FLASH TEMPERATURE FOR W/N CIRCULAR ARC GEARS

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درجة حرارة التماس اللحظي لمسننات ولدهبر - نوفيكوف ذات الأسنان الدائرية الحلزونية

إن درجة حرارة التماس اللحظي تعتبر واحدة من أهم العوامل التي تأخذ في الإعتبار عند تقدير مقاومة التحزيز لأسنان المسننات. وهذا البحث يقدم حل رياضي عددي لتحديد درجة حرارة التماس اللحظي لمسننات ولدهبر-نوفيكوف ذات الأسنان الدائرية الحلزونية وذلك نتيجة إلى مصدر الحرارة المتحرك آخذاً في الإعتبار كيفية تجزئة كمية الحرارة المتولدة لكل سنة وكل متغيرات التصميم لهذا النوع من المسننات وهي الحمل العمودي المؤثر على السنة، السرعة الدورانية للمسننات، الموديول، نصف قطر التقوس للسنة، زاوية اللولبة الحلزونية، نسبة التماس "التغطية"، نسبة التخفيض في عدد الأسنان للمسننات وكذلك عدد الأسنان. النتائج النظرية لدرجة حرارة التلامس اللحظية تم توضيحها ومناقشتها مع كل متغيرات التصميم السابقة وكذلك تم عمل (Curve fitting) لكل المنحنيات وذلك للحصول على معادلات رياضية تمثل العلاقة بين درجة حرارة التماس اللحظي وكل متغير على حده، وكذلك تم إستنتاج معادلة لدرجة حرارة التلامس اللحظي وكل المتغيرات مع بعضها وهي :

$$\theta/\mu = C\, P^{0.9725} \cdot N^{0.389} \cdot m^{0.968} \cdot \rho^{0.994} \cdot CR^{0.706} \cdot \beta^{0.193}$$

هذه المعادلة تمثل أداة بسيطة تسهل على المصممين كيفية حساب درجة حرارة التلامس اللحظي وتحديد سعة التحميل وعمر هذا النوع الجديد من المسننات. كذلك أظهرت النتائج وجود حد أدنى لدرجة حرارة التلامس اللحظي مع تغير نسبة التخفيض في عدد الأسنان للمسننات وكذلك عدد الأسنان نفسها.

ABSTRACT

The flash temperature is one of the important factors for evaluating the scoring resistance of the gear teeth . In the present paper, a numerical solution of the flash temperature for W/N circular - arc gear due to moving heat source is done taking into account the partitioning of the heat in the contact zone to each tooth and all the variables for the design of this type of gears . Tooth load, speed of rotation , module , radii of curvature , helix angle , gear ratio , number of teeth and contact ratio are considered . The theoretical results of the flash temperature are presented and discussed with the above variables . A curve fitting of the results is done and the following formula derived for the flash temperature

$$\theta/\mu = C\, P^{0.9725} \cdot N^{0.389} \cdot m^{0.968} \cdot \rho^{0.994} \cdot CR^{0.706} \cdot \beta^{0.193}$$

This formula represents a simple tool for the designer to calculate the flash temperature the corresponding load carrying capacity and life of the gear are determined . The study also shows that there is a certain minimum value for the flash temperature with the change of the gear ratio.

NOMENCLATURE

English Alphabet

a	semi - major axis of the elliptical area of contact , m
b	semi - minor axis of the elliptical area of contact , m
c	specific heat of gear material, *KJ/ KN . deg C*
c' and c*	constant in the equations ,
C	constant in the flash temperature equation and given in appendix I
CR	contact ratio (overlap ratio)
E	Young's modulus of gear material , *KPa*
F	face width of the gear , m
G	gear ratio
J	mechanical equivalent of heat , *KN. m/KJ*
k	diffusivity of gear material m^2/s
K	constant for the equations
m	module , m
m'	constant depending on the ratio $=\varepsilon_2/\varepsilon_1$
N	speed of rotation of the pinion, *number of revolutions per min*
n'	constant depending on the ratio $=\varepsilon_2/\varepsilon_1$

p Hertz's contact pressure , *KPa*
p_o the maximum pressure exists at the center of the elliptical area of contact , *KPa*
P_t transverse circular pitch, *m*
P normal tooth load , *KN*
q rate of generated heat per unit area, per unit time , $KJ/m^2.s$
Q heat source
r_1 and r_2 pitch radius of the pinion and the wheel , *m*
R_1 , R_2 , R_1' and R_2' principal radii of curvature at the contact point,*m*
t time of mesh, *s*
u_1 and u_2 entrainment velocity , *m/s*
v_s sliding velocity , *m/s*
x,y and z cartesian coordinate system
x_t position of the center of the moving heat source at time t
x' moving coordinate system whose origin is the center of heat source
x^* constant in the equations
z_1 and z_2 number of teeth of the pinion and the wheel.

Greek Alphabet

α pressure angle , *degree*
β helix angle , *degree*
γ specific weight of gear material, Kg/m^3
ε_1 and ε_2 roots of the quadratic equation defining the contact surfaces
η constant in the equations
θ flash temperature, ^oC
λ constant in the equations
μ coefficient of friction
υ Poisson's ratio of the gear material
ξ constant in the equations
ρ_1 profile radius of the pinion tooth, *m*
ρ_2 profile radius of the wheel tooth , *m*
$\Delta\rho$ mismatch in radius of curvature of the tooth profiles of the pinion and of the wheel in transverse plane,
ϕ coefficient of local partition of heat
ψ angle between the planes containing the maximum or the minimum principal radius of curvature
ψ_1 auxiliary angle dependent on ε_1 and ε_2
ω angular velocity of the gear *rad/s*

INTRODUCTION

Gear systems are being used more frequently at high speed and heavy load, and scoring resistance of gears has become an important factor in evaluating their strength . Generally, scoring is considered to be related to the instantaneous temperature rise on tooth surface caused by frictional heat , and this concept of flash temperature is recommended by *AGMA 217.01* as the most reliable means to determine the scor ing resistance . The total temperature in the contact is the sum of the bulk temperature of the gear and the flash temperature .

The first theoretical study on the flash temperature caused by friction between two bodies was done by Blok in *(1937)* . He assumed one- dimensional heat flow which leads to a simple and efficient approximate equation on flash temperature . A similar study was also done by Jaeger *(1942)*.

More detailed studies were done by Holm *(1948)*. Bowden and Tabor *(1950)* , and Nakada and Hashimoto *(1963)*. Archard *(1958-1959)* has also referred to both elastic and plastic contact .The most recent studies were made by Symm *(1967)* who determined the flash temperature and partition of generated heat between two rubbing bodies numerically. Tobe and Kato *(1974)* examined unsteady conditions in line contacts in which the intensity and velocity of the moving heat source change instantaneously at it moves through the contact . Terauchi and Mori *(1974)* considered effects of dynamic load on flash temperature under the influence of different load - speed conditions.

Roylance and Alkaleb *(1987)* determined the surface temperature components, bulk and flash temperatures, during a four - ball operation .

The main purpose of the present work is to determine the formula of the flash temperature of the Wildhaber / Novikov (W/N) circular - arc gears and the partition of heat to both teeth within the contact band . Also the influences of the applied tooth load , speed of rotation , module , helix angle, radii of curvature , gear ratio, number of teeth and contact ratio are studied.

ANALYSIS OF THE FLASH TEMPERATURE ON W/N CIRCULAR - ARC GEARS

1 - W/N Circular - Arc Gears :

Wildhaber Novikov (W/N) circular-arc gears are conformal gears of convex concave tooth profile in transverse plane and convex convex tooth profile in axial plane . Contact is theoretically at a point, which under load becomes an ellipse . The elliptical area of contact moves in axial direction along the tooth face at a fixed height above the root of the teeth as shown in Fig (1) .

