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DRAINAGE OF AN AGRICULTURAL SOIL UNDERLAIN BY AN INCLINED IMPERVIOUS LAYER

BY

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ABSTRACT

The problem of draining a clay layer , overlying an inclined impervious substratum , by means of a system of parallel drain tubes is mathematically solved in this paper .

The complex potential , the velocity potential and the stream function of the system are established .

A new drain discharge formula satisfying the proper boundary conditions is also deduced . Finally the effect of the angle of inclination of the impervious layer , on the drain discharge is investigated .

INTRODUCTION

Many theories treated the problem of draining an agricultural clay soil underlain by a horizontal flat impervious layer[7] , are Spottle , Walker[8] , 1911 , Kozeny[6] , 1931 , Houghoudt[4] 1940 , Kirkham[5] , 1949 , Van Deemter[9] , 1950 , Glover[1] , 1954 , Hammad[2] , 1965 and Hathoot[3] , 1979.

None of the above treatments took into account the case in which the imperious sublayer is inclined , as shown in Fig. 1. The present paper presents a mathematical solution of the above problem which is based on a hydrodynamical basis .

MATHEMATICAL MODEL

To study the effect of the inclined impervious substratum on a single tile drain B, Fig.2, the effect of the two neighbouring drains A and C is taken into account . If drain B is hydrodynamically represented by a point sink of strength m, the two drains A and C may be similarly represented by two point sinks of strengths m_1 and m_2 , respectively . To represent the inclined impervious substratum , the latter is considered as a mi[illegible] the images of points A,B and C , whi[illegible] and G are introduced , Fig.2.

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COMPLEX POTENTIAL

The complex potential of the three sinks at points A, B, and C and their images at points E, F and G is:

$$\begin{aligned} W = {} & m \ln Z + m_1 \ln [Z+L] + m_2 \ln [Z - L] \\ & + m \ln [Z + 2D \sin\alpha + 2iD \cos\alpha] \\ & + m_1 \ln [Z + L + 2(D - L \sin\alpha) \sin\alpha + 2i(D - L \sin\alpha)\cos\alpha] \\ & + m_2 \ln [Z - L + 2(D + L \sin\alpha) \sin\alpha + 2i(D + L \sin\alpha)\cose\alpha] \end{aligned} \quad (1)$$

Rearranging

$$\begin{aligned} W = {} & m \{\ln Z + \ln [Z + 2D \sin\alpha + 2iD \cos\alpha]\} \\ & + m_1 \{\ln [Z+L] + \ln [Z+L+2(D-L \sin\alpha)\sin\alpha+2i(D-L \sin\alpha)\cose\alpha]\} \\ & + m_2 \{\ln [Z-L] + \ln [Z-L+2(D+L \sin\alpha)\sin\alpha+2i(D+L \sin\alpha)\cos\alpha]\} \end{aligned} \quad (2)$$

Substituting $Z = x + iy$ where $i = \sqrt{-1}$ and simplifying :

$$\begin{aligned} W = {} & m \{\ln (x+iy) + \ln [x+2D \sin\alpha + i(y+2D \cos\alpha)]\} \\ & + m_1 \{\ln (x+L+iy) + \ln [x+L+2(D-L \sin\alpha) \sin\alpha \\ & + i(y + 2(D - L \sin\alpha)\cos\alpha)]\} + m_2 \{\ln (x-L+iy) \\ & + \ln [x - L + 2 (D + L \sin\alpha) \sin\alpha \\ & + i (y + 2 (D + L \sin\alpha) \cos\alpha)]\} \end{aligned} \quad (3)$$

Setting $W = \phi + i\psi$, where ϕ is the velocity potential and ψ is the stream function, and equating real to real and imaginary to imaginary in both sides of Eq. (3) results in

$$\phi = m (\ln R_1 + \ln R_2) + m_1 (\ln R_3 + \ln R_4) + m_2(\ln R_5 + \ln R_6) \quad (4)$$

