THE TRANSIENT MOTION OF A CURRENT LOOP ALONG A CONDUCTING CYLINDRICAL SHELL

الحركة العابرة لأنبوب تيار خلال قشرة اسطوانية موصلة

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ملخـــــص

خلاصة :

في هذا البحث جرى معالجة رياضية لتوزيع التيارات الدوامة في أنبوب موصـــل ذو مقطع دائري تحت تأثير محفز خارجي.لقد أعتبر المؤثر الخارجي حلقة تيـــار دائرية حول الأنبوب فيها تيار كهربائي ذو تغير زمني ثـــابت وتســـير بســـرعة متغيرة على طول الأنبوب. وقد تم الحصول على معادلة التيارات الدوامـــة عـــن طريق حل معادلة تفاضلية زمنية باستخدام تحويل لابلاس. وقد أعطـــى توزيـــع التيارات الدوامة في الأنبوب والقوة العاملة على الحلقة علـــى شـــكل تكـــاملات متشابكة ومرسمات متعددة.

Abstract

A mathematical and an analytical procedure for calculating the induced surface eddy currents in a cylindrical shell due to an arbitrary moving concentric current loop is developed. The consideration involves both uniform and accelerated motions. The accelerated motion, has the form of a finite ramp function, by which the current loop moves with a constant acceleration during a given time interval to a certain velocity. The surface currents induced in the cylindrical shell are governed by a time-dependent linear differential equation, which is then solved by the method of Laplace transforms. A solution in the form of Fourier and convolution integrals is achieved. The analytical solution is demonstrated in a graphical form in which the induced surface currents for both uniform and accelerated motion are presented. The solution so obtained is then extended to an arbitrary motion. The force acting on the moving current loop is also calculated and the results are presented in analytical and graphical forms.

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1. Introduction

There are many applications in today's technology for eddy currents. Some of these applications fall in the following categories: Cylindrical geometry with excitation caused by concentric circular current loops, cylindrical geometry with excitation originated from rotating fields, and finally cylindrical geometry with excitation coming from moving circular current loops. These cases arise in applications such as non-destructive testing of materials, induction heating, and magnetic levitation [11-13].

Many interesting aspects of magnetic levitation have led to the examination of problems related to the motion of exciting sources near conducting media. The induced eddy currents within a conducting medium interact with the excitation sources, resulting in forces which can be used for levitation of the moving part carrying the excitation circuit. As a result of the motion of the loop, eddy currents develop within the conducting cylindrical shell in the form of surface currents, while a lift force is exerted on the suspension system.

Many books [1-4] and papers [5-9] have dealt with the analytical study of eddy currents in solid conducting cylinders and cylindrical shells, in which the excitation is due to axially directed parallel wires, stationary and rotating around the cylinder. The problem of interest in this investigation however, is related to a current loop surrounding a cylindrical shell and moving along the axes.

2. Formulation of the problem

A concentric circular current loop of radius a carries a time-independent current I_0 and moves along a thin-walled conducting cylindrical shell with an arbitrarily velocity $v = v(t)$. The cylindrical shell is of radius a, conductivity σ , and thickness h much less than the skin depth δ, see figure 1.

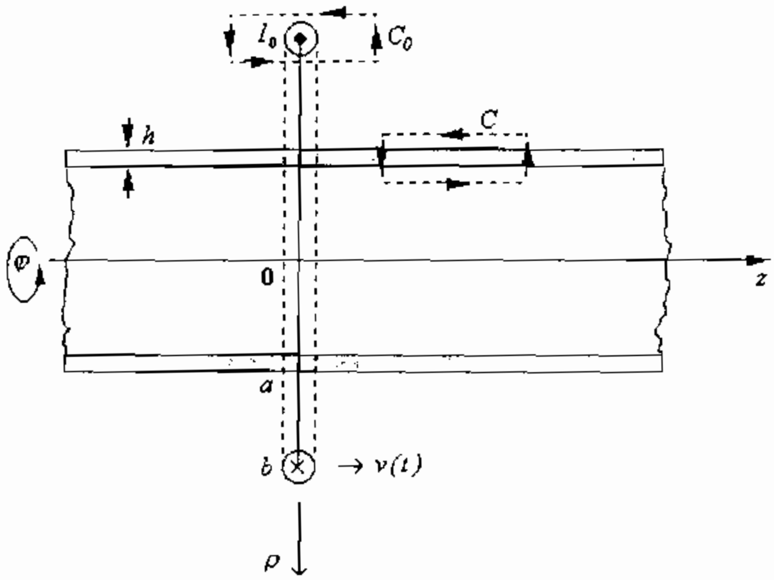

Fig. 1: A circular concentric current loop of radius b around a conducting thin cylindrical shell of radius a