2 - Flash Temperature Equations :

Let the x- axis be the moving direction of the ellipsoidal heat source, y- axis along the tooth height . The z- axis towards the inside of a semi-infinite body, and the surface of the teeth are $z = 0$. The rate of heat q generated per unit area in unit time distributes from $x = -a$ to $x = a$, and $y = -b$ to $y = b$. The heat source is assumed to move along the tooth face in x- direction .
The quantity of the heat source is given by the Hertzian elastic contact stress $P(x,y)$, sliding velocity between gear teeth v_g and the coefficient of frictionμ.

$$q(x,y) = 1/J \;\; \mu \;\; p(x,y) \, |v_g| \tag{1}$$

Consider the case where the heat source moves on the surface of the semi - infinite body , and assume the bulk temperature to be zero . When a heat source Q is given on the surface $x=\bar{x}$, $y=y$ at time $t=\bar{t}$, the temperature θ at point (x, y, z) and at time t=t is expressed in the form

$$\theta = \frac{Q}{2\pi\gamma ck(t-\bar{t})} \exp\left\{-\frac{(x-\bar{x})^2+y^2+z^2}{4k(t-\bar{t})}\right\} \tag{2}$$

Fig (2) shows a heat source moving on the surface of the tooth along the path of contact. If we take the center of the ellipsoidal heat source at the starting of meshing ($t = 0$) as the origin of the x - axis , the center will be at $x = x_t$ at $t=t$. Using equation (2), the temperature of the point P (x,y, z) at t=t is obtained by summation of temperature rise caused by the heat source q $d\bar{x}$ dy dt at each instant from t=o to t=t

$$\theta(x,y,z,t) = \frac{1}{2\pi\gamma ck}\int_0^t \frac{d\bar{t}}{t-\bar{t}} \int_{x_t-a}^{x_t+a}\int_{y_t-b}^{y_t+b} q(\bar{x}-\bar{x}_t, y, \bar{t}) \exp\left\{-\frac{(x-\bar{x})^2+y^2+z^2}{4k(t-\bar{t})}\right\} dy\, d\bar{x} \tag{3}$$

The surface temperature at the point p(x,y) (z=o) at time t=t

$$\theta(x,y,t) = \frac{1}{2\pi\gamma ck}\int_0^t \frac{d\bar{t}}{t-\bar{t}} \int_{x_t-a}^{x_t+a}\int_{y_t-b}^{y_t+b} q(\bar{x}-\bar{x}_t, y, \bar{t}) \exp\left\{-\frac{(x-\bar{x})^2+y^2}{4k(t-\bar{t})}\right\} dy\, d\bar{x} \tag{4}$$

Introducing the following new variables due to an imaginary singular point at $\bar{t}=t$

a - Let $\lambda = (t-\bar{t})^{1/2}$, then $d\bar{t} = -2\lambda\, d\lambda$,

b- Let $\xi = \frac{x-\bar{x}}{\sqrt{4k\lambda}}$, then $d\bar{x} = -2\sqrt{k}\lambda d\xi$,

c- Let $\eta = \frac{y}{\sqrt{4k\lambda}}$, then $dy = -2\sqrt{k}\lambda d\eta$,

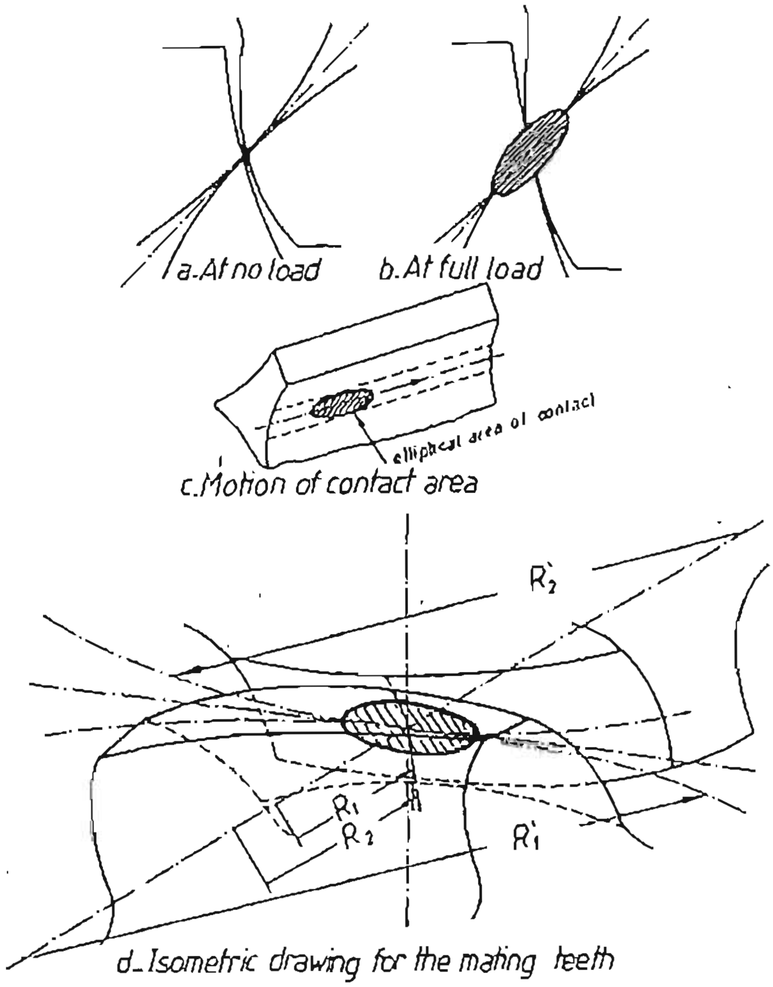
By substituting these variables in the equation (4) and determining the limits of integrations, we can obtain the following equation

$$\xi_u = \frac{x-x_t+a}{2\sqrt{k}\lambda}, \quad \xi_l = \frac{x-x_t-a}{2\sqrt{k}\lambda}$$

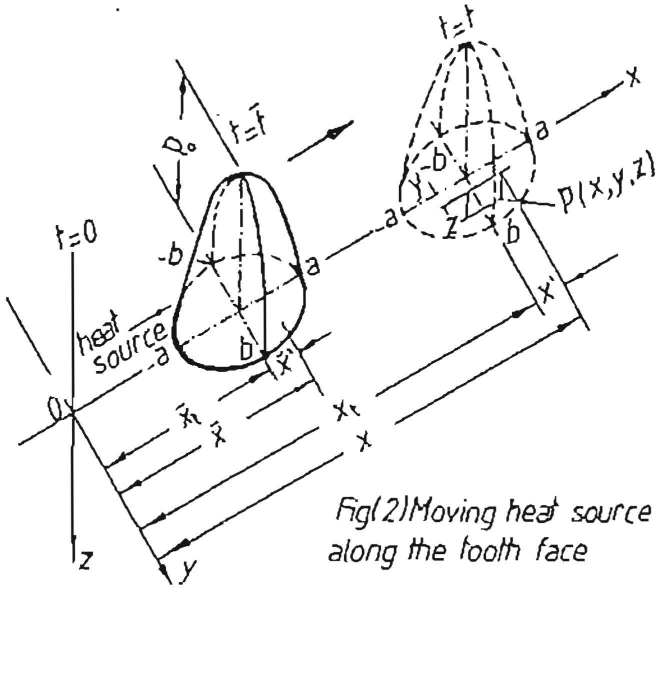
$$\eta_u = \frac{y_t+b}{2\sqrt{k}\lambda}, \quad \eta_l = \frac{y_t-b}{2\sqrt{k}\lambda}$$

$$\therefore \theta(x,y,t) = \frac{4}{\pi\gamma c}\int_0^{\sqrt{t}} \lambda d\lambda \int_{\xi_l}^{\xi_u}\int_{\eta_l}^{\eta_u} q(x-\bar{x}_t-2\sqrt{k}\lambda\xi,\ 2\sqrt{k}\lambda\eta,\ t-\lambda^2) \exp\{-(\xi^2+\eta^2)\}\, d\eta\, d\xi \tag{5}$$

If q is the rate of generated heat between the contacting teeth and ϕ is the local partition of heat. the ϕ q is the amount of heat flowing into tooth 1 (pinion) and the remainder $(1-\phi)$ q



Fig(1) Geometry of teeth profiles before and after loading



Fig(2) Moving heat source along the tooth face flows into tooth 2 (wheel). Then the surface temperature rises of two teeth are obtained from equation (5) as

$$\theta_1(x', y, t) = \left[\frac{4}{\pi\gamma c}\int_0^{\sqrt{t}}\lambda d\lambda \int_{\xi_l}^{\xi_u}\int_{\eta_l}^{\eta_u}\phi\, q\, exp\{-(\xi^2+\eta^2)\}\, d\eta\, d\xi\right]_1 \quad (6)$$

$$\theta_2(x', y, t) = \left[\frac{4}{\pi\gamma c}\int_0^{\sqrt{t}}\lambda d\lambda \int_{\xi_l}^{\xi_u}\int_{\eta_l}^{\eta_u}(1-\phi)\, q\, exp\{-(\xi^2+\eta^2)\}\, d\eta\, d\xi\right]_2 \quad (7)$$

where $x'=x-x_l$

The surface temperatures of both bodies are required to be equal at each point in contact. The bulk temperatures of both bodies are assumed to be zero, so the imposed condition is

$$\theta_1(x',y,t) = \theta_2(x',y,t) \quad (8)$$

over the contact band. This equation determines the unknown function ϕ.

