

Semi θ – Compactness in Intuitionistic Fuzzy Topological Spaces

الإحكام من النوع semi θ - علي الفراغات التوبولوجية الفازية الحدسية

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الملخص العربي:

يهدف هذا البحث إلى إدخال مفهوم جديد من مفاهيم الإحكام هو semi θ -compactness وذلك علي الفراغات التوبولوجية الفازية الحدسية. كما تم تعريف الإحكام المحلي من النوع semi θ - وكذلك تم استنتاج بعض الخصائص لهذه المعارف الجديدة كما تم دراسة علاقاتها ببعض مثيلاتها المعرفة من قبل وتأثير بعض الدوال عليها.

Abstract:

The purpose of this paper is to construct the concept of semi θ -compactness in intuitionistic fuzzy topological spaces. We give some characterizations of semi θ -compactness, locally semi θ -compactness. A comparison between these concepts and some other types of compactness in intuitionistic fuzzy topological spaces are established.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy topological space, Intuitionistic fuzzy semi θ -compact.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11], and later Atanassov [1,2] generalized this idea to intuitionistic fuzzy sets. On the other hand, Coker [3] introduced the notions of intuitionistic fuzzy topological spaces, fuzzy continuity and some other related concepts. In this paper, we introduce the concepts of semi θ -compactness, locally semi θ -compactness in intuitionistic fuzzy topological spaces. We give some characterizations and basic properties for these concepts. For definitions and results not explained in this paper, we refer to the papers

[1, 3, 5, 6, 8], assuming them to be well known. The words "neighbourhood", "continuous" and "irresolute" will be abbreviated as respectively "nbd", "cont." and "i".

2. Preliminaries

First, we present the fundamental definitions.

Definition 2.1[2]. Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS, for short) U is an object having the form $U = \{(x, \mu_U(x), \gamma_U(x)) : x \in X\}$ where the functions $\mu_U : X \rightarrow I$ and $\gamma_U : X \rightarrow I$ denote respectively the degree of membership (namely

$\mu_u(x)$ and the degree of nonmembership (namely $\gamma_u(x)$) of each element $x \in X$ to the set U , and $0 \leq \mu_u(x) + \gamma_u(x) \leq 1$ for each $x \in X$.

The reader may consult [3, 4, 6] to see several types of relations and operations on IFS's, intuitionistic fuzzy points (IFP's, for short) and some properties of images and preimages of IFS's.

Definition 2.2[3]. Let X be a nonempty set and let the IFS's U and V be in the form $U = \{(x, \mu_u(x), \gamma_u(x)) : x \in X\}$, $V = \{(x, \mu_v(x), \gamma_v(x)) : x \in X\}$ and let $\{U_j : j \in J\}$ be an arbitrary family of IFS's in X . Then

- (i) $U \leq V$ iff $\mu_u(x) \leq \mu_v(x)$ and $\gamma_u(x) \geq \gamma_v(x), \forall x \in X$;
- (ii) $\bar{U} = \{(x, \gamma_u(x), \mu_u(x)) : x \in X\}$;
- (iii) $\cap U_j = \{(x, \wedge \mu_{U_j}(x), \vee \gamma_{U_j}(x)) : x \in X\}$;
- (iv) $\cup U_j = \{(x, \vee \mu_{U_j}(x), \wedge \gamma_{U_j}(x)) : x \in X\}$;
- (v) $\underline{1} = \{(x, 1, 0) : x \in X\}$ and $\underline{0} = \{(x, 0, 1) : x \in X\}$;
- (vi) $\bar{\bar{U}} = U, \bar{\underline{0}} = \underline{1}$ and $\bar{\underline{1}} = \underline{0}$;
- (vii) $\coprod U = \{(x, \mu_u(x), 1 - \mu_u(x)) : x \in X\}$;
- (viii) $\langle U = \{(x, 1 - \gamma_u(x), \gamma_u(x)) : x \in X\}$.

Definition 2.3[3]. An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family Ψ of IFS's in X containing $\underline{0}, \underline{1}$ and closed under finite infima and arbitrary suprema.

In this case the pair (X, Ψ) is called an intuitionistic fuzzy topological space (IFTS, for short) and each IFS in Ψ is known as an intuitionistic fuzzy open set (IFOS, for short) in X . The complement \bar{U} of an IFOS U in an IFTS (X, Ψ) is called an intuitionistic fuzzy closed set (IFCS, for short), in X .

