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BEHAVIOUR OF BUILDINGS SUPPORTED ON SOILS WITH NON-LINEAR PROPERTIES

By

Sherief Abu-El-Magd

Lecturer, Struct. Eng. Dept., Faculty of Eng. Mansoura Univ.

Summary:

This paper investigates the effect of neglecting the non-linear behaviour of the soil in a structure-soil interaction analysis of plane walls with openings. In most cases, the use of elastic properties for the soil was found to be conservative. Only when the soil stiffness increases with the applied load, considering the non-linear soil properties can yield higher stresses in the frame. The basis of the study is a plane-strain finite element analysis of the soil which supports a wall idealised as a frame. The wall has elastic properties whereas the soil is represented both by an elastic half-plane and by a hyperbolic relationship between stress and strain.

An interesting observation made was that while the rate of increase of settlement decreased with increase of load for dense sand it increased for lean clay. This type of behaviour is not commonly recognised for sand in laboratory tests. The behaviour of lean clay is probably due to its incomplete confinement

Introduction:-

In most cases when a structure-soil interaction analysis is carried out, the soil is considered to have elastic properties. However, most soils have non-linear relationships between stress and strain. The effect of considering the non-linear properties on both the contact pressure and maximum stresses in the frame is studied in this paper.

For the problem under consideration, the soil behaviour is three-dimensional rather than plane-strain. Three-dimensional non-linear finite element solutions are, however, very expensive and can be subject to stability problems. Consideration is given to extrapolating the plane strain results to the three-dimensional case.

Choice of model for Non-linear Analysis:

A Hyperbolic relationship between stress and strain is used to model the soil characteristics. The Hyperbolic model takes into account three important haracteristics of the stress-strain behaviour of soils.
They are non-linearity, stress dependency and in-elasticity. In this model the tangent modulus of elasticity is given by equation(1) and the tangent value of Poisson's ratio is given by equation(2), ref.(1).

$$E_t = \left[1 - \frac{Rf\,(1-\sin\Phi)\,(\sigma_1-\sigma_3)}{2c\cos\Phi + 2\sigma_3\sin\Phi}\right]^2 k\,P_a\left(\frac{\sigma_3}{P_a}\right)^n \quad \ldots\ldots(1)$$

$$\nu_t = \frac{G - F\log\left(\dfrac{\sigma_3}{P_a}\right)}{\left[1 - \dfrac{d(\sigma_1-\sigma_3)}{kP_a\left(\dfrac{\sigma_3}{P_a}\right)^n\left[1 - \dfrac{R_f(\sigma_1-\sigma_3)(1-\sin\Phi)}{2c\cos\Phi + 2\sigma_3\sin\Phi}\right]}\right]^2} \quad \ldots\ldots(2)$$

where

σ_1 and σ_3 are the major and minor principal stresses, P_a is the atmospheric pressure.

the definition and role of each of the hyperbolic parameters are given in Table (1)

The hyperbolic relationships are chosen because they have proven quite useful for a wide variety of practical problems for the following reasons:-

(1) The parameter values can be determined from the results of conventional triaxial compression tests.

(2) The same relationships can be used for effective stress analyses (using data from drained tests) and total stress analyses (using data from unconsolidated - undrained tests)

(3) Values of the parameters have been calculated for 150 different soils by Wong and Duncan.[1]

The incremental method is chosen to carry out the non-linear analysis because it provides a knowledge of the displacements, stresses and strains after different stages of loading which is quite useful.
The Runge-Kutta scheme is used to improve the accuracy of the incremental method[2]

The incremental stress-strain relationship for an isotropic material under plane-strain conditions is given by:

$$\begin{bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{bmatrix} = \frac{E_t}{(1+\nu_t)(1-2\nu)} \begin{bmatrix} (1-\nu_t) & \nu_t & 0 \\ \nu_t & (1-\nu_t) & 0 \\ 0 & 0 & (1-2\nu_t)/2 \end{bmatrix} \begin{bmatrix} \Delta\epsilon_x \\ \Delta\epsilon_y \\ \Delta\gamma_{xy} \end{bmatrix} \quad \ldots (3)$$

The modulus of elasticity and Poisson's ratio for each element during each increment are re-evaluated in accordance with the stresses in the element. Thus, the non-linear stress-strain relationship is approximated by a series of linear relationships

In order to represent post-failure behaviour of soils more accurately, Clough and Woodward (3) suggested that it is desirable tn express the stress-strain relationship in an alternative form:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} M_B+M_D & M_B-M_D & 0 \\ M_B-M_D & M_B+M_D & 0 \\ 0 & 0 & M_D \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad \ldots\ldots (4)$$

in which M_B is the Bulk Modulus = $E/2(1+\nu)(1-2\nu)$ and M_D is the shear modulus = $E/2(1+\nu)$. The fact that soils have high resistance to volumetric compression after failure but very low resistance to shearing may be represented by reducing the value of M_D to zero after failure, while the value of M_B is maintained at the same value as it had in the increment before failure.