3 - Semi - Major and Semi - Minor Axes of the Elliptical Area of Contact :

The elliptical area of contact is a function of the geometry of the surfaces in contact, the elastic constants of the material of the gears and the normal load on the gear teeth. The semi - major and semi - minor axes of the ellipse are obtained from Hertz's contact stress equations. The following equations for the principal radii of curvature of the surfaces at the contact point are obtained. If $\Delta\rho$ is considered; $\Delta\rho=(R_2-R_1)$

$$R_1 = \rho_1 = c^* m = \rho \quad (9)$$

$$R_2 = \rho_2 = c\, \rho_1 \quad (10)$$

$$R'_1 = \frac{r_1\,(1+\tan\beta.\cos\alpha)^{3/2}}{\tan^2\beta.\,\sin\alpha(1+\sin\alpha.r_1/\rho_1)} \quad (11)$$

$$R'_2 = \frac{r_2\,(1+\tan\beta.\cos\alpha)^{3/2}}{\tan^2\beta.\,\sin\alpha(1-\sin\alpha.r_2/\rho_2)} \quad (12)$$

The roots ε_1 and ε_2 of the quadratic equation defining the elliptical area of contact, are dependent on R_1, R_2, R'_1, R'_2, and the angle ψ between the planes containing the maximum or minimum principal radii of curvature and are given by the following equations

$$\varepsilon_1 = \frac{1}{2}\left[\frac{1}{R_1}+\frac{1}{R'_1}+\frac{1}{R_2}+\frac{1}{R'_2}\right] \quad (13)$$

$$\varepsilon_2 = \frac{1}{2}\left[\left(\frac{1}{R_1}-\frac{1}{R'_1}\right)^2+\left(\frac{1}{R_2}-\frac{1}{R'_2}\right)^2+2\left(\frac{1}{R_1}-\frac{1}{R'_1}\right)\left(\frac{1}{R_2}-\frac{1}{R'_2}\right)\cos2\psi\right]^{0.5} \quad (14)$$

For Wildhaber - Novikov gears, $\psi = 0$

ε_1 and ε_2 are related by the auxiliary angle ψ_1, given by

$$\psi_1 = \cos^{-1}(\varepsilon_2/\varepsilon_1) \quad (15)$$

From the values of ψ_1, the constants m' and n' in the following equations are obtained (Roark, 1965; Timoshenko and Goodier, 1984)

$$a = m'\left[\frac{1.5\,PA}{\varepsilon_1}\right]^{1/3} \quad (16)$$

$$b = n'\left[\frac{1.5\,PA}{\varepsilon_1}\right]^{1/3} \quad (17)$$

$$A = \frac{1}{2}\left[\frac{1-\upsilon_1^2}{E_1}+\frac{1-\upsilon_2^2}{E_2}\right], \text{ for the same material} \quad A = \frac{1-\upsilon^2}{E}$$

4. Contact Pressure Distribution :

According to Hertz, the intensity of pressure, p, over the surface of contact is represented by the ordinates of a semi - ellipsoid constructed on the surface of contact. thus

$$p = p_0\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} \quad (18)$$

The maximum pressure (p_0) exists at the center of the surface of contact. Since the total tooth load P. is equal to the volume of the semi - ellipsoid.

$$P = \frac{2}{3}\pi a b p_0 \quad (19)$$

from this expression the maximum pressure. p_0 is found to be

$$p_0 = 1.5\frac{P}{\pi a b} \quad (20)$$

5 - Sliding and Entrainment Velocities :

According to Dyson, Evans and Snidle *(1986)* the entrainment velocity is the mean of the surface velocities relative to the point of contact ; thus

$$\left.\begin{aligned} u_1 &= \omega r\ (\cot^2\beta + \cos^2\alpha)^{1/2}, \\ u_2 &= \frac{\omega}{2}(1 - G^{-1})(\rho_1 - x^*) - \omega r \sin\alpha \end{aligned}\right\} \quad (21)$$

are the components of the entrainment velocity. The absolute magnitude of the sliding velocity is

$$vs = \omega(1 - G^{-1})(\rho_1 - x^*) \quad (22)$$

6 - Time of Meshing :

Contact ratio for W/N circular - arc gears is equal to tooth advance / transverse circular pitch (Fig(3-a))

$$CR\ .\ P_t = F\tan\beta \quad (23)$$

By dividing the equation (23) by the velocity

$$CR\ .\ t_p = t_t \quad (24)$$

t_p = time of meshing for single pair = $1/Nz$

t_t = total time of meshing = time of meshing for single pair + total time of meshing for double pair ($2t^*$)

$$CR\ .\ t_p = t_p + 2t^*$$

$$\therefore 2t^* = CR\ .\ t_p - t_p = 1/Nz\ (CR - 1)$$

Then, time of meshing for a double pair in contact at the start or end of contact is .

$$t^* = 1/2Nz\ (CR - 1) \quad (25)$$

and the time of meshing at any zone of contact is given in Fig (3 -b).

CALCULATION OF THE FLASH TEMPERATURE

The flash temperature on W/N circular-arc gear teeth caused by frictional heat can be obtained by substituting the equations (1),(16), (17),(18), (22) and (24) to equations (6) and(7) and then relating equation (8). This solution is found numerically. The meshing time is divided into short intervals Δt and the unknown function ϕ is determined step by step from $t=0$ to any time $t=t$; that is ,in the case of $t = j\Delta t$ ($j=1,2,3,\ldots\ldots$), equation (6) for pinion can be approximated in the following polynominals (Cheney and Kincaid (1980) and Gerald (1978)).

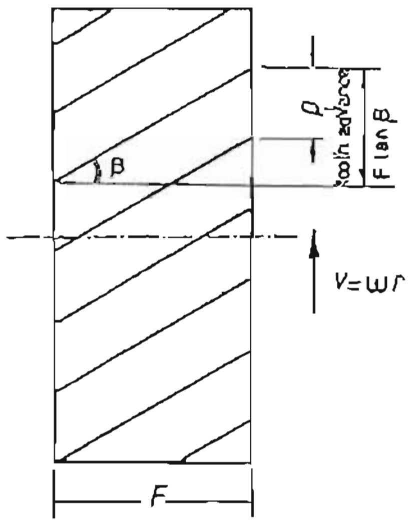
$$\theta_1(x^*, y, j\Delta t) = \left[\frac{2}{\pi\gamma c}\sum_{r=1}^{j}\{(K_{r-1} + K_r)(\sqrt{r\Delta t} - \sqrt{(r-1)\Delta t})\}\right]_1 \quad (26)$$

$$\left[K_r = 4\int_0^{\xi_u}\int_0^{\eta_u}\sqrt{(j-r)\Delta t}\,\phi\{\bar{x}^*, y, (j-r)\Delta t\}\,q\{\bar{x}^*, y, (j-r)\Delta t\}\exp\{-(\xi^2 + \eta^2)\}\,d\eta\,d\xi\right]_1 \quad (27)$$

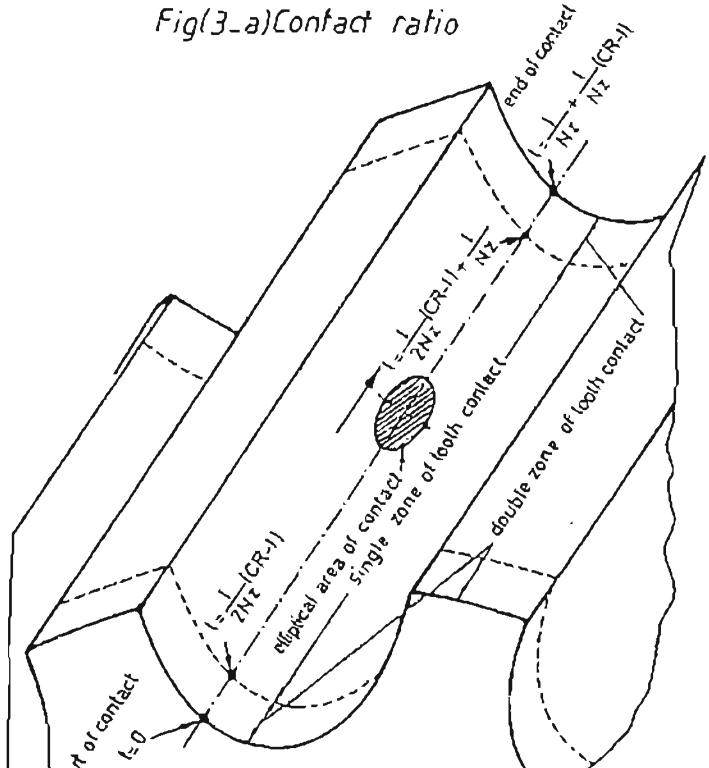
$$\left[K_o = 4\sqrt{\pi}\sqrt{j\Delta t}\,\phi(x^*, y, j\Delta t)\,q(x^*, y, j\Delta t)\right]_1 \quad (28)$$

Applying the trapezoidal rule to the equation of $[K_r]_1$.

$$\left[K_r = 2h_1\sum_{l=1}^{Nl}(S_l + S_{l-1})\right]_1 \quad (29)$$



Fig(3_a)Contact ratio



Fig(3_b)Time of meshing along the tooth face where

$$\left[S_l = \int_0^{\xi_u} \sqrt{(j-r)\Delta t}\ \phi\{\bar{x}', lh_1, (j-r)\Delta t\}\ q\{\bar{x}', lh_1, (j-r)\Delta t\} \exp\{-(\xi^2 + (lh_1)^2)\}\ d\xi \right]_1$$