Proposition 2.4[3]. Let (X, Ψ) be an IFTS

on X . Then, we can construct the following two IFTS's:

- (i) $\Psi_{0,1} = \{\coprod U : U \in \Psi\}$;
- (ii) $\Psi_{0,2} = \{\langle U : U \in \Psi\}$.

Definition 2.5[8]. Let X, Y be nonempty sets and $U = (x, \mu_U(x), \gamma_U(x))$, $V = (y, \mu_V(y), \gamma_V(y))$ IFS's of X and Y , respectively. Then $U \times V$ is an IFS of $X \times Y$ defined by: $(U \times V)(x, y) = \{(x, y), \min(\mu_U(x), \mu_V(y)), \max(\gamma_U(x), \gamma_V(y))\}$.

Definition 2.6[8]. Let $(X, \Psi), (Y, \Phi)$ be IFTS's and $A \in \Psi, B \in \Phi$. We say that (X, Ψ) is product related to (Y, Φ) if for any IFS's U of X and V of Y , whenever $(\bar{A} \not\leq U$ and $\bar{B} \not\leq V) \Rightarrow (\bar{A} \times \underline{1} \cup \underline{1} \times \bar{B} \geq U \times V)$, there exist $A_1 \in \Psi, B_1 \in \Phi$ such that $\bar{A}_1 \geq U$ or $\bar{B}_1 \geq V$ and $\bar{A}_1 \times \underline{1} \cup \underline{1} \times \bar{B}_1 = \bar{A} \times \underline{1} \cup \underline{1} \times \bar{B}$.

Definition 2.7[9]. An IFP $c(a, b)$ is said to be intuitionistic fuzzy θ -cluster point (IF θ -cluster point, for short) of an IFS U iff for each $A \in N_\epsilon^q(c(a, b)), cl(A) q U$.

The set of all IF θ -cluster points of U is called the intuitionistic fuzzy θ -closure, of U and denoted by $cl_\theta(U)$. An IFS U will be called IF θ -closed (IF θ CS, for short) iff $U = cl_\theta(U)$. The complement of an IF θ -closed set is IF θ -open (IF θ OS, for short).

Lemma 2.8.[10] Let X, Y are IFTS's such that X is product related to Y . Then the product $U \times V$ of IF θ OS U of X and IF θ OS V of Y is an IF θ OS of $X \times Y$.

Definition 2.9. An IFS U of an IFTS X is called ϵ -nbd [4] ($\epsilon\theta$ -nbd) [10] of an IFP $c(a, b)$, if there exists an IFOS (IF θ OS) U in X such that $c(a, b) \in U \leq U$.

The family of all ϵ -nbd ($\epsilon\theta$ -nbd) of an IFP $c(a, b)$ will be denoted by $N_\epsilon(N_\epsilon^\theta)(c(a, b))$.

Definition 2.10.[7] An IFS U of an IFTS X is said to be an IFsemiopen (IFSO),

for short)(IFpreopen(IFPOS, for short)) iff $U \leq cl(int(U))(U \leq int(cl(U)))$.

Definition 2.11. Let (X, Ψ) and (Y, Φ) be two IFTS's. A function $f : X \rightarrow Y$ is said to be.

- (i) IF-cont.[3](IFsemi-cont.(IFS-cont., for short)[7]) if the preimage of each IFOS in Y is IFOS(IFSOS) in X .
- (ii) IFi (IFsuper i) function if the preimage of each IFOS in Y is IFOS (IFOS) in X [10].
- (iii) IFstrongly θ -(resp. IF θ -, IFfaintly, IF θ -)cont. if the preimage of each IFOS(resp. IF θ OS, IF θ OS, IFSOS) of Y is IF θ OS(resp. IF θ OS, IFOS,IF θ OS) in X [9,10].
- (iv) IFweakly cont.[7] if for each IFOS V of Y , $f^{-1}(V) \leq int(f^{-1}(cl(V)))$.
- (v) IF-[10](resp. IFsemi-[10], IFpre-, IFsuper semi-, IF θ -, IFfaintly-[10])open if the image of each IFOS(resp. IFOS, IFOS, IFSOS, IF θ OS, IF θ OS) of X is IFOS(resp. IFSOS, IFPOS, IFOS, IF θ OS, IFOS) in Y .