3. Development of the solution

The present skin effect problem is solved using the solenoidal magnetic vector potential $\vec{A}=\vec{A}(\vec{r},t)$ with $div\,\vec{A}=0$. The magnetic flux density $\vec{B}=\vec{B}(\vec{r},t)$ is then obtained from $\vec{B}=curl\,\vec{A}$. According to the configuration, figure (1), the exciting magnetic vector potential of the current loop $\vec{A}_e=\vec{A}_e(\vec{r},t)$ is rotationally symmetric and has only one φ-directed component in the cylindrical coordinate system (ρ,φ,z), which is a function of the space coordinates ρ and z as well as of time t: $\vec{A}_e(\vec{r},t)=\vec{e}_\varphi A_{\varphi e}(\rho,z,t)$. Because of the thin cylindrical shell, the induced eddy currents due to the motion of the loop, are surface currents with surface current density $\vec{K}=\vec{e}_\varphi K_\varphi(z,t)$ The electric field intensity $\vec{E}=\vec{E}(\vec{r},t)$ is related to the surface current density $\vec{K}$ by Ohm's law $\vec{E}=\vec{K}/\sigma h$. The induced surface currents cause a distributed field in the surrounding non-conducting medium, which has a φ-directed vector potential as a function of the space coordinates ρ and z as well as time t: $\vec{A} = \vec{A}(r,t) = \vec{e}_\varphi A_\varphi(\rho, z, t)$. This vector potential will be called the excited vector potential. The relation between the exciting vector potential $\vec{A}_e$ and the excited vector potential $\vec{A}$ can be obtained by applying Faraday's law at the cylinder surface $\rho = a$:

$$\oint_C \vec{E}\, d\vec{\ell} = -\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S (\vec{B} + \vec{B}_e)\, d\vec{S} = -\frac{d}{dt}\iint_S curl\, (\vec{B} + \vec{B}_e)\, d\vec{S} \qquad (1)$$

The surface integral on the right hand-side of the above relation can be converted into a contour integral by applying Stoke's theorem:

$$\oint_C \frac{\vec{K}}{\sigma h} d\vec{\ell} = -\frac{d}{dt}\oint_C (\vec{A} + \vec{A}_e) d\vec{\ell}$$

Since all the vector quantities in the above relation are φ-directed, one can write at once:

$$-\frac{d}{dt}[A_\varphi(a,z,t) + A_{\varphi e}(a,z,t)] = \frac{1}{\sigma h} K_\varphi(z,t) \qquad (2)$$

In order to satisfy the boundary conditions at the cylinder surface, it is necessary to expand the surface current density K_φ in terms of the orthogonal functions of the exciting field $A_{\varphi e}$.

The above differential equation is best solved by Laplace transformation. The differential equation is therefore considered in the frequency domain. The solution in the time domain t is then obtained by taking the inverse transform.

3.1 The field of the moving current loop

In the absence of the conducting cylinder, the exciting field is a field in a non-conducting medium, which can be determined by the exciting vector potential $\vec{A}_e$, for which the following vector field equation is valid: $curl\ curl\ \vec{A}_e = 0$. The exciting vector potential is solenoidal, φ- directed, and rotationally symmetric: $\vec{A}_e = \vec{e}_\varphi A_{\varphi e}(\rho, z, t)$.