Also applying the trapezoidal rule to the equation of s_l

$$\left[S_l = \frac{1}{2} h_2 \sum_{m=1}^{N2} (q_m + q_{m-1}) \right]_1 \qquad (30)$$

$$\left[q_m = \sqrt{(j-r)\Delta t}\ \phi\{mh_2, lh_1, (j-r)\Delta t\}\ q\{mh_2, lh_1, (j-r)\Delta t\} \exp\{-[(mh_2)^2 + (lh_1)^2]\} \right]_1$$

Similarly, for the wheel

$$\theta_2(x', y, j\Delta t) = \left[\frac{2}{\pi\gamma c} \sum_{r=1}^{j} \{(K_{r-1} + K_r)(\sqrt{r\Delta t} - \sqrt{(r-1)\Delta t})\} \right]_2 \qquad (31)$$

$$\left[K_r = 4 \int_0^{\xi_u}\int_0^{\eta_u} \sqrt{(j-r)\Delta t}\ \{1 - \phi(\bar{x}', y, (j-r)\Delta t)\}\ q\{\bar{x}', y, (j-r)\Delta t\} \exp\{-(\xi^2+\eta^2)\}\ d\eta\ d\xi \right]_2 \qquad (32)$$

$$\left[K_o = 4\sqrt{\pi}\ \sqrt{j\Delta t}\ \{1 - \phi(x', y, (j\Delta t))\}\ q(x', y, j\Delta t) \right]_2 \qquad (33)$$

Applying the trapezoidal rule to the equation of $[Kr]_2$

$$\left[K_r = 2 h_1 \sum_{l=1}^{N1} (S_l + S_{l-1}) \right]_2 \qquad (34)$$

where

$$\left[S_l = 4 \int_0^{\xi_u} \sqrt{(j-r)\Delta t}\ \{1 - \phi(\bar{x}', lh_1, (j-r)\Delta t)\}\ q\{\bar{x}', lh_1, (j-r)\Delta t\} \exp\{-(\xi^2 + (lh_1)^2)\}\ d\xi \right]_2$$

Also applying the trapezoidal rule to the equation of $[S_l]_2$

$$\left[S_l = \frac{1}{2} h_2 \sum_{m=1}^{N2} (q_m + q_{m-1}) \right]_2 \qquad (35)$$

where

$$\left[q_m = \sqrt{(j-r)\Delta t}\ \{1 - \phi(mh_2, lh_1, (j-r)\Delta t)\}\ q\{mh_2, lh_1, (j-r)\Delta t\} \exp\{-[(mh_2)^2 + (lh_1)^2]\} \right]_2$$

From these equations and equation (8), $\phi(x', y, j\Delta t)$ can be determined.

A flow chart of the calculations is shown in Fig (4).The loop concerning J is for the meshing time, the loop concerning M is for the position over the elliptical area of contact along the tooth face x-direction and loop L concerning the second dimension of the elliptical area of contact along the tooth height in y-direction . In the case of calculations J=400,M=50 and L=50 to insure a maximum accuracy . This work is programmed with Fortran and run on the VAX Computer system under VMS operating system.

THEORETICAL RESULTS AND DISCUSSION

1- Effect of Tooth load:

Fig (5) shows the change of the flash temperature divided by the coefficient of friction with the change of the applied tooth load at different running conditions and gear variables (speed, module, radii of curvature, contact ratio, helix angle, gear ratio and number of teeth). From this figure it is clearly shown that the flash temperature increases with increase of the applied tooth load for all running conditions and gear variables. It is noticed that the quantity of the heat generated and temperature increased with increase of applied tooth load and this could be attributed to the increase of Hertzian area of contact and contact pressure under load. This is indicated in Fig (6).

A curve fitting for these results has been found using Grapher Soft ware which gives the following equation

$$\theta/\mu = (0.01\text{-}0.177)\ P^{(0.802\text{-}0.979)} \qquad (36)$$

Ranges of constants given in this equation depend on the variety of the running conditions and gear variables. The fitting equation for each curve is indicated in Fig (5).

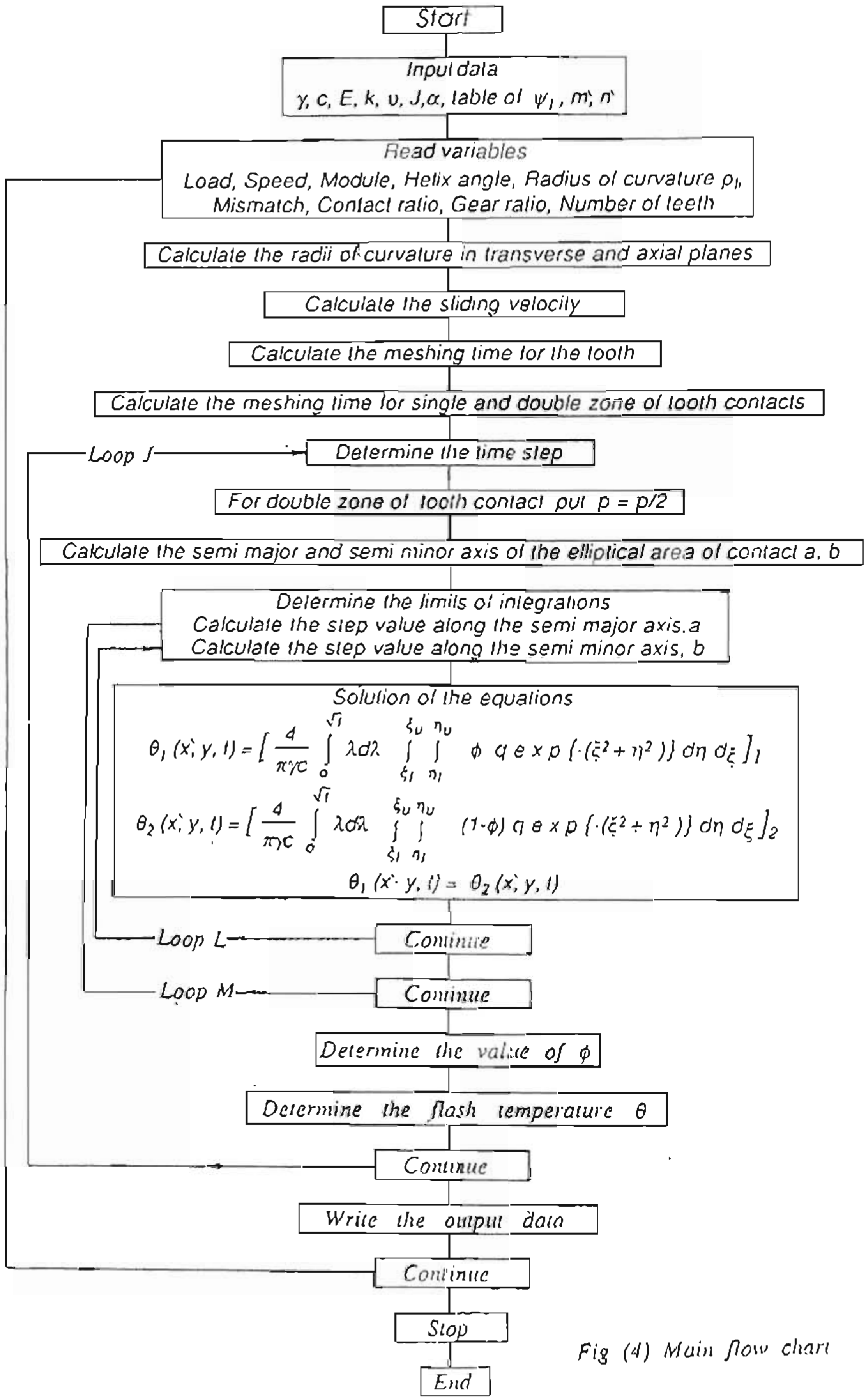
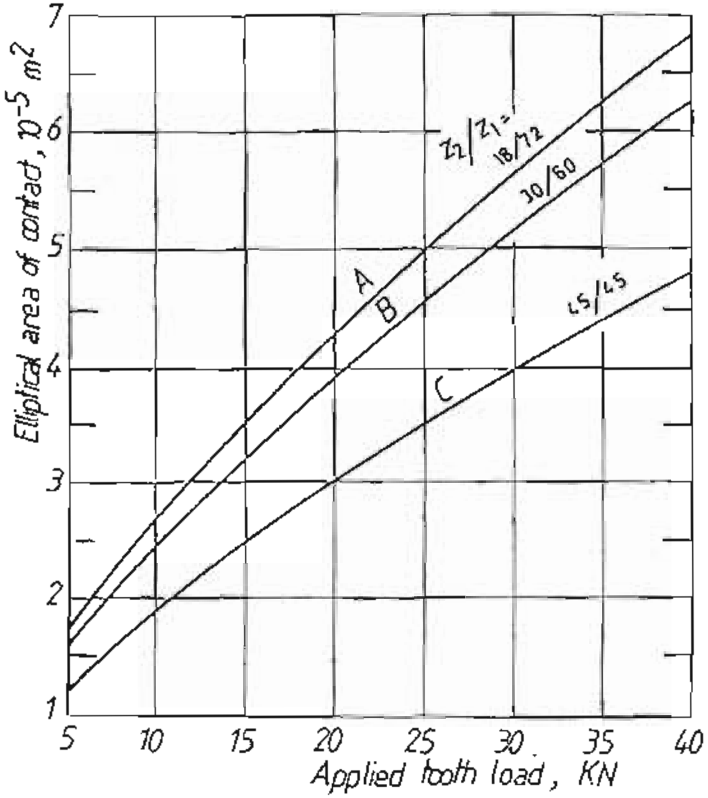
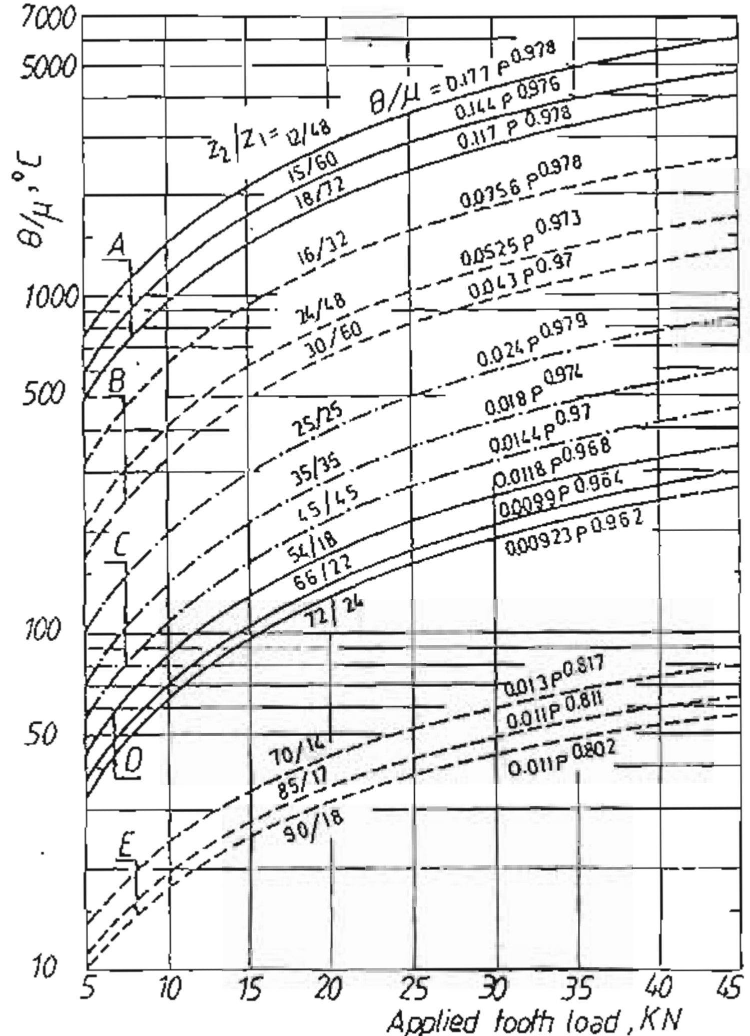