Definition 2.12. An IFS U of an IFTS (X, Ψ) is said to be an IF[3](IF θ -)compact relative to X iff every an IF(θ -)open cover of U has a finite subcover.

Definition 2.13. An IFTS (X, Ψ) is called :

- (i) IFcompact[3](resp. IFS-compact, IF λ -compact, IF θ -compact) iff every an IFopen (resp. semiopen, λ -open, θ -open) cover of X has a finite subcover which covers X .
- (ii) Locally IF(θ -)compact if for each IFP $c(a, b)$ in X , there is $U \in N_c(c(a, b))$ such that $\mu_U(c) = 1$, $\gamma_U(c) = 0$ and U is an IF(θ -)compact relative to X .
- (iii) IF-submaximal if each dense subset of X is IFOS.
- (iv) IFS-closed iff every an IFsemiopen cover of X has a finite subfamily whose closures cover X .
- (v) IF-regular iff for each $U \in \Psi$, $U = \bigvee \{U_j : U_j \in \Psi, cl(U_j) \leq U\}$.

Lemma 2.14. Let $f : X \rightarrow Y$ be an IFS-cont. and IFpreopen function, then $f^{-1}(V)$ is an IFSOS in X for each an IFSOS V in Y .

Proof. Let V be an IFSOS in Y , then there exists an IFOS U of X such that $U \leq V \leq cl(U)$. Now, $f^{-1}(U) \leq f^{-1}(V) \leq f^{-1}(cl(U))$, since f is an IFpreopen function we have, $f^{-1}(U) \leq f^{-1}(V) \leq f^{-1}(cl(U)) \leq cl(f^{-1}(U))$. Since f is an IFS cont., $f^{-1}(U)$ is an IFSOS in X , implies there is an IFOS G of X such that $G \leq f^{-1}(U) \leq f^{-1}(V) \leq cl(f^{-1}(U)) \leq cl(G)$. Hence $f^{-1}(V)$ is an IFSOS in X .

3. Semi θ -compactness in IFTS's

Definition 3.1. (i) A family $\{(x, \mu_{U_j}, \gamma_{U_j}) : j \in J\}$ of IFOS's(IF θ OS's) in X such that $\bigvee \{(x, \mu_{U_j}(x), \gamma_{U_j}(x)) : x \in X\} = 1$, is called an IFsemi(θ -)open cover of X .

(ii) A finite subfamily $\{(x, \mu_{U_j}, \gamma_{U_j}) : j = 1, 2, \dots, n\}$ of an IFsemi(θ -)open cover, which is also a semi(θ -)open cover, i.e. $\bigvee_{j=1}^n \{(x, \mu_{U_j}, \gamma_{U_j})\} = 1$, is called a finite subcover of $\{(x, \mu_{U_j}, \gamma_{U_j}) : j \in J\}$.

Definition 3.2. A family $\{(x, \mu_{U_j}, \gamma_{U_j}) : j \in J\}$ of IFS's satisfy the θ -finite intersection property (θ -FIP, for short) iff for every finite subfamily $\{(x, \mu_{U_j}, \gamma_{U_j}) : j = 1, 2, \dots, n\}$ of the family, we have $\bigwedge_{j=1}^n \{(x, \mu_{U_j}, \gamma_{U_j}) : j \in J\} \neq 0$.

Definition 3.3. An IFTS (X, Ψ) is called fuzzy semi θ -compact(IF θ -compact, for short) iff every an IFsemiopen cover of X has a finite subcollection(subcover)of IF θ OS's, which covers X .

Definition 3.4. An IFS U of an IFTS (X, Ψ) is said to be an IFS θ -compact

relative to X if for every family $\{U_j : j \in J\}$ of IFOS's in X such that $U \subseteq \bigvee_{j \in J} U_j$, there is a finite subfamily $\{U_j : j = 1, 2, \dots, n\}$ of IFOS's such that $U \subseteq \bigvee_{j=1}^n U_j$.

Remark 3.5. From the above definition and some other types of IF compactness, one can illustrate the following implications:

IFS θ -compact \Rightarrow IFS-compact \Rightarrow IF λ -compact \Rightarrow IF-compact \Rightarrow IF θ -compact

Theorem 3.6. (X, Ψ) is an IFS θ -compact iff every family $U = \{U_j : j \in J\}$ of IFSCS's in X having the θ -FIP, $\bigwedge_{j \in J} U_j \neq \emptyset$.