The vector field equation is then reduced to the following scalar partial differential equation:

$$\frac{\partial^2 A_{\varphi e}}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial A_{\varphi e}}{\partial \rho}-\frac{A_{\varphi e}}{\rho^2}+\frac{\partial^2 A_{\varphi e}}{\partial z^2}=0 \qquad (3)$$

By the separation of variables, the solution will consist of trigonometric functions of argument (mz) and modified Bessel functions of the first and second kind of the argument $(m\rho)$ and order *one*. For a circular current loop of radius b, which passes the coordinate $z=0$ at a certain time $t=0$, the exciting potential can be expressed in the form of a Fourier integral as follows:

$$A_{\varphi e}=\mu_0 I_0\int_0^\infty C(m)\, I_1(m\rho)\cos(mz)\,dm \qquad \text{for} \qquad 0\le\rho\le b \qquad (4\text{ a})$$

$$A_{\varphi e}=\mu_0 I_0\int_0^\infty D(m)\, K_1(m\rho)\cos(mz)\,dm \qquad \text{for} \qquad b\le\rho\le\infty \qquad (4\text{ b})$$

$I_1(\zeta)$ and $K_1(\zeta)$ denote the modified Bessel functions of the first and second kind of the argument ζ and order *one* respectively. $C(m)$ and $D(m)$ are unknown functions to be determined from the boundary conditions, which require as first, the continuity of the potential $A_{\varphi e}$ at $\rho=b$. The second boundary condition can be obtained from Ampere's law by performing the contour integral

$$\oint_{C_o}\frac{1}{\mu_0}\bar{B}_e\,d\bar{\ell}=I_0 \qquad (5)$$

as indicated in figure 1. Therefore, the potential $A_{\varphi e}$ will be:

$$A_{\varphi e}=\mu_0 I_0\frac{b}{\pi}\int_0^\infty K_1(mb)\, I_1(m\rho)\cos(mz)\,dm \quad \text{for } 0\le\rho\le b \qquad (6\text{ a})$$

$$A_{\varphi e}=\mu_0 I_0\frac{b}{\pi}\int_0^\infty I_1(mb)\, K_1(m\rho)\cos(mz)\,dm \quad \text{for } b\le\rho\le\infty \qquad (6\text{ b})$$

The time dependence of the exciting potential $A_{\varphi e}$ of the moving current loop can be taken into account, by replacing the argument z in the cosine function by $z'=z-t\,v(t)$. The exciting potential of the moving current loop will be:

$$A_{\varphi e}=\mu_0 I_0\frac{b}{\pi}\int_0^\infty K_1(mb)\, I_1(m\rho)\cos\{m[z-t\,v(t)]\}\,dm \quad \text{for } 0\le\rho\le b \qquad (7\text{ a})$$

$$A_{\varphi e}=\mu_0 I_0\frac{b}{\pi}\int_0^\infty I_1(mb)\, K_1(m\rho)\cos\{m[z-t\,v(t)]\}\,dm \quad \text{for } b\le\rho\le\infty \qquad (7\text{ b})$$

3.2 The distributed field

With the presence of the conducting cylindrical shell, eddy currents are induced in the shell and cause a distributed field in the surrounding medium, for which the potential $\vec{A} = \vec{e}_\varphi A_\varphi$ is determined by the solution of the vector field equation: $curl\ curl\ (\vec{e}A_\varphi) = 0$. Since the distributed field is solenoidal and rotationally symmetric, the vector field equation is also reduced to a scalar partial differential equation of the same form as (3). In order to satisfy the boundary conditions at the shell surface $\rho = a$, the trigonometric functions which appear in the solution, must be of the same variation as for the exciting field. Therefore, the potential of the distributed field can be written in the form of the following Fourier integral:

$$A_{\varphi i} = \mu_0 \int_0^{\infty} D(m) K_1(ma) I_1(m\rho) \cos\{m[z - t\,v(t)]\}\, dm \quad \text{for } 0 \le \rho \le a \tag{8 a}$$

$$A_{\varphi u} = \mu_0 \int_0^{\infty} D(m) I_1(ma) K_1(m\rho) \cos\{m[z - t\,v(t)]\}\, dm \quad \text{for } a \le \rho \le \infty \tag{8 b}$$

D(m) is an unknown function to be determined from the boundary conditions at the shell surface $\rho = a$. The form given in (8), already fulfills the continuity condition at the shell surface.