If the stress level decreases in an element at some increment compared with the previous increment, the unloading-reloading modulus, E_{ur} given by equation (5) is used

$$E_{ur} = K_{ur} P_a \left(\frac{\sigma_3}{P_a}\right)^n \quad \ldots\ldots (5)$$

where
K_{ur} is the unloading-reloading modulus number

The Analytical Approach

a) Wall: The wall is treated as a frame with rigid arms as shown in Fig.(1) Further details of this technique are givenin ref (4).

Walls with two opening ratios were used in the analysis. A wall with (bw/b) = (hw/h) = 0.8, see Fig.1, represents a flexible wall, and a wall with (hw/b) = (hw/h) = 0.4 represente a rigid one. Both walls are 4 storeys high and have a length/height ratio of 2.

b) Foundation: A strip footing is represented by line elements. Full contact is assumed between the footing and the soil.

c) Soil: Plane-strain quadratic hybrid elements, with three degrees of freedom per node are used to represent the soil. The finite element mesh used is shown in Fig.(2). A very thin column of elements is used next to the four columns of elements under the wall in order to obtain a more accurate estimate of the contact pressure under the edge of the wall.

The non-linear soil behaviour is represented in the model as explained in the previous section. Two types of soils were considered in this analysis. The hyperbolic parameters of each type are given in Table 2. The dense sand, taken from ref.(6), was chosen to represent a very stiff soil, while the lean clay, taken from ref.(1), was chosen to represent a very flexible one.

d) Loading: The loads were applied in four equal increments of 50 KN/m^2 each. The first load increment was applied at ground level, the second at first storey level and so on, in order to represent the increase of loads during construction. The total applied load (200 KN/m^2) is probably higher than the maximum practical loads on such buildings. However, it was chosen in order to reach a state of shear failure in the soil under the edges of the footing. The effect of such failure in the soil on the behaviour of the settling walls must be investigated.

Although the practical loads on walls with big openings could be less than those on walls with small openings (as the loads in practice are mainly dead loads), the same loads were applied to all walls so that comparison could be more readily carried out.

Behaviour of Soil under Load:

Before considering the results of the interaction analysis, the basic behaviour of the soil model used under load should be investigated. For this purpose, the soil model was loaded, incrementally, with a loaded area at the surface. The results of this analysis show the basic difference between the behaviour of the two types of soils used. The rate of increase of settlement with the increase of load decreased for dense sand and increased for lean clay, Fig. 3. This means that the stiffness of dense sand increased while the stiffness of lean clay decreased with the increase of applied load in the range considered. It has to be noted here that the total applied load (200 kN/m^2) is very small compared with the bearing capacity of dense sand, while it is almost equal to half the bearing capacity of the lean clay. The behaviour of dense sand under load will change to that of lean clay as the load approaches its bearing capacity.

The distribution of elastic moduli for both types of soil is shown in Fig. 4. It can be seen from the contours in this figure that, near the suface, the values of the elastic modulus under the loaded area are much greater than those outwith it. However at a depth $\geq$ a, the lateral distribution of the elastic moduli becomes more even (where a is half the length of the loaded area)

Discussion of Results :

The results for contact pressure under the walls and the maximum stresses in them, obtained from the non-linear analysis, are compared with those from an equivalent linear analysis in Fig. 5 to 8. The basis of the equivalency is that the linear analysis would have a central settlement equal to that obtained by a non-linear analysis, under the same total load. According to this definition, the modulus of elasticity equivalent to dense sand and lean clay was found to be 52.3 N/mm^2 and 1.63 N/mm^2, respectively.

The contact pressure distributions, Fig. 5 to 8. for each load increment are due to this increment of loads only, so that the results for different load increments can be readily compared. The results at the end of the non-linear analysis are also given. ref (5)

Comparing the results of the linear and non-linear analysis, the following conclusions can be drawn :

1. The contact pressure distributions under walls supported by lean clay are different from those under walls supported by dense sand, While the contact pressure distribution for the former case tends to become more uniform than that predicted by linear analysis, Fig. 6 and Fig. 8, the contact pressure for the latter case tends to concentrate towards the edges, Fig 5 and Fig. 7.

   The reduction in the edge contact pressure under the walls on lean clay (as compared with the elastic case) could be caused by the distribution of elastic moduli near the surface of the soil, The values of elastic moduli under a loaded area are much greater than those outwith it, as can be seen in Fig. 4. The same trend will occur under loaded walls. This will reduce the con tribution of the soil outwith the wall in resisting the applied loads, i.e. decrease the edge contact pressure. The tendency of lean clay to be more flexible as the applied load increases could be another factor in decreasing the contact pressure under the edges of the footing. A further reduction in the edge contact pressure can occur if the soil under the edges of the footing reaches shear failure because of the high values of contact pressure there. The reduction in the edge contact pressure is about 35% under walls with openings ratios of 0.4 and 0.8. Shear failure started after the application of the second load increment where shear failure occurred in the surface element under the edge of the footing. As the load increased the shear failure spread from the failed surface element. The condition of shear failure is :

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$$\tfrac{1}{2}(\sigma_1 - \sigma_3) \geq (c \cos \phi + \sigma_3 \sin \phi)(1 - \sin \phi)$$

Where σ_1 and σ_3 are the major and minor principal stresses.

c and ϕ are the cohesion intercept and friction angle.