Proof. (\Rightarrow): Let $U = \{U_j : j \in J\}$ be a family of IFSCS's in X having the θ -FIP. Suppose that $\bigwedge_{j \in J} U_j = \emptyset$, then $\bigvee_{j \in J} \overline{U_j} = \mathbb{1}$.

From the IFS θ -compactness and $\{\overline{U_j} : j \in J\}$ is IFOSs, there is a finite subfamily $\{\overline{U_j} : j = 1, 2, \dots, n\}$ of IFOS's such that $\bigvee_{j=1}^n \overline{U_j} = \mathbb{1}$. Then $\bigwedge_{j=1}^n U_j = \overline{\bigvee_{j=1}^n \overline{U_j}} = \emptyset$, which is a contradiction to the θ -FIP. Hence $\bigwedge_{j \in J} U_j \neq \emptyset$.

(\Leftarrow): Let $U = \{U_j : j \in J\}$ be an IFsemiopen cover of X . Hence $\{\overline{U_j} : j \in J\}$ is a family of IFSCSs having the θ -FIP. Then from the hypothesis, we have $\bigwedge_{j \in J} \overline{U_j} \neq \emptyset$ which implies $\bigvee_{j \in J} U_j \neq \emptyset$ and hence a contradiction with that $\{U_j : j \in J\}$ is an IFsemiopen cover of X .

Theorem 3.7. An IFTS (X, Ψ) is an IFS θ -compact iff $(X, \Psi_{0,1})$ is an IFS θ -compact.

Proof. (\Rightarrow) Let $\{\bigcup U_j : j \in J\}$ be an IFsemiopen cover of X in $(X, \Psi_{0,1})$. Hence $\bigvee(\bigcup U_j) = \mathbb{1} \Rightarrow \bigvee \mu_{U_j} = \mathbb{1}, \bigwedge \gamma_{U_j} = \mathbb{1} - \bigvee \mu_{U_j} = \emptyset$. Since (X, Ψ) is an IFS θ -compact,

there is $\{U_j : j = 1, 2, \dots, n\}$ of IFOS's such that $\bigvee_{j=1}^n U_j = \mathbb{1}$. Now we have, $\bigvee_{j=1}^n \mu_{U_j} = \mathbb{1}$ and $\bigwedge_{j=1}^n (1 - \mu_{U_j}) = \mathbb{1} - \bigvee_{j=1}^n \mu_{U_j} = \emptyset$. Hence $\{\bigcup U_j : j \in J\}$ has a subcover of IFOS's and then $(X, \Psi_{0,1})$ is an IFS θ -compact.

(\Leftarrow) Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X in (X, Ψ) . Since $\bigvee U_j = \mathbb{1}$, we have $\bigvee \mu_{U_j} = \mathbb{1}, \bigwedge \gamma_{U_j} = \mathbb{1} - \bigvee \mu_{U_j} = \emptyset$. Since $(X, \Psi_{0,1})$ is an IFS θ -compact, there is a subfamily $\{U_j : j = 1, 2, \dots, n\}$ of IFOS's such that $\bigvee_{j=1}^n (\bigcup U_j) = \mathbb{1}$ i.e.

$\bigvee_{j=1}^n \mu_{U_j} = \mathbb{1}$ and $\bigwedge_{j=1}^n (1 - \mu_{U_j}) = \emptyset$. Hence $\mu_{U_j} = \mathbb{1} - \gamma_{U_j} \Rightarrow \bigvee_{j=1}^n \mu_{U_j} = \bigvee_{j=1}^n (1 - \gamma_{U_j}) \Rightarrow \mathbb{1} = \mathbb{1} - \bigwedge_{j=1}^n \gamma_{U_j} \Rightarrow \bigwedge_{j=1}^n \gamma_{U_j} = \emptyset \Rightarrow \bigvee_{j=1}^n U_j = \mathbb{1}$ i.e. $\{U_j : j \in J\}$ has a finite subcover of IFOS's. Hence (X, Ψ) is an IFS θ -compact.

Theorem 3.8. An IFTS (X, Ψ) is an IFS θ -compact iff $(X, \Psi_{0,2})$ is an IFS θ -compact.

Proof. Similar to the proof of Theorem 3.7.