Fig (4) Main flow chart



Fig(6) Change of the elliptical area of contact with the applied tooth load for different gear variables



Fig(5) Change of the flash temperature divided by the coefficient of friction (θ/μ) with tooth load at different running conditions and gear variables. Conditions. A, B, C, D and E see appendix I.

2- Effect of Speed of Rotation :

Fig (7) shows the change of the flash temperature divided by the coefficient of friction with the change of the speed of rotation at different running conditions and gear variables (tooth load, module, radii of curvature, helix angle, contact ratio, gear ratio and number of teeth). From this figure it can be shown that the flash temperature increases with a decreasing rate of increase with increase of the speed of rotation. This could be explained as follows :
With the increase of rotating speed, the sliding velocity increases which accordingly increases the amount of heat (equation1) and the flash temperature. On the other hand, with the increase of speed of rotation the time of contact between teeth along the path of tooth contact decreases, which decreases the flash temperature.

.A curve fitting for these results has been found using Grapher soft ware and which gives the following equation

$$\theta/\mu = (\ 1.1\text{-}152.2)\ N^{(0.321\text{-}0.391)} \qquad (37)$$

Ranges of constants given in this equation depend on he variety of the gear variables and running conditions. The fitting equation for each curve is indicated in Fig(7) .

3- Effect of Module :

Fig (8) shows the change of the flash temperature divided by the coefficient of friction with the change of the module at different running conditions and gear variables . It is noticed that with increase of module the flash temperature and the amount of heat increrase for all running conditions and gear variables. This may be due to the increase of Hertzian contact pressure and area of contact Fig(9) (heat generation increases).

Curve fitting for the obtained results was done and the following relationship between the flash temperature divided by the coefficient of friction and the module has been obtained.

$$\theta/\mu = (\ 1494\text{-}259172)\ m^{(0.8\text{-}0.979)} \qquad (38)$$

The ranges of constants in this equation depend on the variety of the running conditions and gear variables and the equation for each condition is indicated on the appropriate curve shown in Fig(8).

4-Effect of Helix Angle :

Fig(10) shows the change of the flash temperature divided by the coefficient of friction with the change of the helix angle at different running conditions and gear variables . These curves show that flash temperature slightly increases with increase of the helix angle of W/N circular arc gear at all running conditions and gear variables . This may be due to the following:
With increase of helix angle the length of tooth face increases, accompanied by an increase of accumulated heat generation and temperature due to increase of total time of mesh. On the other hand, with increase of helix angle, the Hertzian contact pressure and area of contact decreases as shown in Fig(11) which decreases the heat generation and the flash temperature. Accordingly, the flash temperature slightly increases with the increase of the helix angle .

A curve fitting for these results has been found giving the following equation

$$\theta/\mu = (\ 63.9\text{-}2463)\ \beta^{(0.0619\text{-}0.279)} \qquad (39)$$

Ranges of constants given in this equation depend on the variety of the gear variables and the running conditions. The fitting equation for each curve is indicated in Fig(10).

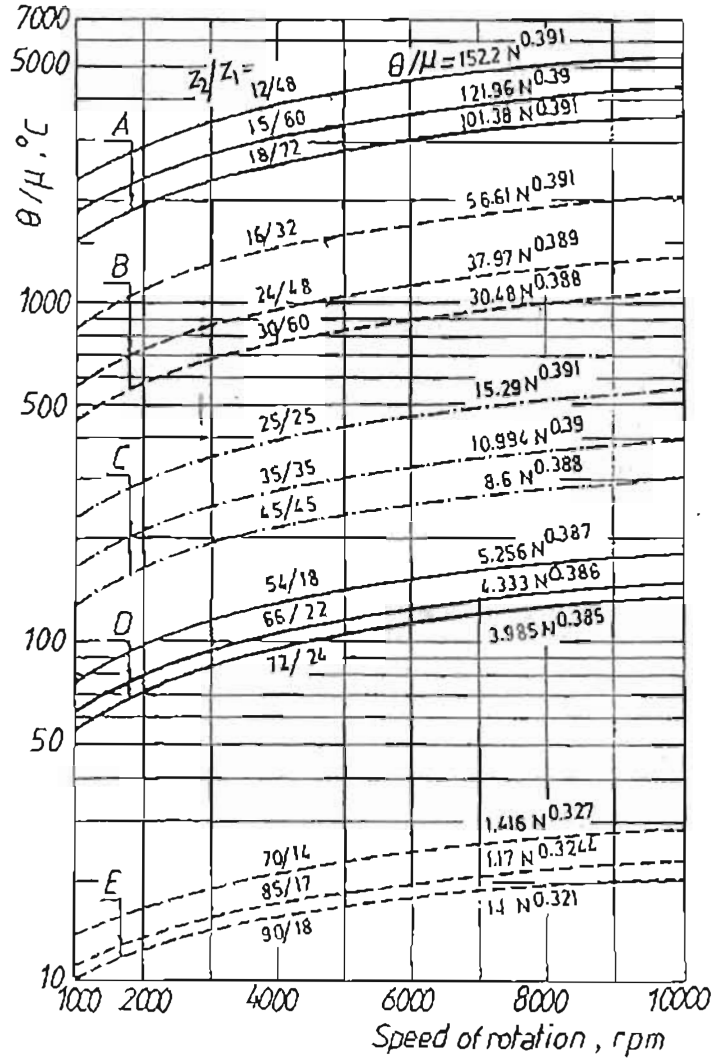
5- Effect of Radius of Curvature :

Fig(12) shows the change of the flash temperature divided by the coefficient of friction with the change of the radius of curvature of the pinion tooth in the transverse plane for W/N circular - arc gears at different running conditions and gear variables. It is very clear that the flash temperature increases with the increase of the radius of curvature. The rate of increase is high for the smallest value of the radius of curvature and decreases with an increase of the radius of curvature. This may be because, with increase of the radius of curvature of the pinion tooth, all radii of curvature of the pinion and wheel teeth increase, dimensions of the tooth increase, and the contact pressure and Hertzian area of contact increase as shown in Fig(13). Accordingly, the quantity of the heat generation and the flash temperature increase.

