $(\bar{c} - c_\tau) / c_\tau$ is function of y/δ only. Hence, $(\bar{c} - c_x)^2 / c_\tau^2$, is a similar unique function and thus in such a boundary layer the defect shape factor, I, defined as :

$$I = \frac{\int_0^\infty \left[\frac{\bar{c} - c_x}{c_\tau}\right]^2 . \, dy}{\int_0^\infty \frac{\bar{c} - c_x}{c_\tau} \, dy} = \frac{1 - \delta^* / \delta^{**}}{\sqrt{c_f / 2}} \, , \qquad \ldots (11.a)$$

will be a constant. Through algebric manibulation the convential shape factor, $H_{1/2} = \delta^*/\delta^{**}$, can be expressed in terms of the defect shape factor, I, by :

$$H_{1,2} = [1 - (c_f/2)^{0.5} \, . \, I]^{-1} \qquad \ldots (11.b)$$

which is constant in an equilibrium turbulent boundary layer as long as the local skin friction, c_f, is invariant [10 , 11]. This agrees with the previous assumed boundary condition for integrating the momentum integral equation.

For characterising the shear stress layers the proper value is the dimensionless shear stress tangent at the wall [12] ,

$$\frac{\partial (\tau/\tau_w)}{\partial (y / \delta^*)_{y=o}} = \frac{\partial p}{\partial x} \frac{\delta^*}{\tau_w} = - \frac{1}{\bar{c}} \frac{d\bar{c}}{dx} \frac{\delta^*}{c_f/2} \qquad \ldots (12)$$

Developing the shear stress distribution across the flow in Mac-Laurin series, so it comes as first and important term. This magnitude will be evaluated in the same time through the pressure gradient at the outer edge of the boundary layer.

The relationship between velocity defect-profile and shear stress profile, i.e., between both parameter, Eq. (11 . b) and Eg. (12) is evaluated from experiments [13]. For equilibrium boundary layers it follows from **Mellor-Gibson** theory [14]. Fig. 4, illustrates this relationship between velocity defect profile and the dimensionless shear stress tangent at the wall. The exprimental data originated from boundary layers which has steady deceleration till separation exhibit .

3- PRESENTATION AND DISCUSSIONS OF RESULTS

The two equation formerly obtained ;

$$\frac{\delta^{**}}{x_o} = \frac{\delta_o^{*}}{x_o} + 0.0033\,\frac{x}{x_o} \qquad \ldots (5)$$

and

$$\frac{c}{c_o} = \left[\, 1 + 0.0033\,\frac{x/x_o}{\delta_o^{**}/x_o} \,\right]^{-0.5714} \qquad \ldots (10)$$

are evaluated and plotted in figures 2 and 3. Equation (5) emphathises that the momentum thickness increases lineary. In figure (2) indicates the linear increase in momentum thickness which obtained from equation (5).

Equation (10) for the velocity distribution shows that the velocity at the boundary layer edge decreases according to a power function. This relation given by equation (10) is illustrated in figure (3). Moreover, figure (3) shows the marked effect of the initial value of momentum thickness (suffix o), while smaller thickness of boundary layer results in higher values of deceleration than these corresponding to thicker values of boundary layer thickness.

Figure 4 describes the relation between the velocity profile and shear-stress profile. For comparison the curve for equilibrium boundary layer is drawn,which indicates a unequivocal join both parameter. The deviation between both curves is due to simplification in the calculation. In addition, boundary layers with essential another type of boundary conditions have remarkable deviation from the drawn curve.

Fig. (5) shows the relation between Λ values and momentum thickness Reynolds number, $Re_{\delta^{**}}$. This illustration agrees and in harmony with the physical perception, after-which in near separation essentialy pressure and inertia forces are only effective.

For satisfying the boundary condition for flows near separation a constant Λ values is selected, e.g. 0.00049 that guarantee the attached flow.

4- CONCLUSIONS

The conclusions of the present investigation may be listed as follows :

i) Formulas have been obtained [equations (5) and eq. (10)] which represent adequately the theoretical data for the influence of the boundary layer growth in momentum thickness at diffuser inlet on its performance. Increasing the velocity at diffuser inlet increases the momentum thickness and decreases the recovery factor remarkably. So, to obtain maximum deceleration the initial value of momentum thickness must be decreased.

ii) An independent, also indirect, check has been made on the procedure of velocity profile matching to determine the local shear stress. It shows a reasonable agreement with that given by Mellor-Gibson theory (Fig. 5) [14] .

iii) Near separation pount the pressure and inertia forces are only effective.