In which :

$$R_1 = (x^2 + y^2)^{1/2} \quad (5)$$

$$R_2 = [(x+2D \sin\alpha)^2 + (y+2D \cos\alpha)^2]^{1/2} \quad (6)$$

$$R_3 = [(x+L)^2 + y^2]^{1/2} \quad (7)$$

[illegible] $= \{[x+L+2(D-L \sin\alpha)\sin\alpha]^2 + [y+2(D-L \sin\alpha)\cos\alpha]^2\}^{1/2}$ (8)

[illegible] $[(x-L)^2 + y^2]^{1/2}$ (9)

and

$$R_6 = \left\{ \left[x - L + 2(D + L \sin\alpha)\sin\alpha \right]^2 + \left[y + 2(D + L \sin\alpha)\cos\alpha \right]^2 \right\}^{1/2} \qquad (10)$$

and

$$\psi = m(\theta_1 + \theta_2) + m_1(\theta_3 + \theta_4) + m_2(\theta_5 + \theta_6) \qquad (11)$$

in which

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) \qquad (12)$$

$$\theta_2 = \tan^{-1}\left[(y + 2D\cos\alpha)/(x + 2D\sin\alpha) \right] \qquad (13)$$

$$\theta_3 = \tan^{-1}\left[y/(x + L) \right] \qquad (14)$$

$$\theta_4 = \tan^{-1}\left\{ \left[y + 2(D - L\sin\alpha)\cos\alpha \right] / \left[x + L + 2(D - L\sin\alpha)\sin\alpha \right] \right\} \qquad (15)$$

$$\theta_5 = \tan^{-1}\left[y/(x - L) \right] \qquad (16)$$

and

$$\theta_6 = \tan^{-1}\left\{ \left[y + 2(D + L\sin\alpha)\cos\alpha \right] / \left[x - L + 2(D + L\sin\alpha)\sin\alpha \right] \right\} \qquad (17)$$

DISCHARGE FORMULA

The velocity potential , ϕ , may be written in the form :

$$\phi = K\left(\frac{P}{\rho g} + y\right) \qquad (18)$$

in which K is the hydraulic conductivity of the soil , P is the gauge pressure, ρ is the density of water and g is the acceleration due to gravity .

Combining Eqs. (4) and (18) :

$$K\left(\frac{P}{\rho g} + y\right) = m\left[\ln R_1 + \ln R_2\right] + m_1\left[\ln R_3 + \ln R_4\right] + m_2\left[\ln R_5 + \ln R_6\right] \qquad (19)$$

At points B and C , Fig. 3, assuming that drains are running just full , the pressures are atmospheric . Also at point 0 on the free water surface the pressure is atmospheric .

Applying Eq. (19) to point B :

$$K \cdot \frac{d}{2} = m(\ln R_{11} + \ln R_{21}) + m_1(\ln R_{31} + \ln R_{41}) + m_2(\ln R_{51} + \ln R_{61}) \qquad (20)$$

in which

$$R_{11} = \frac{d}{2} \qquad (21)$$

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$$R_{21} = \left[(2D \sin\alpha)^2 + \left(\frac{d}{2} + 2D \cos\alpha\right)^2\right]^{1/2} \quad (22)$$

$$R_{31} = \left[L^2 + \left(\frac{d}{2}\right)^2\right]^{1/2} \quad (23)$$

$$R_{41} = \left\{\left[L+2(D-L \sin\alpha)\sin\alpha\right]^2 + \left[\frac{d}{2} +2(D-L \sin\alpha)\cos\alpha\right]^2\right\}^{1/2} \quad (24)$$

$$R_{51} = \left[L^2 + \left(\frac{d}{2}\right)^2\right]^{1/2} \quad (25)$$

and

$$R_{61} = \left\{\left[-L +2(D+L \sin\alpha)\sin\alpha\right]^2 - \left[\frac{d}{2} + 2 (D + L \sin\alpha) \cos\alpha\right]^2\right\}^{1/2} \quad (26)$$

applying Eq. (19) to point O :

$$KH = m (\ln R_{12} + \ln R_{22}) m_1 (\ln R_{32} + \ln R_{42}) + m_2 (\ln R_{52} + \ln R_{62}) \quad (27)$$