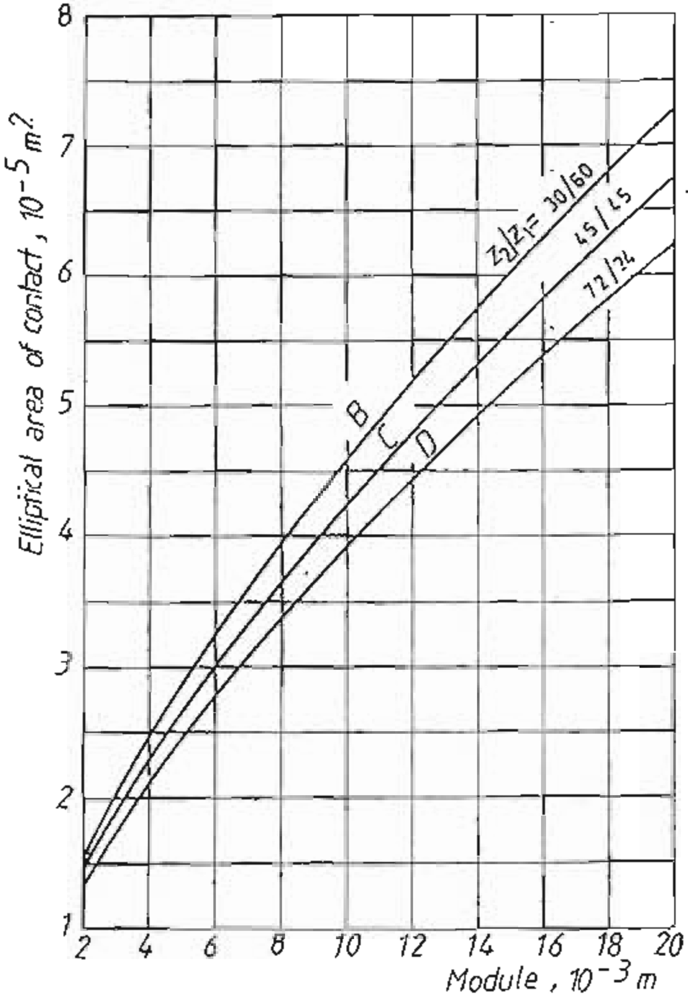
Curve fitting for the obtained results was done and the following equation was obtained

$$\theta/\mu = (\ 1322\text{-}89710)\ \rho^{(0.735\text{-}1)} \qquad (40)$$

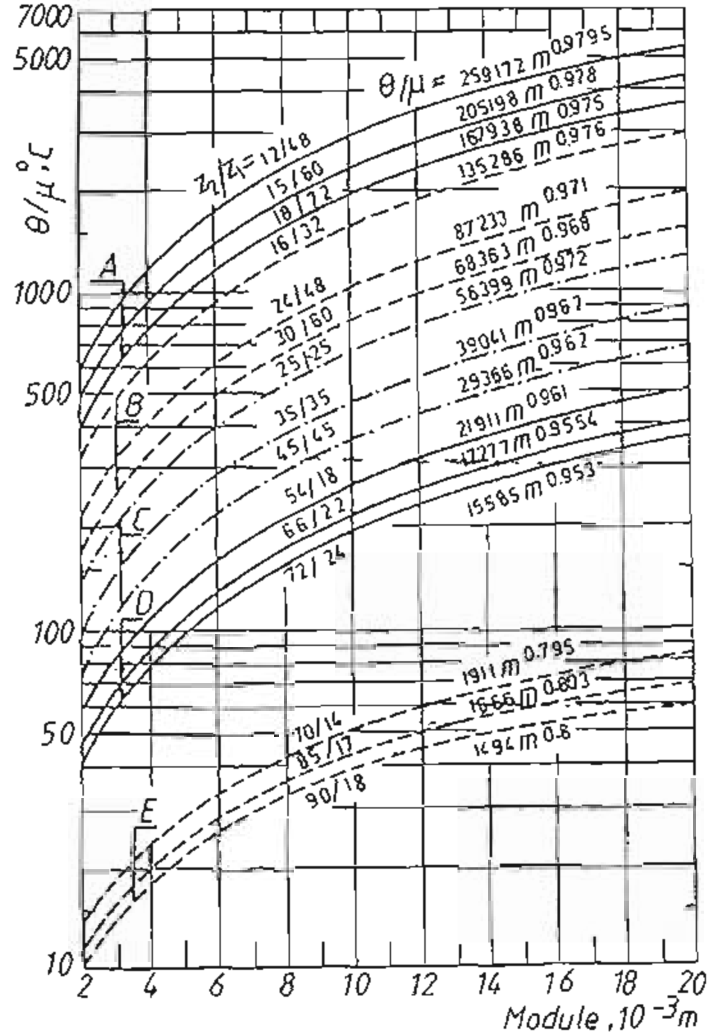
The ranges of constants in this equation depend on the variety of the running conditions and gear variables and the equation for each condition is indicated at its allotted curve shown in Fig(12).



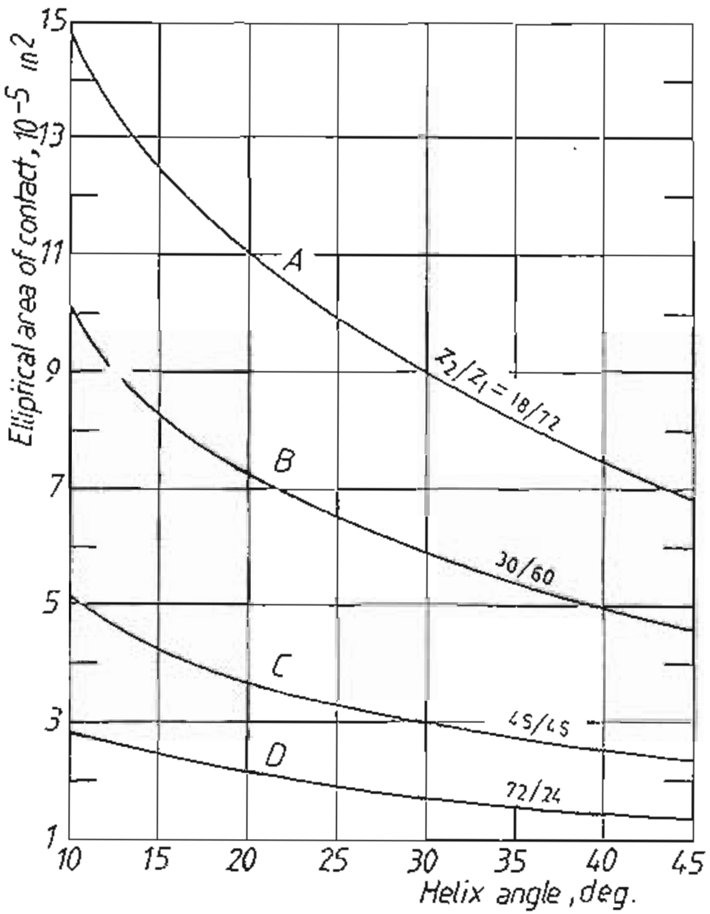
Fig(7)Change of the flash temperature divided by the coefficient of friction(θ/μ) with speed of rotation at different running conditions and gear variables.
Conditions A, B, C, D and E are given in appendix 1



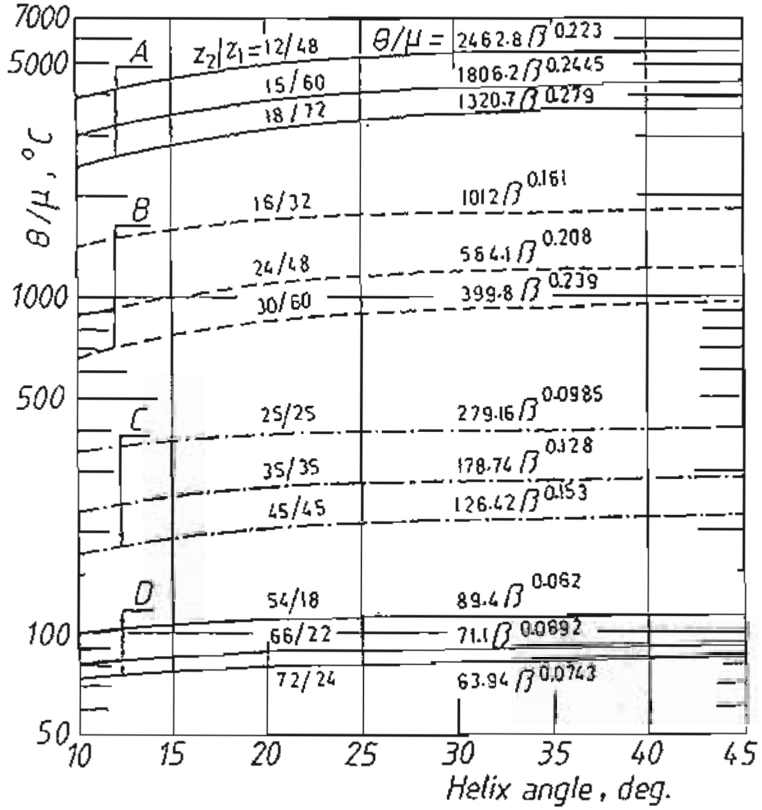
Fig(9)Change of the elliptical area of contact with the module at different running conditions.