When this condition is satisfied the shear modulus of the element is reduced to zero. No dramitic change occurred in the edge contact pressure after failure because the bulk modulus is maintained after shear failure as mentioned earlier.

For the cases of walls on dense sand, however, the edge contact pressure of the non-linear case increased instead of decreasing relative to that given by the linear analysis. The reason may be that the stiffness of the dense sand increases with the increase of the applied loads, as discussed in the previous section. This behaviour under load means that the sand under the edges of the footing will be stiffer than that under the interior parts because of the concentration of the contact pressure there. As the stiffness of the sand under the edges increase , its share of the load will increase and so will the edge contact pressure. Hence, the tendency of the edge contact pressure to decrease because of the distribution of the elastic moduli near the surface, as explained above, is counteracted by the tendency of the edge contact pressure to increase because of the increase of sand stiffness with load. The result for the walls analysed was a slight increase in the edge contact pressure (5% for the wall with openings ratios = 0.8 and 15% for that with openings ratios = 0.4)

2. As a result of the decrease in the edge contact pressure for walls on lean clay the maximum stresses in these walls decrease, Fig.9. The reduction is about 49% for both rigid and flexible walls. On the other hand, the maximum stresses for walls on dense sand increase slightly as a result of the increase in the edge contact pressure. This increase ranges from 20% for rigid walls to 8% for flexi ble ones.

3. The non-linear analysis yields similar differential settlement results as compared with the linear analysis. As mentioned above, the modulus of elasticity of the linear analysis was chosen such that the total settlement is equal to that predicted by the non-linear analysis under the same total load. Using the equivalent modulus, not only the total settlements under the walls supported by linear and non-linear soils are equal, but also the differential settlements in both cases are nearly equal. In other words, the effect of considering the non-linear properties on the differential settlement is negligible when compared with an equivalent linear analysis. The reason for this may be due to the case of loading considered. Because the load was uniformly distributed, the elastic moduli underneath the wall's footing were reasonably uniform, i.e. similar to the linear case.

4. The increase in the edge contact pressure under walls on dense sand is more evident for rigid walls than flexible ones. The contact pressure is more concentrated towards the edges of the footing under the rigid walls (bw/b=hw/h=0.4) than under fléxible ones (bw/b = hw/h = 0.8) using a linear analysis, compare Fig. 5 with Fig. 7. Hence, the effect of sand stiffening under the edges of the footing, which is the cause of the increase in the edge contact pressure in the non-linear analysis, will be stronger for the rigid wall case. The effect of soil non-linearity would be more evident for rigid walls on lean clays as well, i.e. the decrease in the edge contact pressure would be greater under them than under flexible walls. However, the decrease in the edge contact pressure under the rigid and flexible walls analysed is almost the same. The reason for this is that the lean clay used in the analysis is so soft that both walls (with openings ratios of 0.4 and 0.8) are rigid relative to it. This can be seen if the contact pressure under both walls using linear analysis is compared, Fig.6 and Fig. 8.

CORRELATION BETWEEN LINEAR TWO-AND THREE-DIMENSIONAL ANALYSIS

In order that the results of the two-dimentional analysis can be extrapolated to the more realistic three-dimentional case, a correlation between the results of both cases is investigated. For this purpose, the same walls under the same loads are analysed with two-dimentional plane strain finite element for the soil (as described in this paper) and using an elastic half space model having the same soil properties. A comparison between the results of the half-space and half-plane cases is given in Table 3. These results indicate that:

1. Central settlement of the half-plane model is greater than that of the half-space model by a factor of about 10.

2. Differential settlement of the half-plane model is greater than that of the half-space model by a factor ranging between 1.75 and 4.4. The reason that the factor of increase of central settlement is constant while that of differential settlement varies may be because the former is a function of soil stiffness only, while the latter is a function of both structural and soil stiffnesses. This may also be the reason for the difference in magnitude of the factor of increase for both settlements.

3. If the maximum stresses in the walls on the half-plane are factorised to the half-space case (divided by the factor for differential settlement) the Resulting stresses will be smaller than those in the walls on the half-space model by 18-39%
The half-plane elements were 1 metre thick (equal to the width of the footing). If, instead, the thickness is increased to 10 m (in order to obtain the same central settlement results as those given by the half-space model) the results would not have to be factorized. Computer runs with finite elements of thickness = 10 m indicate that the half-plane model would yield similar differential settlement and maximum stresses as the half-space model for flexi ble walls. The half-plane model, however, would overestimate the differential settlement by not more than 70% and the maximum stresses by not more than 30% for rigid walls. More work is needed to determine the thickness of the half-plane model for different footing widths such that its stiffness would be equivalent to the half-space model.

