كهرباء

Faculty of Engineering,
Math. &Eng. Physics Dept.

Mathematics (3)
final Exam.

First Semester 2013/2014
Time 3hours.

Solve the following questions

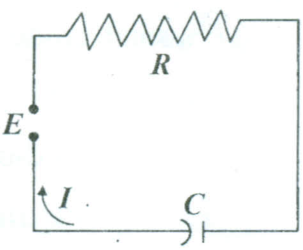
Question 1 (27 marks)

(a) Solve the initial value problem

$$\left(y e^{-y} \cos x\right)dx + \left(1+e^{-y} \sin x\right)dy = 0, \quad y(0)=1.$$

(b) Solve the differential equation $\cos x \; y''' + \cos x \; y' = \sec x$.

(c) An RC-circuit has e.m.f. given (in volts)by $400e^{t}\cos 2t$, a resistance of 100Ω, and a capacitance of 0.01 $Farads$. Initially there is no charge on the capacitor. Find **(i)** the current in the circle at any time, **(ii)** the steady state current in the circuit.



Question 2 (28 marks)

(a) Solve the integro-differential equations

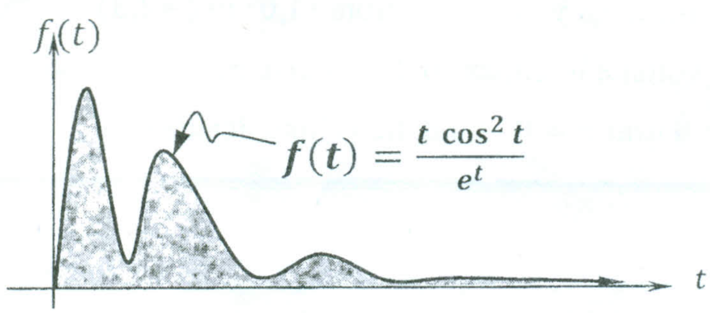
$$y'(t)=e^{t} - \int_0^t y(x)\cosh(t-x)\,dx, \quad y(0)=1.$$

(b) Find $L^{-1}\left\{e^{-\pi s} \times \ln\left(\dfrac{e^{\tan^{-1} s}}{\sqrt{s^2+1}}\right)\right\}$

(c) Find Laplace transform of $f(t)$, where

$$f(t)=\begin{cases} e^{t}, & 0 \le t \le \pi \\ \sin t, & t > \pi \end{cases}$$

(d) Evaluate the shaded area.



[3] (a) [6 pts] The domain of Ali's garden is describe by the domain of the function

$$f(x,y) = \sqrt{x - |y|} + \sqrt{4x - x^2 - y^2}$$

i) Find the domain of Ali's garden and sketch

ii) Using double integration prove that area of garden $= 2\pi + 4$

(b) [9 pts] If z, u, v are three **positive** real numbers satisfy equation

$z^2u - uvz + u^2 + v^2 = 8v$ where $u = (x+1)e^y$ and $v = (y+1)\cos(x)$

prove that $z_x = -1.6$ and $z_y = 0.2$ at ($x = 0$ and $y = 0$)

(c) [8 pts] Suppose that the elevation z of a hill is given by

$$z = f(x,y) = 39 + 10x - x^2 + 12y - y^2$$

i) If a small stone moves from site $(6,8)$ to $(9,12)$. Find the rate of change of elevation in that direction

ii) Using second-order approximation, find the elevation z at point $(6.01, 8.02)$ use $[x_0 = 6 \text{ and } y_0 = 8]$

[4] (a) [7 pts] Find $I = \int_0^1 \int_0^1 \frac{1}{1-(xy)^2}\, dx\, dy$ using transformation

$x = \frac{\sin(u)}{\cos(v)}$ and $y = \frac{\sin(v)}{\cos(u)}$ (note : this transformation transform square $0 \le x \le 1$, $0 \le y \le 1$ into triangle $0 \le u \le \frac{\pi}{2} - v$, $0 \le v \le \frac{\pi}{2}$)

(b) [7 pts] Find the center of mass of the lamina for the shape inside curve $r = 2 - 2\sin(\theta)$ shown in figure 1.
If the mass density given by $\rho(x,y) = 1$

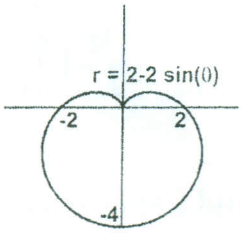


Figure 1

(c) [4 pts] For $\vec{F} = (x^2y)\,\hat{\imath} + (3x - yz)\,\hat{\jmath} + (z^3)\,\hat{k}$. Find Curl $\vec{F}$ and Div $\vec{F}$

(d) [7 pts] Compute the work done by the force field $F(x,y) = (y)\,\hat{\imath} + (-x)\,\hat{\jmath}$ acting on object as it moves along parabola $y = x^2 - 1$ from $(1,0)$ to $(-2,3)$

(e) [7 pts] Ali has a tent its volume is similar to the volume of the solid bounded by $z = 4 - y^2, x + z = 4, x = 0$ and $z = 0$. Find the volume inside tent