in which

$$R_{12} = \left[\left(\frac{L}{2}\right)^2 + H^2\right]^{1/2} \quad (28)$$

$$R_{22} = \left[\left(\frac{L}{2} + 2D \sin\alpha\right)^2 + (H+2D \cdot \cos\alpha)^2\right]^{1/2} \quad (29)$$

$$R_{32} = \left[\left(3 \frac{L}{2}\right)^2 + H^2\right]^{1/2} \quad (30)$$

$$R_{42} = \left\{\left[3 \frac{L}{2} + 2 (d-L \sin\alpha) \sin\alpha\right]^2 + \left[H + 2 (D - L \sin\alpha) \cos\alpha\right]^2\right\}^{1/2} \quad (31)$$

$$R_{52} = \left[\left(\frac{L}{2}\right)^2 + H^2\right]^{1/2} \quad (32)$$

and

$$R_{62} = \left\{\left[-\frac{L}{2} + 2(D + L \sin\alpha) \sin\alpha\right]^2 + \left[H + 2(D + L \sin\alpha) \cos\alpha\right]^2\right\}^{1/2} \quad (33)$$

and applying Eq. (19) to point C :

$$K \frac{d}{2} = m (\ln R_{13} + \ln R_{23}) + m_1 (\ln R_{33} + \ln R_{43}) + m_2(\ln R_{53} + \ln R_{63}) \quad (34)$$

in which :

$$R_{13} = \left[L^2 + \left(\frac{d}{2}\right)^2\right]^{1/2} \quad (35)$$

$$R_{23} = \left[(L + 2D \sin\alpha)^2 + \left(\frac{d}{2} + 2D \cos\alpha\right)^2\right]^{1/2} \quad (36)$$

$$R_{33} = \left[(2L)^2 + \left(\frac{d}{2}\right)^2\right]^{1/2} \quad (37)$$

$$R_{43} = \left\{\left[2L + 2\,(D - L \sin\alpha) \sin\alpha\right]^2 + \left[\frac{d}{2} + 2\,(D - L \sin\alpha) \cos\alpha\right]^2\right\}^{1/2} \quad (38)$$

$$R_{53} = \frac{d}{2} \quad (39)$$

and

$$R_{63} = \left\{\left[2\,(D + L \sin\alpha) \sin\alpha\right]^2 + \left[\frac{d}{2} + 2\,(D + L \sin\alpha) \cos\alpha\right]^2\right\}^{1/2} \quad (40)$$

Equations (20) , (27) and (34) may be put in the forms :

$$Am + A_1 m_1 + A_2 m_2 - K\frac{d}{2} = 0 \quad (41)$$

$$BM + B_1 m_1 + B_2 m_2 - KH = 0 \quad (42)$$

and

$$Cm + C_1 m_1 + C_2 m_2 - K\frac{d}{2} = 0 \quad (43)$$

respectively .
in which :

$$A = \ln R_{11} + \ln R_{21} \quad (44)$$

$$A_1 = \ln R_{31} + \ln R_{41} \quad (45)$$

$$A_2 = \ln R_{51} + \ln R_{61} \quad (46)$$

$$B = \ln R_{12} + \ln R_{22} \quad (47)$$

$$B_1 = \ln R_{32} + \ln R_{42} \quad (48)$$

$$B_2 = \ln R_{52} + \ln R_{62} \quad (49)$$

$$C = \ln R_{13} + \ln R_{23} \quad (50)$$

$$C_1 = \ln R_{33} + \ln R_{43} \quad (51)$$

and

$$C_2 = \ln R_{53} + \ln R_{63} \quad (52)$$

Solving Eqs.(41) , (42) and (43) simultaneously , the strength m becomes :

$$m=\frac{K(\frac{d}{2A_1}-\frac{H}{B_1})(1-\frac{B_1C_2}{B_2C_1})+(\frac{H}{B_1}-\frac{d}{2C_1})(1-\frac{B_1A_2}{B_2A_1})}{\left[(\frac{A}{A_1}-\frac{B}{B_1})(1-\frac{B_1C_2}{B_2C_1})+(\frac{B}{B_1}-\frac{C}{C_1})(1-\frac{B_1A_2}{B_2A_1})\right]} \qquad (53)$$