CONCLUSIONS:-

The effect of considering non-linear properties on the interaction between the structure and its supporting soil is examined in this paper. This effect depends on the load-settlement characteristics of the soil. If the soil stiffness decreases with the applied loads (i.e. as in Fig. 3a), considering soil non-linearity will lead to a considerable reduction in the maximum stresses in the wall when compared with linear analysis. On the other hand, an increase in soil stiffness with applied load (Fig. 3b), can yield slightly higher maximum stresses. That latter type of behaviour is not often discussed in the soil mechanics literature . This may be because it is not easy to produce in laboratory tests.
In general the non-linear analysis indicates that an elastic analysis would give results which tend to be conservative as for as prediction of stress due to differential settlement is concerned. The only exceptions are the cases of long buildings (relative to the depth of the compressible stratum) or cohesionless soils when the confining pressure is relatively high.

Considering soil non-linearity has little effect on the ratio between central and differential settlement. In other words, a linear analysis with an equivalent elastic modulus can yield similar differential settlement results as a non-linear one if the central deflection of both analyses are equal.

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Parameter	Name	Function
k, k_{ur}	Modulus Number	Relate E_i and E_{ur} to σ_3
n	Modulus exponent	
c	Cohesion itercept	Relate $(\sigma_1 - \sigma_3)$ to σ_3
ϕ	Friction angle	
R_f	Failure ratio	Relates $(\sigma_1 - \sigma_3)_{ult}$ to $(\sigma_1 - \sigma_3)_f$
G	Poisson's ratio parameter	Value of ν_i at $\sigma_3 = P_A$
F	ditto	Decrease in ν_i for ten-fold increase in σ_3.
d	ditto	Rate of increase of ν_t with strain

<u>TABLE (1)</u> - SUMMARY OF HYPERBOLIC PARAMETERS

γ kN/m^3	k	k_{ur}	n	d	G	F	C KN/m^2	ϕ degrees	R_f	k_o
Dense sand:										
17.5	1600	1900	1.0	0.25	0.45	0.17	0.0	43	0.9	0.3
Lean clay:										
17	80	200	0.85	4.2	0.3	0.25	40	10	0.8	0.7

k_o is the coefficient of earth pressure at rest.

<u>TABLE (2)</u> - VALUES OF NON-LINEAR PARAMETERS OF THE TWO TYPES OF SOILS USED IN THE ANALYSIS.

case considered	½-space model	½-plane model	factor
1. $hw/h = bw/h = 0.4$			
$E_s = 4.45\ MN/m^2$			
settlement (m)	0.1	1.084	10.8
differential settlement (mm)	2.73	4.9	1.75
maximum stress (N/mm^2)	0.85	1.22	18%
2. $hw/h = bw/b = 0.4$			
$E_s = 22.25\ MN/m^2$			
settlement (m)	0.021	0.224	10.7
differential settlement (mm)	2.2	4.7	2.14
maximum stress (N/mm^2)	0.732	1.1	26%
3. $hw/h = bw/b = 0.8$			
$E_s = 4.45\ MN/m^2$			
settlement (m)	0.109	1.11	10
differential settlement (mm)	21.7	56.4	2.56
maximum stresses (N/mm^2)	2.76	5.15	27%
4. $hw/h = bw/b = 0.8$			
$E_s = 22.25\ MN/m^2$			
settlement (m)	0.023	0.24	10
differential settlement (mm)	8.0	35.2	4.4
maximum stress (N/mm^2)	1.257	3.36	39%

- The percentages are the ratios with which the maximum stresses will be underestimated if the ½-plane results are factorised to give the same differential settlement as the ½-space case.
- for definition of symbols see Fig (1)
  Es is the elastic modulus of the soil .