Theorem 3.9. Every an IFS θ -compact space X which is submaximal regular is an IFS-closed.

Proof. Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X . Then $cl(U_j) = cl(H_j)$ where H_j is an IFOS in X . Since X is submaximal regular space, then $\{cl(U_j) : j \in J\}$ is an IFopen cover of X and consequently an IFsemiopen cover of X . Since X is an IFS θ -compact, then there is a subfamily $\{cl(U_j) : j = 1, 2, \dots, n\}$ of IFOS's such that $\bigvee_{j=1}^n cl(U_j) = \mathbb{1}$. Hence X is an IFS-closed.

Theorem 3.10. Every an IF θ -compact space X which is submaximal regular is an IFS θ -compact.

Proof. Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X . Since every an IFSOS in an IFsubmaximal regular X is an IF θ OS, then $\{U_j : j \in J\}$ is an IF θ -open cover of X . Since X is IF θ -compact, then there is a subfamily $\{U_j : j = 1, 2, \dots, n\}$ of IF θ OS's such that $\bigvee_{j=1}^n U_j = 1$. Hence X is an IFS θ -compact.

Corollary 3.11. Every an IF-(IFS-, IF λ -)compact space X which is submaximal regular is an IFS θ -compact.

Lemma 3.12. If U is an IF θ CS of an IFTS (X, Ψ) and $c(a, b) \notin U$, then there is an IFOS V of X such that $c(a, b) \in cl(V)$, for each $a, b \in (0, 1)$.

Proof. Let U be an IF θ CS and $c(a, b) \notin U$. Hence $c(1-a, 1-b)q\bar{U}$, for each $a, b \in (0, 1)$. From the definition of IF θ OS \bar{U} , there is an IFOS $V = (x, \mu_V, \gamma_V)$ of x such that $c(1-a, 1-b)qcl(V) \leq \bar{U}$, where $cl(V) = (x, \wedge\mu_G, \vee\gamma_G)$ and $\{(x, \mu_G, \gamma_G) : j \in J\}$ is the family of IFCS's containing V . Hence $1-a > \vee\gamma_G$, or $1-b < \wedge\mu_G$, which implies $a < \wedge\mu_G$, and $b > \vee\gamma_G$. Hence $c(a, b) \in cl(V)$.

Theorem 3.13. If U is an IF θ -closed of an IFS θ -compact space X , then U is an IFS θ -compact relative to X .

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{(y, \mu_{V_j}, \gamma_{V_j}) : j \in J\}$, be an IFsemiopen cover of U . For $x(a, b) \notin U$ and by Lemma 3.12, there is an IFOS G of X such that $x(a, b) \in G$. Hence $\{V_j : j \in J\} \vee \{cl(G(x)) : x \in \bar{U}\}$ is an IFsemiopen cover of X . Since X is IFS θ -compact, there is a finite subfamily $\{V_j : j = 1, 2, \dots, n\} \vee \{cl(G(x_i)) : i = 1, 2, \dots, m\}$ of IF θ OS's which covers X and consequently $\{V_j : j = 1, 2, \dots, n\}$ covers U . Hence U is an IFS θ -compact relative to X .

Theorem 3.14. If (X, Ψ_1) and (Y, Ψ_2) are IFS θ -compact spaces and (X, Ψ_1) is product related to (Y, Ψ_2) , then the product $X \times Y$ is IFS θ -compact.

Proof. Let $\{U_j \times V_j : j \in J\}$ be an IFsemiopen cover of $X \times Y$, where U_j 's and V_j 's are IFSOS's in X and Y , respectively. Then $\{U_j : j \in J\}$ and $\{V_j : j \in J\}$ are IFsemiopen covers of X and Y , respectively. Thus there exist subfamilies $\{U_j : j = 1, 2, \dots, n\}$ and $\{V_j : j = 1, 2, \dots, n\}$ of IF θ OS's such that $\bigvee_{j=1}^n U_j = 1_X$ and $\bigvee_{j=1}^n V_j = 1_Y$. From the product related of X, Y and Lemma 2.8, we have $\bigvee_{j \in J_1 \vee J_2} U_j \times V_j \equiv \bigvee_{j \in J_1 \vee J_2} U_j \times \bigvee_{j \in J_1 \vee J_2} V_j = 1_X \times 1_Y$. Thus $X \times Y$ is IFS θ -compact.