Fig(8)Change of the flash temperature divided by the coef. ficient of friction with the module at different running condi. tions and gear variables. Conditions A,B,C,D,E are given in appendix I



Fig(11)Change of the elliptical area of contact with the helix angle at different running conditions.



Fig(10)Change of the flash temperature divided by the coef. ficient of friction(θ/μ)with the helix angle at different running conditions and gear variables. Conditions A,B,C and D are given in appendix I.

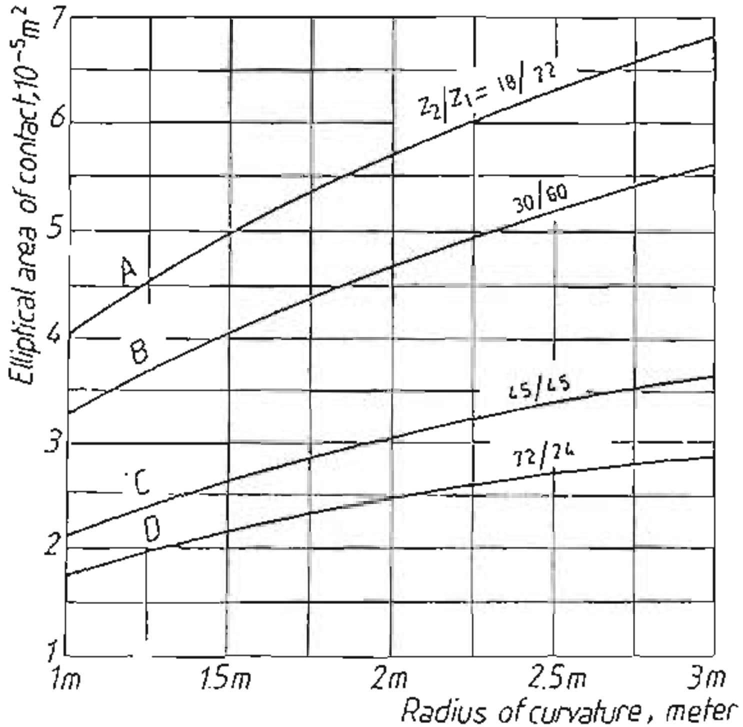


Fig (13) Change of the elliptical area of contact with the radius of curvature of the pinion tooth at different running conditions.

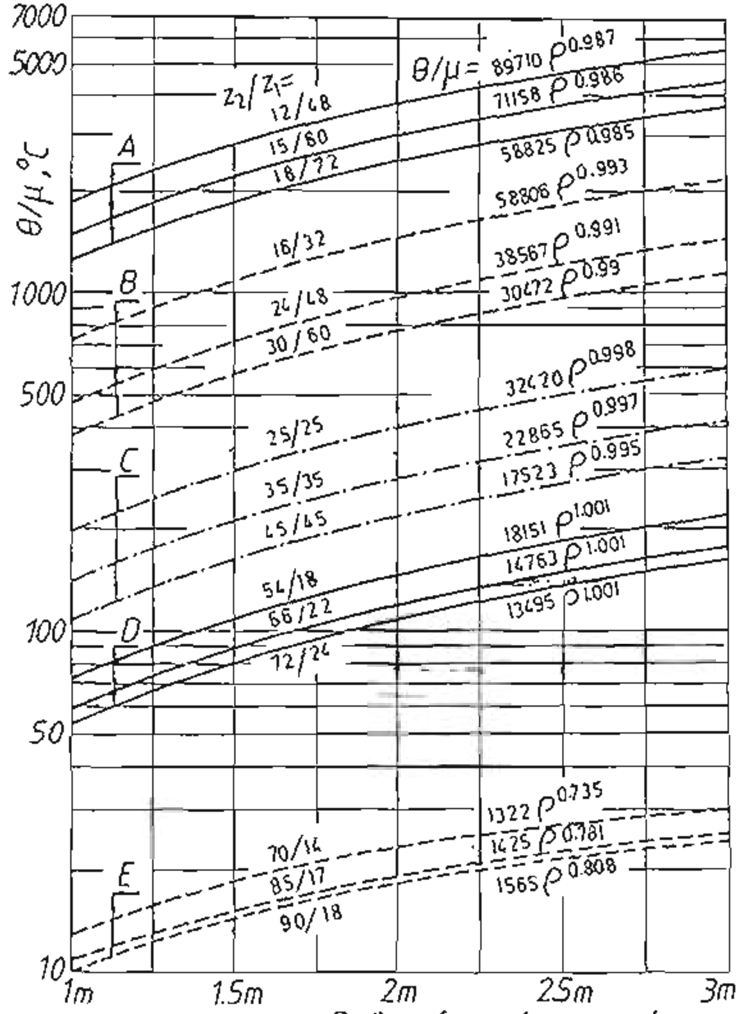
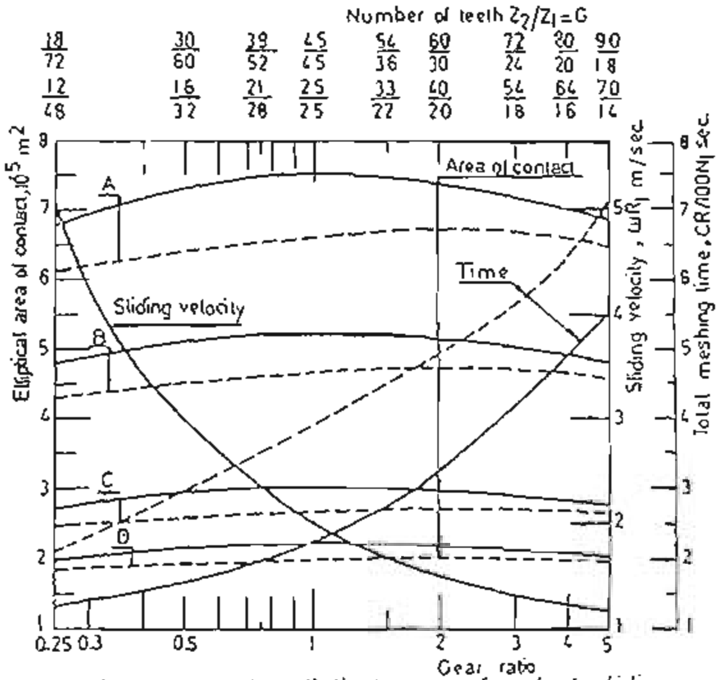
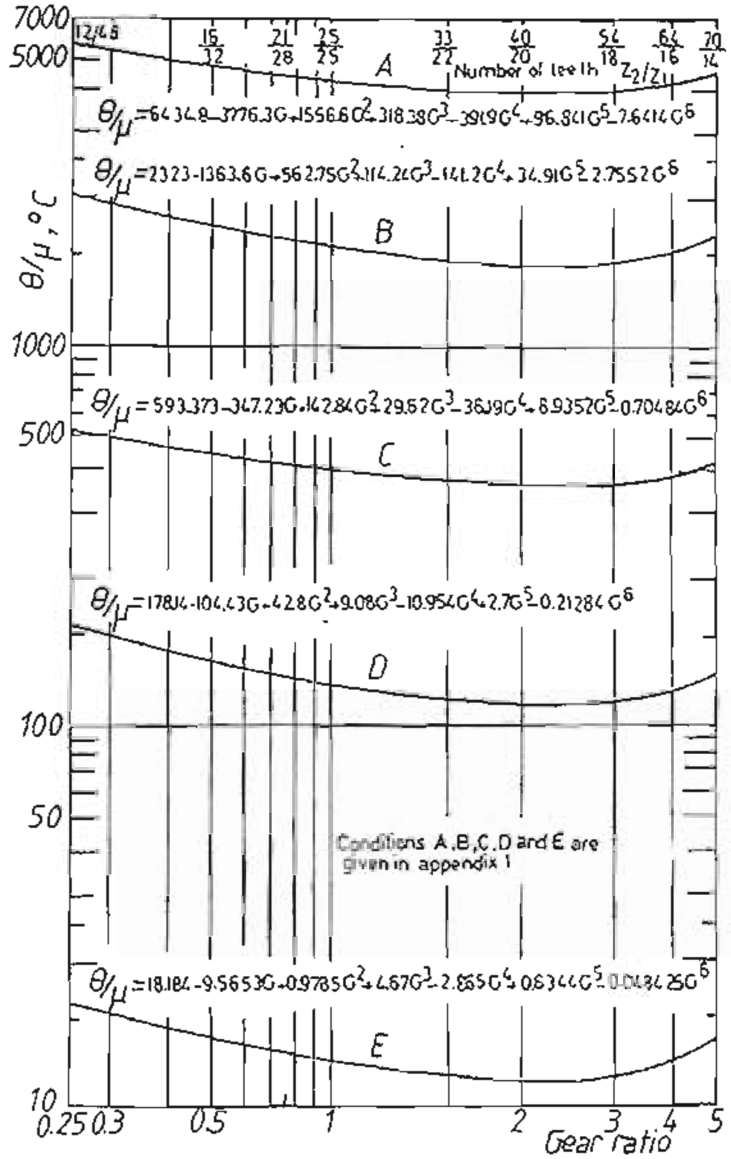


Fig (12) Change of the flash temperature divided by the coefficient of friction (θ/μ) with the radius of curvature of the pinion tooth at different running conditions and gear variables.