TABLE (3) - RELATION BETWEEN HALF-SPACE AND HALF-PLANE MODELS

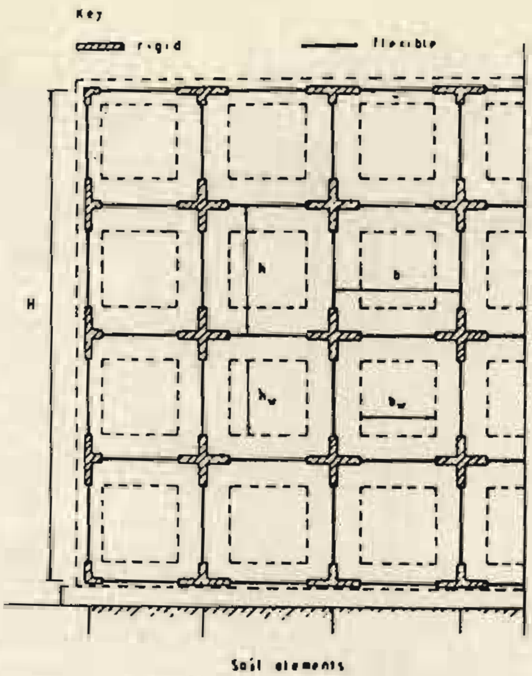

FIG (1)- FRAME MODEL

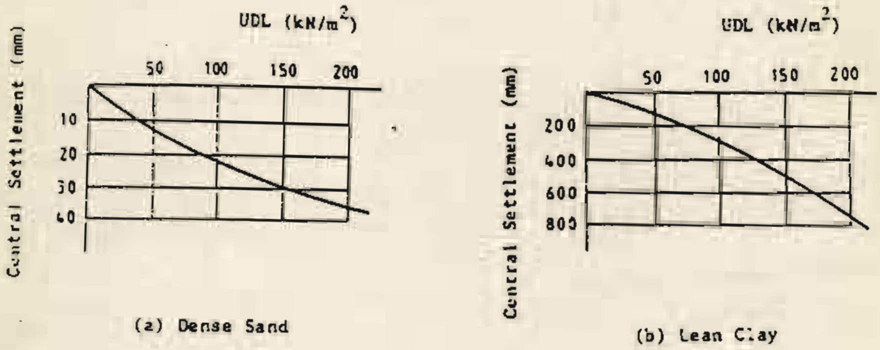

FIG (3) - BEHAVIOUR OF SOIL UNDER LOAD

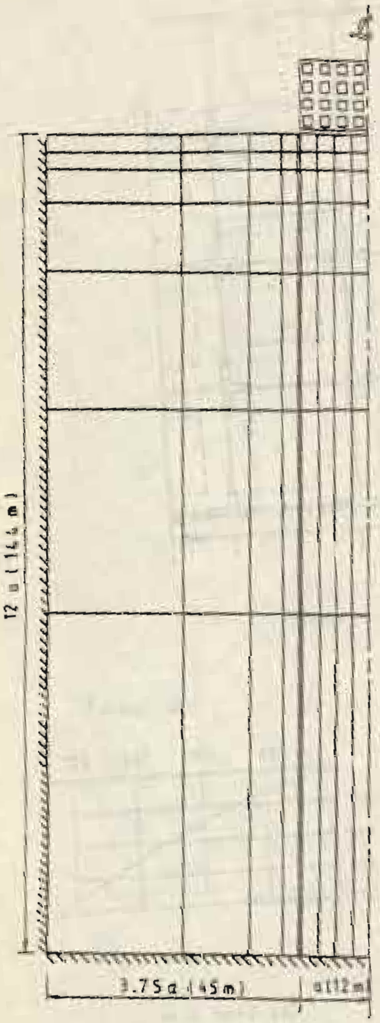

FIG (2) - FINITE ELEMENT MESH FOR WALLS ON NON-LINEAR SOILS (the thickness of the elements is equal to the width of the footing of one metre)

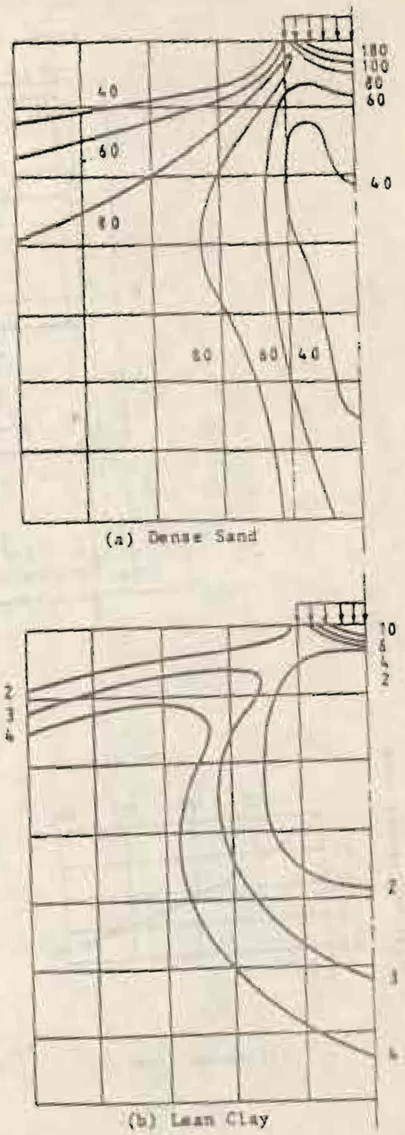

FIG (4) - DISTRIBUTION OF MODULI OF ELASTICITY (N/mm^2) (under the final load of 200 kN/m^2)