4. Functions and IFS θ -compact space

Theorem 4.1. If $f : X \rightarrow Y$ is an IFS-cont. surjection function and U is an IFS θ -compact relative to X , then $f(U)$ is an IF-compact relative to Y .

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{(y, \mu_{V_j}, \gamma_{V_j}) : j \in J\}$, be an IFopen cover of $f(U)$. Since f is IFS cont., then $\{f^{-1}(V_j) : j \in J\}$ is an IFsemiopen cover of U . Since U is IFS θ -compact, there is a finite subfamily $\{V_j : j = 1, 2, \dots, n\}$ of IF θ OS's such that $U \subseteq \bigvee_{j=1}^n V_j$, which implies that $f(U) \subseteq \bigvee_{j=1}^n f f^{-1}(V_j) = \bigvee_{j=1}^n V_j$. Hence $f(U)$ is an IF-compact relative to Y .

Corollary 4.2. If $f : X \Rightarrow Y$ is an IFS-cont. surjection function and X is an IFS θ -compact, then Y is an IF-compact.

Corollary 4.3. If $f : X \rightarrow Y$ is an IF-cont. surjection function and U is an IFS θ -compact relative to X , then $f(U)$ is an IF-compact relative to Y .

Corollary 4.4. If $f : X \rightarrow Y$ is an IF-cont. surjection function and X is an IFS θ -compact, then Y is an IF-compact.

Theorem 4.5. Let $f : X \rightarrow Y$ be an IFS θ -cont. and IF θ -open function. If U is an IF θ -compact relative to X , then $f(U)$ is an IFS θ -compact relative to Y .

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{(y, \mu_{V_j}, \gamma_{V_j}) : j \in J\}$, be an IFsemiopen cover of $f(U)$. Since f is IFS θ -cont., then the family $\{f^{-1}(V_j) : j \in J\}$ of IF θ OS's covers U [Note θ -open \Rightarrow open \Rightarrow semiopen]. Since U is an IF θ -compact, there is a finite subfamily $\{f^{-1}(V_j) : j = 1, 2, \dots, n\}$ of IF θ OS's such that $U \subseteq \bigcup_{j=1}^n f^{-1}(V_j)$. Since f is an IF θ -open, we have $f(U) \subseteq f(\bigcup_{j=1}^n f^{-1}(V_j)) \equiv \bigcup_{j=1}^n f f^{-1}(V_j) \equiv \bigcup_{j=1}^n V_j$. Hence $f(U)$ is an IFS θ -compact relative to Y .

Corollary 4.6. Let $f : \bar{X} \rightarrow \bar{Y}$ be an IFS θ -cont. and IF θ -open function. If U is an IFS θ -compact relative to X , then $f(U)$ is an IFS θ -compact relative to Y .

Corollary 4.7. Let $f : X \rightarrow Y$ be an IFS θ -cont. and IF θ -open function. If X is an IF θ -compact, then Y is an IFS θ -compact.

Corollary 4.8. Let $f : X \rightarrow Y$ be an IFS θ -cont. and IF θ -open function. If X is an IFS θ -compact, then so is Y .

Theorem 4.9. If $f : X \rightarrow Y$ is an IFfaintly cont. function and U is an IFS θ -compact relative to X , then $f(U)$ is an IF θ -compact relative to Y .

Proof. Similar to the proof of Theorem 4.1.

Corollary 4.10. If $f : X \rightarrow Y$ is an IFfaintly cont. function and X is an IFS θ -compact, then Y is an IF θ -compact.

Theorem 4.11. If $f : X \rightarrow Y$ is an IFsuper i function and U is an IFcompact relative to X , then $f(U)$ is an IFS θ -compact relative to Y .

Proof. Similar to the proof of Theorem 4.1.

Corollary 4.12. If $f : X \rightarrow Y$ is an IFsuper i function and X is an IFcompact, then Y is an IFS θ -compact.

Theorem 4.13. Let $f : X \rightarrow Y$ be an IFsuper semiopen and IFstrongly θ -cont. bijective function. If Y is an IF-compact, then X is an IFS θ -compact.