REFERENCES

[1] Ackeret, J "Grenzschichten in geraden und gekrümmten Diffusoren. Grenzschichtforschung (Berlin, Götingen, Heidelberg 1967)

[2] Albring, w. et. al. Entwicklung eines radiallüfter mit hohem wirkungsgrad-Maschinenbautechnik, H.8.

[3] Liepe und Jahn "Einfluss des Dralles auf ruhende und rotierende Radialdiffusoren. Bericht des Inst. für angew. Strömungslehre der TU-Dresden. (Unveröffentlicht).

[4] Ruchti, O. "Versuche mit Radildiffusoren" Technische Bericht 11, H. 5, S. 129 (1964).

[5] Sörgel, G. "Untersuchungen am schaufellosen und beschaufelten Diffusor eines Radialdiffusors, Diss. TU-Dresden.

[6] Hanna, S. "Informationen über turbulente Grenzschichten bie verschiedene Rauhigkeiten auf Grund Von Messungen an einem Radiäldiffusor". Maschinenbautechnik, H. 6, 1978.

[7] Schlichting, H. "Boundary Layer Theory" Mc. Graw Hill, New York 1968.

[8] Bandyopadhyay, P.R. "Rough wall turbulent boundary layers in the transition regime." J. Fluid Mech. (1987), Vol. 180, pp. 231-266 .

[9] Clauser, F.H. "Turbulent boundary layers in adverse pressure gradients, J. Aeronaut Sci. 20, 468 (153) .

[10] Felsh, K.O. "Beitrag zur Berechnung turbulente Grenzschichten in zweidimensionaler incompressible Strömung". Diss. Karlsruhe 1965 .

[11] Nigim, H. H., Cockrell, D.J, "Effects caused by small discrete two-dimensional rouhness elements immersed in turbulent boundary layers." J. Fluid Mech. (1985) Vol. (153) - pp. 17-30.

[12] Felsh, K.O. "Eine neue Methode zur Berechnung turbulenter Grenzschichten." Wiss. Z. der TU-Dresden, H. 4, 1969

[13] Hanna, S. "Unpublished work."

[14] Millor, G.L., Gibson, D.M. "Equilibrium Turbulent Boundary Layers". J. Fluid Mech. 24 (1966)

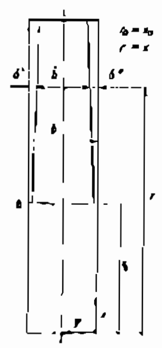

FIG. 1 Illustration of coordinate system on radial diffusor

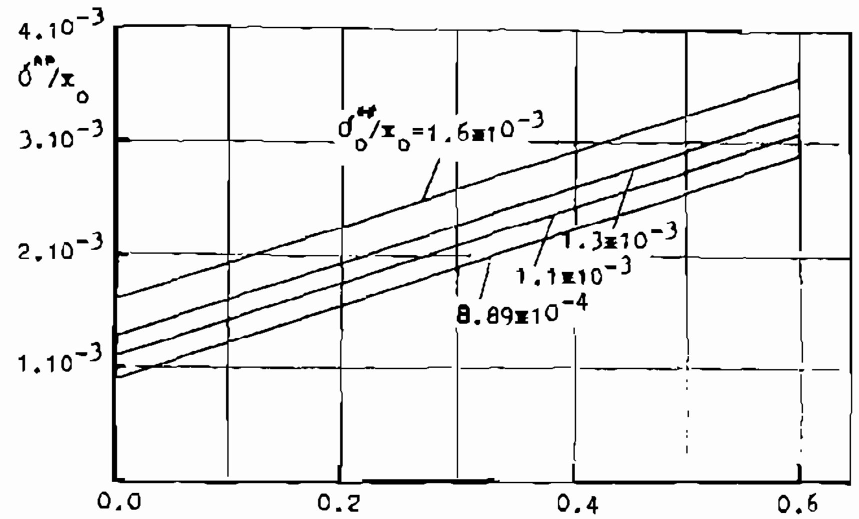

FIG. 2 Incompressible turbulent boundary layer with $c_f \longrightarrow 0$, momentum thickness versus x/x_0