The discharge , Q, of the middle drain is thus given by :

$$Q=\frac{2\pi\cdot K\left[(\frac{d}{2A_1}-\frac{H}{B_1})(1-\frac{B_1C_2}{B_2C_1})+(\frac{H}{B_1}-\frac{d}{2C_1})(1-\frac{B_1A_2}{B_2A_1})\right]}{\left[(\frac{A}{A_1}-\frac{B}{B_1})(1-\frac{B_1C_2}{B_2C_1})+(\frac{B}{B_1}-\frac{C}{C_1})(1-\frac{B_1A_2}{B_2A_1})\right]} \qquad (54)$$

EFFECT OF THE ANGLE α ON THE DISCHARGE

The relation between the drain discharge per unit length , Q, and the angle of inclination of the impervious substratum , α, as given by Eq.(54) , is shown in Fig. 4.

In Fig 4 all the parameters , L,D,H,d, and K are kept constant such that the only variable is α .

CONCLUSIONS

From Fig. 4 , it is clear that the relation between the drain discharge and the angle of inclination of the substratum , α , is more of less linear . As the angle α increases the discharge decreases . It is worthy to note that the drain discharge is slightly affected by the change of the angle of inclination α and hence applying the ordinary discharge formulas (α =0.0) on drains underlain by an inclined impervious substratum will result in neglible errors .

APPENDIX I REFERENCES

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APPENDIX II NOTATION

The following symbols are used in this paper :

A = Quantity defined by Eq. 44 .
A_b = Quantities defined by Eqs. 45 and 46
B = Quantity defined by Eq. 47 .
B_b = Quantities defined by Eqs. 48 and 49 .
C = Quantity defined by Eq. 50 .
C_b = Quantities defined by Eqs. 51 and 52 .
D = The distance from the drain to the impervious layer .
d = Drain diameter .
g = Gravitational acceleration .
H = Height of water table midway between the middle and right drains above drain centres .
i = $\sqrt{-1}$
K = Hydraulic conductivity of clay .
L = Distance between two successive drains .
m = Strength of the sink representing the middle drain .
m_1 = " " " " " " left " .
m_2 = " " " " " " right " .
P = Gauge pressure .
Q = Drain discharge per unit length .
R_e = Quantities defined by Eqs. from 5 to 10 .
R_{en} = " " " " " 21 to 26 , from 28 to 33 and from 35 to 40 .
W = Complex potential
x,y = Cartesian coordinates with origin of coordinates at the centre of the middle drain .
Z = $x + iy$.
α = Angle of inclination of the impervious substratum .
ϕ = Velocity potential .
ψ = Stream function .
ρ = Density of water .

and

θ_e = Quantities defined by Eqs. from 12 to 17 .

Subscripts

b = 1, 2 ;
e = 1 , 2 , ..., 6.

and

n = 1 , 2 , 3 .