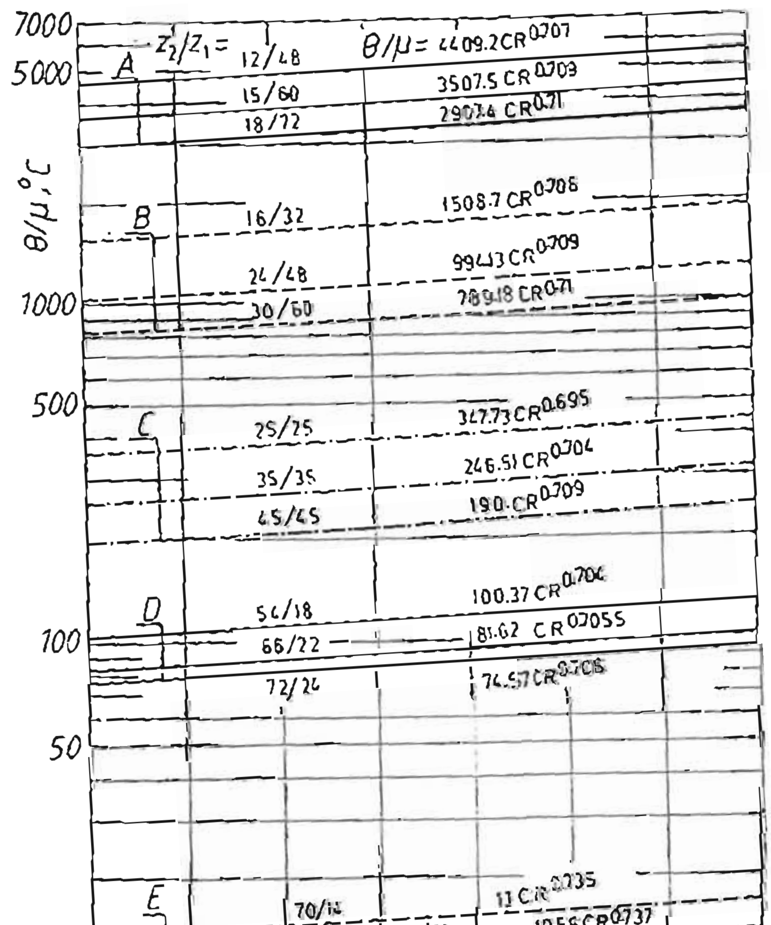
Conditions A,B,C,D and E are given in appendix 1



Fig(15) Change of the elliptical area of contact, sliding velocity and total meshing time with the gear ratio at different running conditions.



Fig(14 c) Change of the flash temperature divided by the coefficient of friction(θ/μ) with the gear ratio at different running conditions.



Fig(16) Change of the flash temperature divided by the coef. ficient of friction (θ/μ) with the contact ratio at different gear variables and running conditions. Conditions A, B, C, D and E are given in appendix 1

Complete form of the flash temperature equation was deriven. This equation contains applied tooth load, speed of rotation, module, radius of curuature of the pinion tooth, contact ratio and helix angle.

$$\theta/\mu = C\ P^{0.9725} \cdot N^{0.389} \cdot m^{0.968} \cdot \rho^{0.994} \cdot CR^{0.706} \cdot \beta^{0.193} \quad (51)$$

REFERENCES

Archard, J. F., " The Temperature of Rubbing Surfaces," WEAR Vol.2, 1958/59, pp.438-455.

Blok, H., "Theoretical Study of Temperature Rise at Surfaces of Actual Contact under Oiliness Lubricating Conditions," Proceedings of the General Discussion on LUBRICATION AND LUBRICANTS, Institution of Mechanical Engineers, Vol.2, 1937, pp.222-235.

Bowden, F. P. and Tabor, D., "The Friction and Lubrication of Solids," Oxford University Press, London, 1950.

Cheney, W. and Kincaid, D., " Numerical Mathematics and Compu-ting " Books / Cole Publishing Company. Monterey, California, 1980.

Dyson, A., Evans, H. P. and Snidle, R. W., " Wildhaber-Novikov Circular Arc Gears: Geometry and Kinematics" Proc.R. Soc. Lond. A, Vol., 403, pp 313- 340, 1986.

Gerald, C. F., " Applied Numerical Analysis, " Addison-Wesley Publishing Company, Singapore, 1978.

Holm, R., "Calculation of the tempertaure Development in a Contact Surface and Application to the problem of the Temperature Rise in a Sliding Contact," JOURNAL OF APPLIED PHYSICS, Vol. 19, No. 4, 1948, pp. 361-366.

Jaeger, J. C., " Moving Sources of Heat and Temperature at Sliding Contacts," Proceedings of ROYAL SOCIETY OF NEW SOUTH WALES, Vol.56,1942,pp. 203-224.

Nakada, T. and Hashimoto, S., "Heat Conduction in a Semi- Infinite Solid Heated by Moving Source Along the Boundary," Bulletin of JSME, Vol.,6, No. 21, 1963, pp. 59-69.

Roark, R. J., " Formulas for Stress and Strain" Fourth Edition, McGraw- Hill Book Co., 1965.

Roylance, B. J. and ALKateb, A. H.," Further Developments in Contact Temperature Determination in Four-Ball Machine Operation and the Tribological Implications," I. Mech. E., C 170/87,1987, pp.399-410

Symm, G., T., "Surface Temperature of Two Rubbing Bodies," Quarterly JOURNAL OF MECHANICS AND APPLIED MATHEMATICS Vol., 20, Pt.3, 1967, pp.381-391.

Terauchi, Y., and Mori, H., " Comparison of theories and experimental results for Surface Temperature of Spur Gear Teeth," Trans. ASME, Journal of Engineering for Industry, february, 1974, pp. 41-50

Timoshenko, S. P. and Goodier, J. N., "Theory of Elasticity" Third Edition, Mc Graw- Hill Book Company, 1984.

Tobe, T. and Kato, M., "A Study on Flash temperatures on the spur gear teeth," Trans. ASME Journal of Engineering for industry, February, 1974, pp.78-84.

APPENDIX 1

Value of the Constant "C" in the Flash Temperature equation

Condition A		Condition B		Condition C		Condition D		Condition E	
G = 0.25 = Z_2/Z_1	C	G=0.5 = Z_2/Z_1	C	G=1 = Z_2/Z_1	C	G=3 = Z_2/Z_1	C	G=5 = Z_2/Z_1	C
12/48	1.428	16/32	2.497	25/25	5.405	54/18	9.444	70/14	25.06
15/60	1.137	24/48	1.649	35/35	3.824	66/22	7.675	85/17	20.43
18/72	0.9434	30/60	1.31	45/45	2.949	72/24	7.014	90/18	18.84

Condition A : P = 40*KN*, N = 10000 *rpm*, m = 0.02*m*, $\beta = 45^0$, $\rho_1 = 0.06m$. CR=1.4 and G=0.25

Condition B : P = 30*KN*, N = 7000 *rpm*, m = 0.012*m*, $\beta = 38^0$, $\rho_1 = 0.03m$. CR=1.3 and G=0.5

Condition C : P = 20*KN*, N = 4000 *rpm*, m = 0.006*m*, $\beta = 30^0$, $\rho_1 = 0.012m$. CR=1.2 and G=1

Condition D : P = 12.5*KN*, N = 2500 *rpm*, m = 0.004*m*, $\beta = 20^0$, $\rho_1 = 0.006m$. CR=1.125 and G=3

Condition E : P = 5*KN*, N = 1000 *rpm*, m = 0.002*m*, $\beta = 10^0$, $\rho_1 = 0.002m$. CR=1.05 and G=5