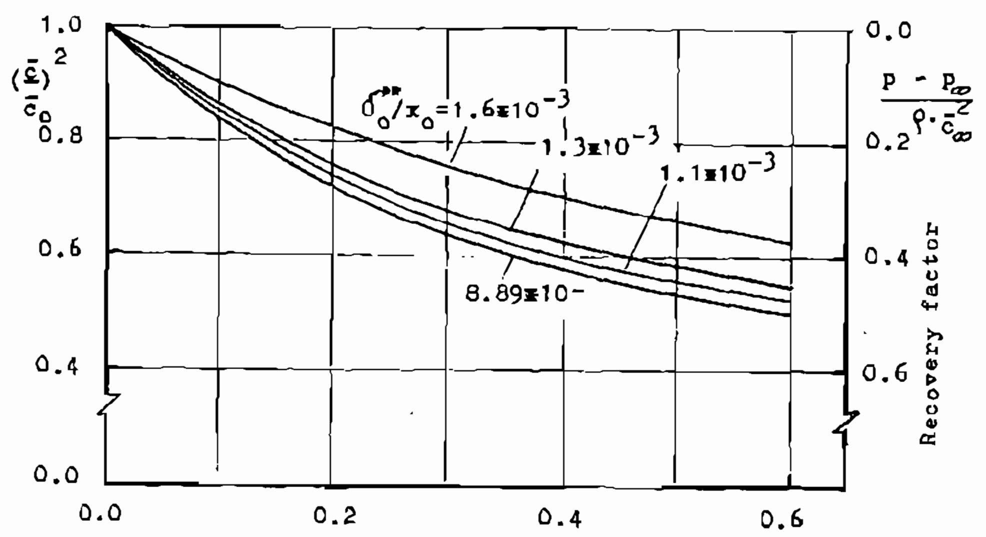

FIG. 3 Recovery factor versus x/x_0 with initial momentum thickness as parameter

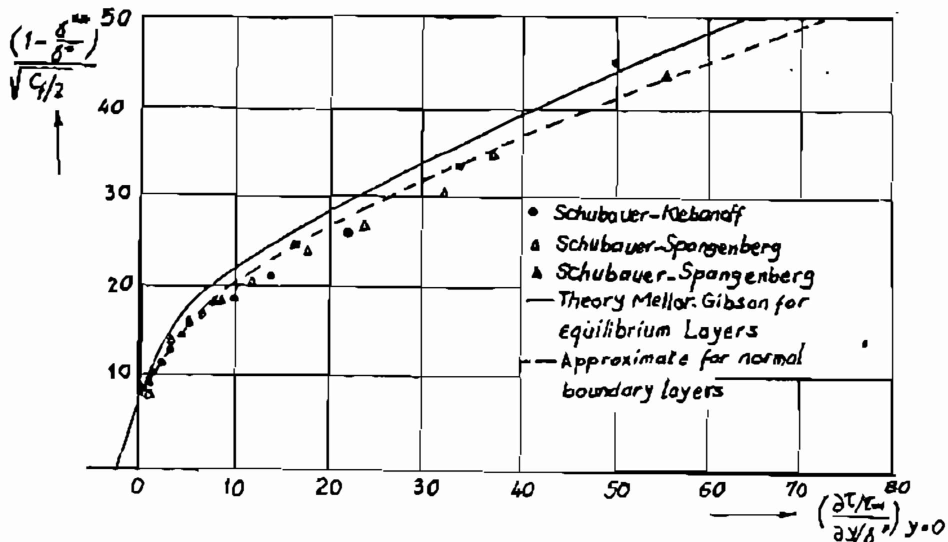

FIG. 4 Relationship between velocity profile and shear-stress profile

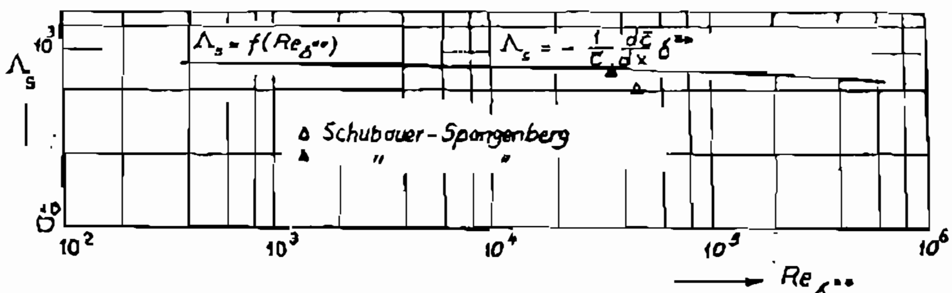

FIG. 5 Dependence of the critical Λ_s-values from $Re_{\delta^{**}}$