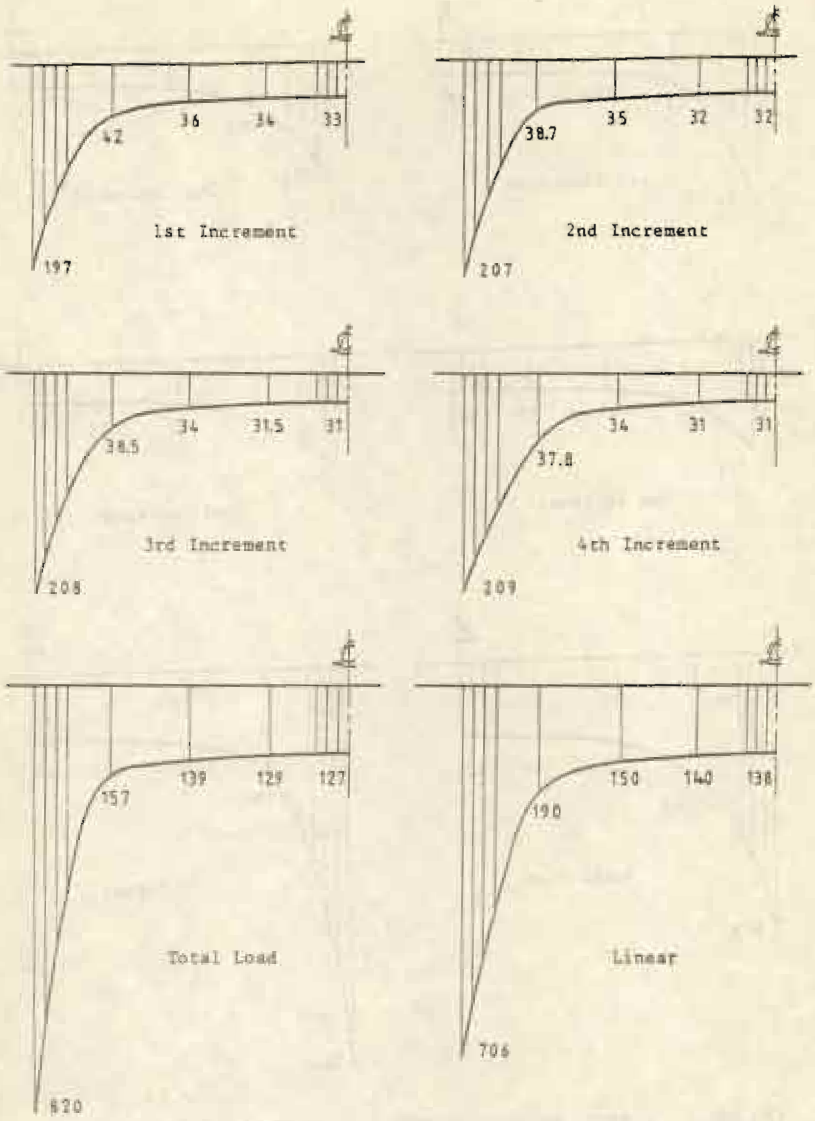

FIG (5) - CONTACT PRESSURE DISTRIBUTION UNDER A WALL WITH (hw/h) = (bw/b) = 0.4 ON DENSE SAND

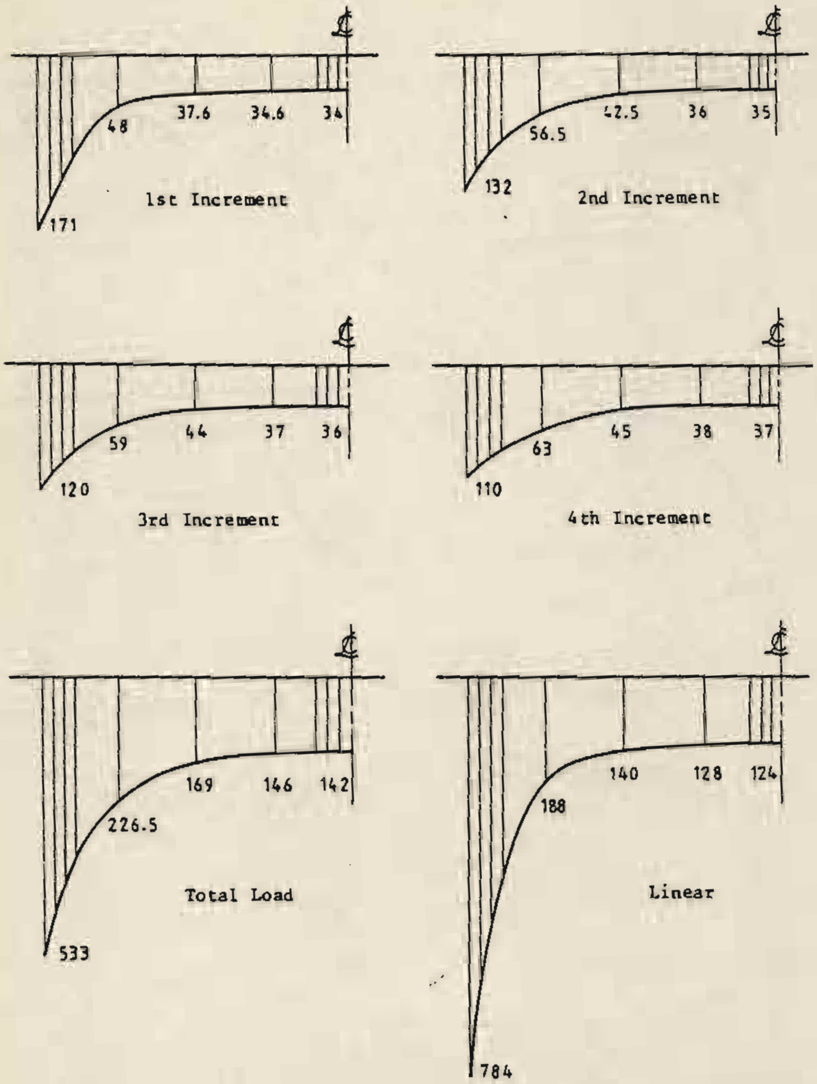

FIG (6) - CONTACT PRESSURE