جامعة المنوفية
كلية هندسة شبين الكوم
قسم العلوم الأساسية الهندسية
الفصل الدراسي الأول ٢٠١٤م

اختبار دراسات عليا مستوى ٦٠٠
دوال المتغير المركب (١) (BES 626)
التاريخ: ٢٠١٤/١/١٨م
الزمن: ثلاث ساعات

Solve all the following questions:

1) Letting the function $v=e^{-x}(x\cos y+y\sin y)$. Prove that v is harmonic, find the conjugate u in such away that $f(z)=u+iv$ is analytic, find the orthogonal trajectories of the family of curves $e^{-x}(x\cos y+y\sin y)=\beta$ and express f(z) in terms of z.

2) If $w=f(z)=u+iv$ is analytic in R, show that u satisfies Laplace's equation

$$\nabla^2 u=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$$

3) Letting the function $u=r^2\cos 2\theta$. Prove that u is harmonic, find the complex conjugate v in such away that $f(z)=u+iv$ is analytic, find the orthogonal trajectories of the curves $r^2\cos 2\theta=\alpha$ and express f(z) in terms of z.

4) In the transformation $w=u+iv=z+z^{-1}$ where $z=re^{i\theta}$, express (u,v) in terms of (r,θ) and deduce that circles $r=c$, $(c\neq 1)$ in the z-plane are transformed into con-focal ellipses in w-plane, plot. Using polar coordinates show that w is analytic.

5) Letting R_z be a region in the z-plane bounded by the straight lines $x=0$, $x=2$, $y=0$, $y=1$. Determine the region R_w in the w-plane in which R_z is mapped under the transformation $w=\sqrt{2}\,e^{(\pi/4)i}z+(1-2i)$, plot R_z and R_w. Using the Jacobean of transformation determine the numerical ratio R_w/R_z, then check your result from the graphs.

6) The circle of radius a in ζ-plane is transformed into the aerofoil section in z-plane by the transformation $z=g(\varsigma)=\varsigma+a\varsigma^{-1}$, deduce the inverse function $\zeta=g^{-1}(z)=f(z)$ for large values of z. If $\zeta=f(z)=\eta+i\xi$, using the Cartesian equations of Cauchy Riemann prove that f(z) is analytic.

7) Solve for x the integral equation $x^2+4x\int_0^{\infty} xe^{-x}\sin x\,dx-3=0$

8) If $k\int_{-\infty}^{\infty}\frac{x^2}{(x+1)(x-1)^2}dx=6\pi i$, using the contour integral and the residues theorem, calculate the constant k where $\int_{\Gamma}\frac{x^2}{(x+1)(x-1)^2}dx=0$.