

Mansoura University Faculty of Engineering Eng. Math. and Phys. Dept.	<i>Final First Term Exam. 2010-2011</i>	Mathematics (1). Preparatory Year. Time allowed: 3 Hours.
Full Mark:130		

Algebra

Question.1 [33 marks]

(a) If $z^2 = -1 + i\sqrt{3}$, then evaluate: [9 Marks]

(i) $3z + 4$ (ii) z^6 (iii) $\frac{z^2 + 1}{3 + i}$

(b) Decompose the fraction $\frac{2x - 1}{x^4 - x^3 - 5x^2 - x - 6}$ into its partial fractions. [12 Marks]

(c) Find the roots of the equation $16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0$ if they form arithmetic sequence. [12 Marks]

Question.2 [37 marks]

(a) Suppose A and B are 4×4 matrices with $\det A = 5$ and $\det(AB^{-1}) = 2$. Then; [15 Marks]

(i) prove that the matrix AB is nonsingular.

(ii) prove that the matrix AA^T is symmetric.

(iii) evaluate $\det(3A^T)$ and $\det(B^2)$.

(b) Find the inverse of the matrix $A = \begin{bmatrix} a & 0 & 0 \\ -a & a & 0 \\ 0 & -a & a \end{bmatrix}$. [7 Marks]

(c) Discuss the type of the solution for the following linear system; [15 Marks]

$$x - y + 2z = 4$$

$$3x + 2y + z = 7$$

$$2x + 3y - z = 3$$

$$x - 6y + 7z = 9$$

Question 3 :

[a] Sketch the function $f(x) = \cos x$ for $0 \leq x \leq \pi$. For the interval $0 \leq x \leq \pi$, find and sketch $\frac{1}{f(x)}$, $f^{-1}(x)$ and $\frac{df(x)}{dx}$. [8 marks]

[b] Find the constants a and b for which the following function is continuous for all values of x

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2} & x < 0 \\ ax + b & 0 \leq x \leq \pi \\ \frac{\sin x}{\pi - x} & x > \pi \end{cases} \quad [8 \text{ marks}]$$

[c] Prove that : $\operatorname{cosech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 + x^2}}{x} \right)$ [6 marks]

[d] Solve for : $\operatorname{cosec}(2x) = 2 + \ln \left(\frac{1}{\operatorname{cosec}^2 x} + e^{2 \ln(\cos x)} \right)$ [6 marks]

Question 4 :

[a] Find $\frac{dy}{dx}$ for : (i) $y = \tan^5(e^{\sinh x}) + \sin^{-1}(\sqrt{\operatorname{sech} x})$ [6 marks]

(ii) $y = \sqrt{\frac{x e^{x^3}}{(\sec^{-1} x)(x^2 + 1)^7}}$ [6 marks]

(iii) $y = 4^{\tanh^{-1} x} + (x)^{\cosh x}$. [6 marks]

[b] (i) What is the equation of tangent to the circle $x^2 + y^2 + 2x + 2y - 11 = 0$ at the point $(1, 2)$? [6 marks]

(ii) Calculate the slope of the normal line to the curve whose parametric equations are $y = t^4 + 2t + 1$ and $x = 2t^3 + 3t^2 + 1$ at $t = 1$. [6 marks]

[c] Evaluate the limit (if it exists) :

(i) $\lim_{x \rightarrow 0} \left(\frac{1}{\sinh x} - \frac{1}{x^2} \right)$

(ii) $\lim_{x \rightarrow e} (\ln x)^{\frac{1}{x-e}}$ [8 marks]

[d] Find the second order Maclourin series of the function $f(x) = e^{x^2}$. [6 marks]

With our best wishes

Prof. Dr. Mostafa Saleh & Dr. Ehab Mostafa