DISTRIBUTION UNDER A WALL WITH (bw/b) = (hw/h) = 0.4 ON LEAN CLAY

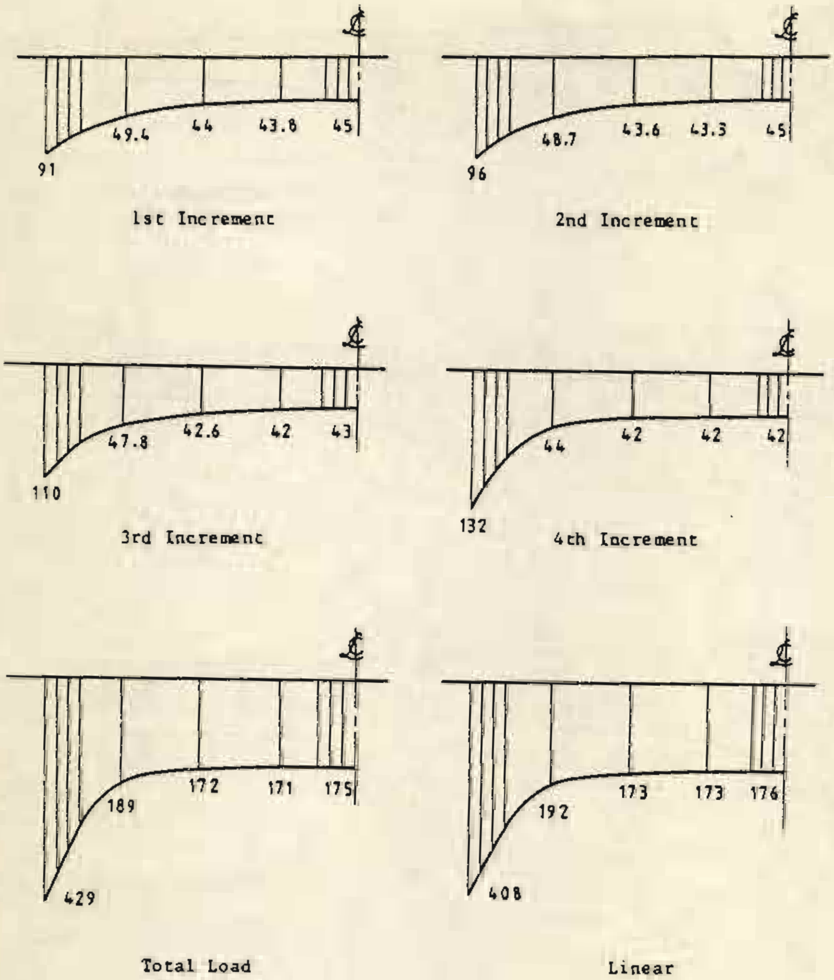

FIG (7) - CONTACT PRESSURE DISTRIBUTION UNDER A WALL WITH (bw/b) = (hw/h) = 0.8 ON DENSE SAND