Proof. Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X . Since f is an IFsuper semiopen, then the family $\{f(U_j) : j \in J\}$ is an IFopen cover of Y . Since Y is an IF-compact, there is a subfamily $\{f(U_j) : j = 1, 2, \dots, n\}$ of IFOS's which cover Y . Now, $\{U_j : j = 1, 2, \dots, n\} \equiv \{f^{-1}(f(U_j)) : j = 1, 2, \dots, n\}$ is an IF θ -open cover in X (since f is IFstrongly θ -cont. bijective function). Hence X is an IFS θ -compact.

Corollary 4.14. Let $f : X \rightarrow Y$ be an IFsuper semiopen and IFstrongly θ -cont. bijective function. If V is an IFcompact relative to Y , then $f^{-1}(V)$ is an IFS θ -compact relative to X .

Theorem 4.15. Let $f : X \rightarrow Y$ be an IFsemiopen and IFfaintly cont. surjection function. If Y is an IFS θ -compact, then X is an IFcompact.

Proof. Similar to the proof of Theorem 4.13.

Corollary 4.16. Let $f : X \rightarrow Y$ be an IFsemiopen and IFfaintly cont. surjection function. If V is an IFS θ -compact relative to Y , then $f^{-1}(V)$ is an IF-compact relative to X .

Corollary 4.17. Let $f : X \rightarrow Y$ be an IFsemiopen and IFfaintly cont. surjection function. If V is an IFS θ -compact relative to Y , then $f^{-1}(V)$ is an IF θ -compact relative to X .

Theorem 4.18. Let $f : X \rightarrow Y$ be an IFfaintly open and IF θ - cont. surjection function. If Y is an IFS θ -compact, then X is an IF θ -compact.

Proof. Similar to the proof of Theorem 4.13.

Corollary 4.19. Let $f : X \rightarrow Y$ be an IFfaintly open and IF θ - cont. surjection function. If V is an IFS θ -compact relative to Y , then $f^{-1}(V)$ is an IF θ -compact relative to X .

Theorem 4.20. Let \bar{Y} be an IF-submaximal regular space and $f : X \rightarrow Y$ be an IFpreopen surjection function. If f is an IFS-cont. and X is an IFS θ -compact, then Y is so.

Proof. Let $\{V_j : j \in J\}$ be an IFsemiopen cover of Y . Since f is an IFS-cont. and IFpreopen, then by Lemma 2.14 the family $\{f^{-1}(V_j) : j \in J\}$ is an IFsemiopen cover of X . Since X is an IFS θ -compact, there is a subfamily $\{f^{-1}(V_j) : j = 1, 2, \dots, n\}$ of IF θ OS's which covers X . Now, $\{V_j : j = 1, 2, \dots, n\} = \{ff^{-1}(V_j) : j = 1, 2, \dots, n\}$ is an IF θ -open cover in Y , since Y is IF-submaximal regular space. Hence Y is an IFS θ -compact.

Corollary 4.21. Let Y be an IF-submaximal regular space and $f : X \rightarrow Y$ be an IFi function. If X is an IFS θ -compact, then so is Y .

Corollary 4.22. Let Y be an IF-submaximal regular space and $f : X \rightarrow Y$ be an IFi function.

If U is an IFS θ -compact relative to X , then $f(U)$ is an IFS θ -compact relative to Y .

5. Locally IFS θ -compact

Definition 5.1. An IFTS (X, Ψ) is said to be locally IFS θ -compact if for each an IFP $c(a, b)$ in X , there is $U \in N_\epsilon(c(a, b))$ such that $\mu_U(c) = 1$, $\gamma_U(c) = 0$ and U is an IFS θ -compact relative to X .

Remark 5.2. Every an IFS θ -compact space is locally IFS θ -compact but the converse may not be true.

Example 5.3. An infinite discrete IFTS is locally IFS θ -compact but not IFS θ -compact.

Remark 5.4. Every locally IFS θ -compact space is locally IF-compact but the converse may not be true.

Theorem 5.5. Let Y be an IF-submaximal regular space and $f : X \rightarrow Y$ be an IF-open surjection function. If f is an IFi function and X is locally IFS θ -compact, then so is Y .

Proof. Let $y(m, n)$ be an IFP in Y . Then $y(m, n) = f(x(a, b))$ for some $x(a, b)$ in X . Since X is locally IFS θ -compact, there is $U \in N_\epsilon(x(a, b))$ such that $\mu_U(x) = 1$, $\gamma_U(x) = 0$ and U is an IFS θ -compact relative to X . Since f is an IF-open function, $f(U) \in N_\epsilon(y(m, n))$ with $(f(U))(y) = \bigvee_{x \in f^{-1}(y)} U(x) = 1$ and by Theorem 3.19, $f(U)$ is an IFS θ -compact relative to Y . Hence Y is locally IFS θ -compact space.