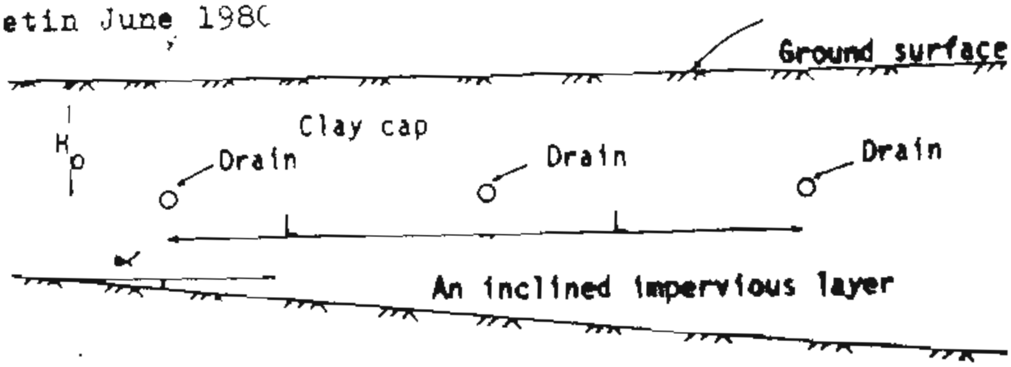

Fig. 1: GEOLOGICAL SECTION

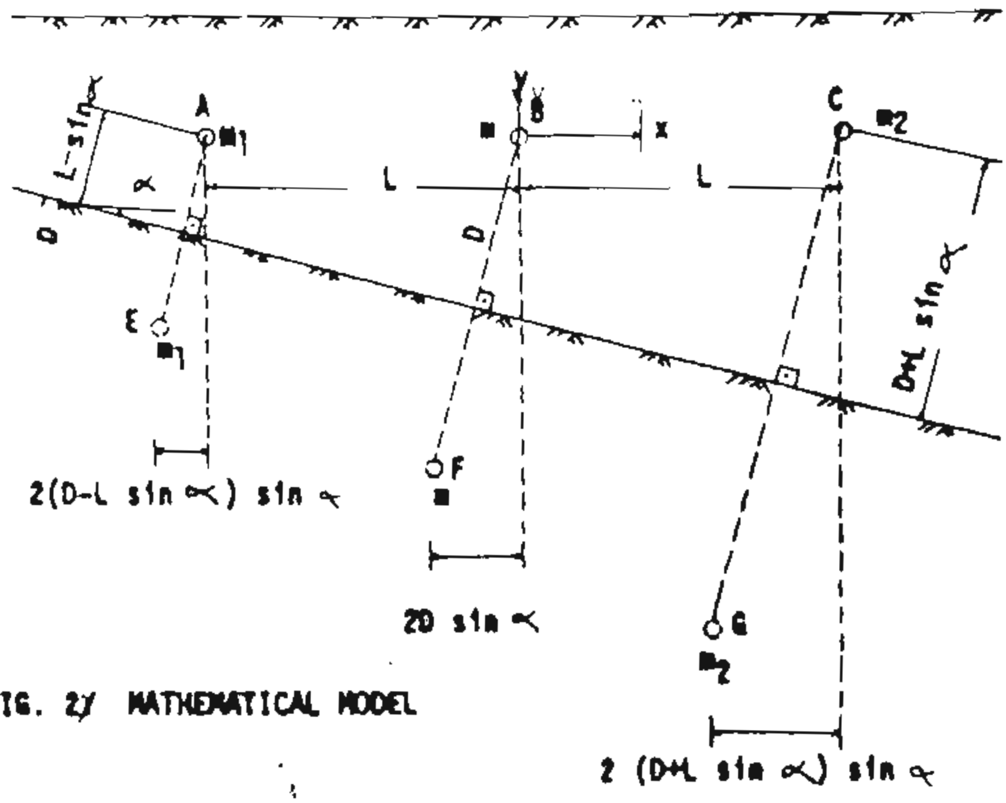

FIG. 2/ MATHEMATICAL MODEL