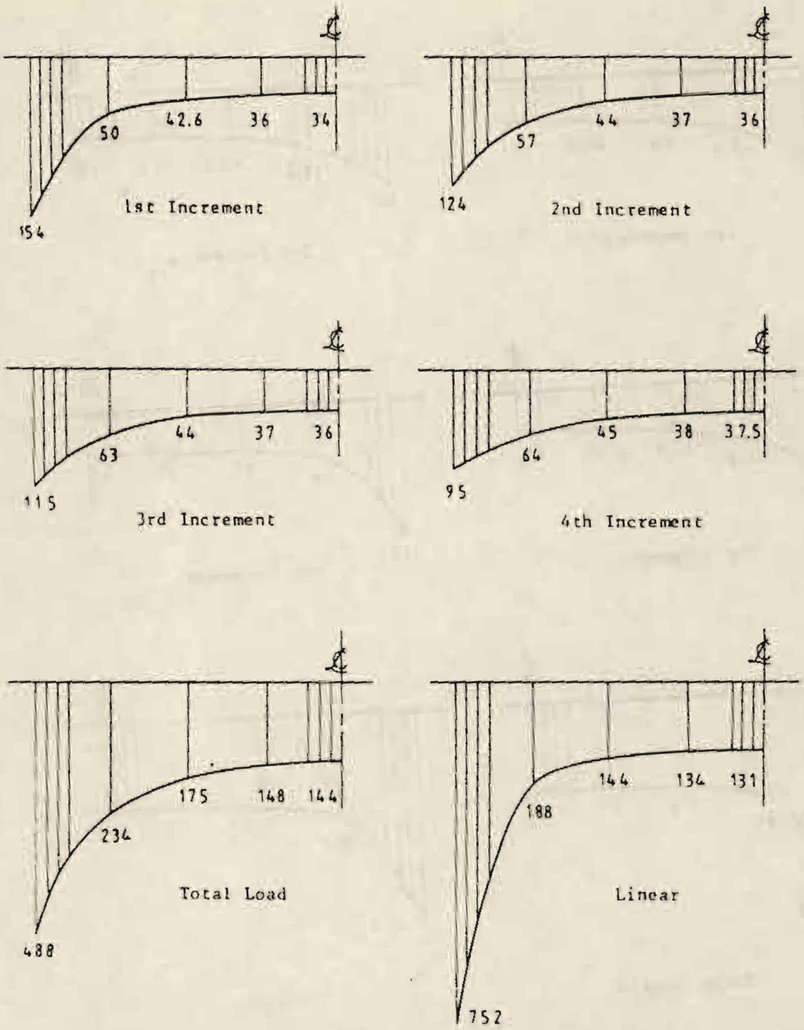

FIG (8) - CONTACT PRESSURE DISTRIBUTION UNDER A WALL WITH (bw/b) = (hw/h) = 0.8 ON LEAN CLAY

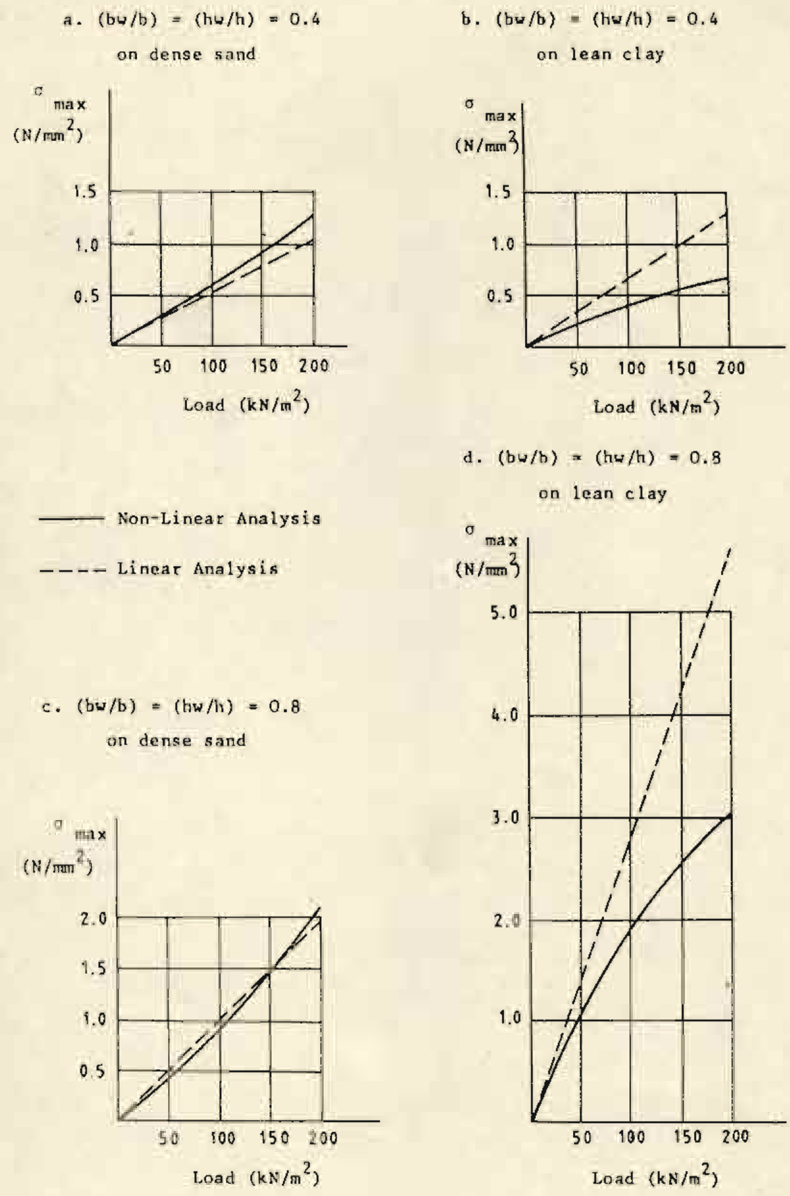

FIG (9) - MAXIMUM STRESSES IN THE ANALYSED WALLS