Corollary 5.6. Let Y be an IF-submaximal regular space and $f : X \rightarrow Y$ be an IF-open surjection function. If f is an IFsuper i function and X is locally IFS θ -compact, then so is Y .

Proof. Since every an IFsuper i function is an IFi and from Theorem 5.5, the proof be obtained.

Theorem 5.7. Let $f : X \rightarrow Y$ be an IF-cont. and IF-open surjection function. If X is locally IFS θ -compact, then Y is locally IF-compact.

Proof. Let $y(m, n)$ be an IFP in Y . Then $y(m, n) = f(x(a, b))$ for some $x(a, b)$ in X . Since X is locally IFS θ -compact, there is $U \in N_\epsilon(x(a, b))$ such that $\mu_U(x) = 1, \gamma_U(x) = 0$ and U is an IFS θ -compact relative to X . Since f is an IF-open function, $f(U) \in N_\epsilon(y(m, n))$ with $(f(U))(y) = \bigvee_{x \in f^{-1}(y)} U(x) = \frac{1}{2}$ and by Corollary 4.3, $f(U)$ is an IF-compact relative to Y . Hence Y is locally IF-compact space.

Corollary 5.8. Let $f : X \rightarrow Y$ be an IF-cont. and IF-open surjection function. If X is locally IFS θ -compact, then Y is locally IF θ -compact.

Proof. Obvious, since every locally IF-compact is locally IF θ -compact.

Corollary 5.9. Let Y be an IF-regular space and $f : X \rightarrow Y$ be an IF-open surjection function. If f is an IFweakly function and X is locally IFS θ -compact, then Y is locally IF-compact.

Proof. It is follows from the above Theorem and the fact that every an IFweakly cont. function is an IF-cont. in an IF-regular space.

Theorem 5.10. Let X be an IF-regular space and $f : X \rightarrow Y$ be an IF θ -open bijective function. If f is an IFS θ -cont. and X is locally IFS θ -compact, then so is Y .

Proof. Using Corollary 4.6, the proof similar to the proof of Theorem 5.5.

Theorem 5.11. Let $f : X \rightarrow Y$ be an IFsuper semiopen and IFstrongly θ -cont. surjection function. If Y is an locally IFcompact, then X is an locally IFS θ -compact.

Proof. Let $x(a, b)$ be an IFP in X . Since f is surjective, there is $y(m, n)$ such that $f(x(a, b)) = y(m, n)$. Since Y is locally IFcompact, there is $V \in N_\epsilon(y(m, n))$ such that $\mu_V(y) = 1, \gamma_V(y) = 0$ and V is an IF-compact relative to Y . Using Theorem 4.13, $f^{-1}(V)$ is an IFS θ -compact relative to X . Since f is an IFstrongly θ -cont, then $f^{-1}(V) \in N_\epsilon^\theta(x(a, b))$ and hence $f^{-1}(V) \in N_\epsilon(x(a, b))$. Therefore $f^{-1}(V)(x) = V(f(x)) = V(y) = \frac{1}{2}$. Hence for $x(a, b)$ in X , there is $f^{-1}(V) \in N_\epsilon(x(a, b))$ such that $f^{-1}(V)(x) = \frac{1}{2}$ and $f^{-1}(V)$ is an IFS θ -compact relative to X . Hence X is an locally IFS θ -compact.

Corollary 5.12. Let $f : X \rightarrow Y$ be an IFsuper semiopen and IFstrongly θ -cont. surjection function. If Y is locally IFcompact, then X is locally IFcompact.

Theorem 5.13. Let $f : X \rightarrow Y$ be an IFsemiopen and IFfaintly cont. surjection function. If Y is locally IFS θ -compact, then X is locally IFcompact.

Proof. Using Corollary 4.17, the proof is smiliar to proof of Theorem 5.5.

Theorem 5.14. Let $f : X \rightarrow Y$ be an IFfaintly open and IF θ -cont. surjection function. If Y is locally IFS θ -compact, then X is locally IF θ -compact.

Proof. Using Corollary 4.19, the proof is smiliar to proof of Theorem 5.5.

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