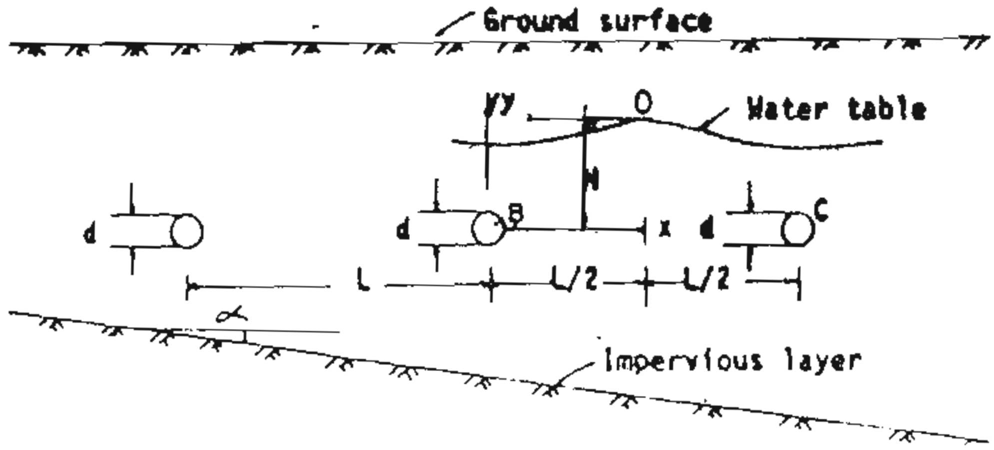

FIG. 3: BOUNDARY CONDITIONS

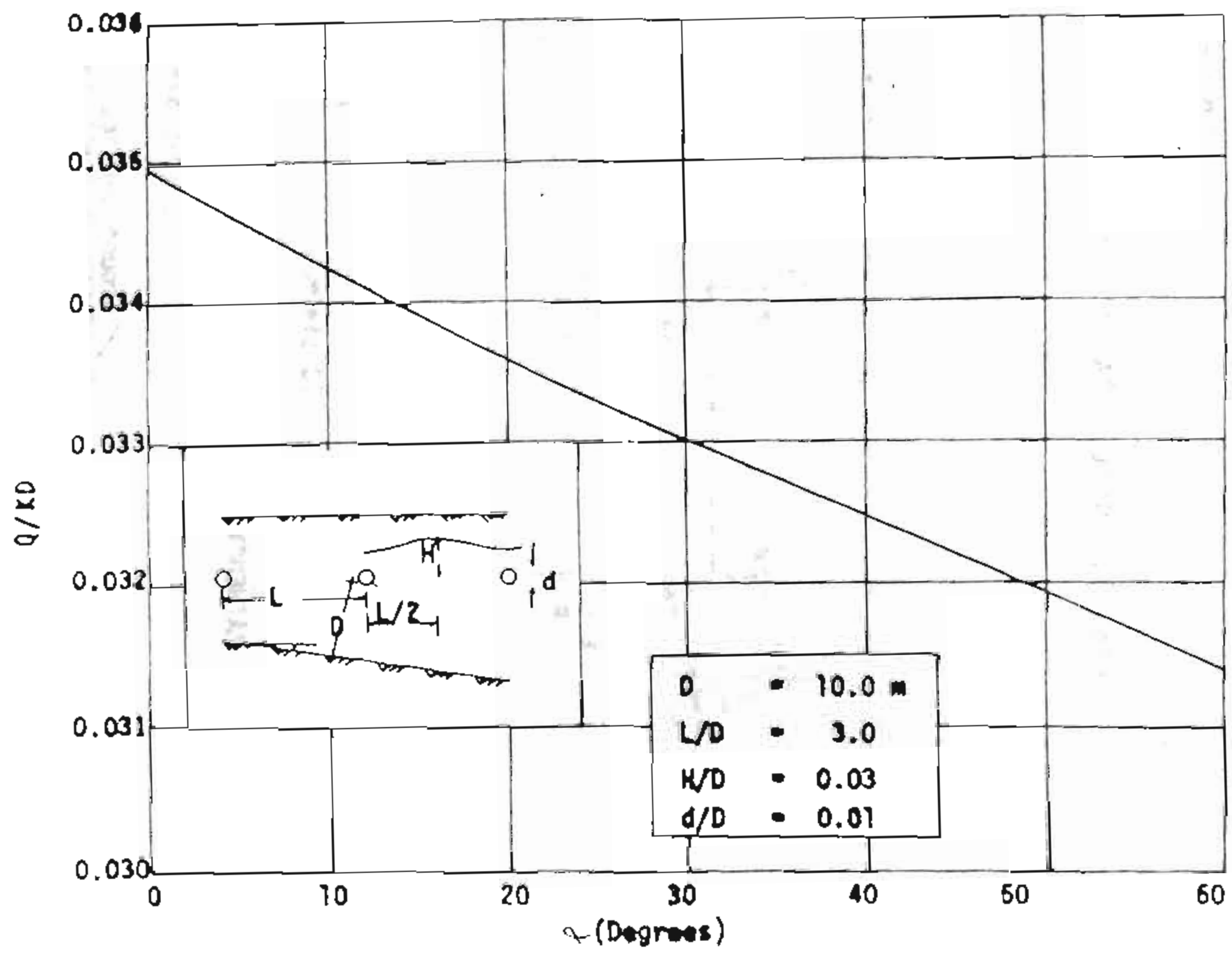

FIG. 4: RELATION BETWEEN DISCHARGE AND ANGLE, α .