A relationship between the potential of the distributed field $\vec{A}$ and the induced surface currents $\vec{K}$ is obtained, by applying Ampere's law (5) at the shell surface. Remembering that, the potential $\vec{A}$ is continuous when passing through the interface $\rho = a$, the condition given below, holds

$$\vec{e}_\varphi \left[\frac{\partial A_{\varphi u}}{\partial \rho} - \frac{\partial A_{\varphi i}}{\partial \rho}\right] = \mu_0 \vec{e}_\varphi K_\varphi \quad \text{for } \rho = a \tag{9}$$

In order to satisfy the boundary condition at the shell surface, the surface current density K_φ is expressed as a Fourier integral

$$K_\varphi(z,t) = \int_0^{\infty} k(m) \cos\{m[z - t\,v(t)]\}\, dm \tag{10}$$

Inserting expressions (8) and (10) in (9), yields the result: $D(m) = a\,k(m)$

Applying Faraday's law (2) at the shell surface, results in the following linear differential equation:

$$\frac{dP_m}{dt} + \alpha_m P_m = -\frac{dQ_m}{dt} \quad \text{with} \quad \alpha_m = \frac{1}{\mu_0 \sigma h a} \frac{1}{I_1(ma) K_1(ma)} \tag{11}$$

and $P_m(z,t) = D(m)\, cos\{m[z - tv(t)]\}$

$$Q_m(z,t) = I_0 \frac{b}{\pi} \frac{K_1(mb)}{K_1(ma)} cos\{m[z - tv(t)]\}$$

The solution of the differential equation (11) is obtained by Laplace transformation and the result is given below as a convolution integral:

$$P_m(z,t) = -\int_0^t \frac{dQ_m(z,t-t')}{dt} exp(-\alpha_m t')\,dt' \tag{12}$$

If the function $Q_m(z,t)$ for $t \le 0$ is not constant, that means, the current loop moves also in the range $z \le 0$, then the upper limit in the convolution integral is equal infinity

For a given predefined motion of the current loop, which is characterized by the velocity function $v(t)$, the required surface current density K_φ is then determined by:

$$K_\varphi(z,t) = -I_0 \frac{b}{a} \frac{1}{\pi} \int_0^\infty \frac{K_1(mb)}{K_1(ma)} f(m,z,t)\,dm \tag{13}$$

with $f(m,z,t) = \int_0^t \frac{d}{dt}\{cos\{m[z - (t-t')\, v(t-t')]\}\}\, exp(-\alpha_m t')\,dt'$

The surface current density K_φ as determined in (13) is valid for any time-variable motion of the current loop $v(t)$. To be more specific, two cases are treated in the next section explicitly: the uniform and the accelerated motion.

4.1 The uniform motion of the current loop

In this case, the current loop moves with constant velocity v_0 from $z = -\infty$ to $z = \infty$. The upper limit in the convolution integral (13) becomes infinity so that, the integral can be evaluated in closed form. The surface current density K_φ with the reference surface current $K_0 = I_0/2\pi a$ is then determined by:

$$\frac{K_\varphi(z,t)}{K_0} = -2\frac{b}{a}\int_0^\infty \frac{K_1(mb)}{K_1(ma)} f_0(m,z,t)\, d(ma) \tag{14}$$

with $f_0(m,z,t) = \dfrac{(mv_0)^2 \cos[m(z-v_0t)]}{(mv_0)^2+\alpha_m{}^2} + \dfrac{(mv_0)\alpha_m \sin[m(z-v_0t)}{(mv_0)^2+\alpha_m{}^2}$

A plot of surface current density with $k=\mu_0\sigma h v_0$ as a parameter is shown in figure(2). It can be seen that the surface current density is an odd symmetric function of the moving coordinate $(z-v_0t)$ for relatively small values of the parameter k, which represents small velocities. As the values of k increase, the zero crossing of the function moves to the left in the region where $(z-v_0t)\langle 0$ and the odd symmetry of the function is distorted. Hence, the extreme value of the function in the region $(z-v_0t))0$ becomes greater than in region $(z-v_0t)\langle 0$, such that for $k\to\infty$ the surface current density function becomes symmetric about the origin. At very high velocities where the term $\alpha_m/mv_0 << 1$, the surface current density reduces to:

$$\frac{K_\varphi(z,t)}{K_0} = -2\frac{b}{a}\int_0^\infty \frac{K_1(mb)}{K_1(ma)} cos[m(z-v_0t)]\, d(ma) \tag{15}$$

A plot of this function is shown in figure (3).

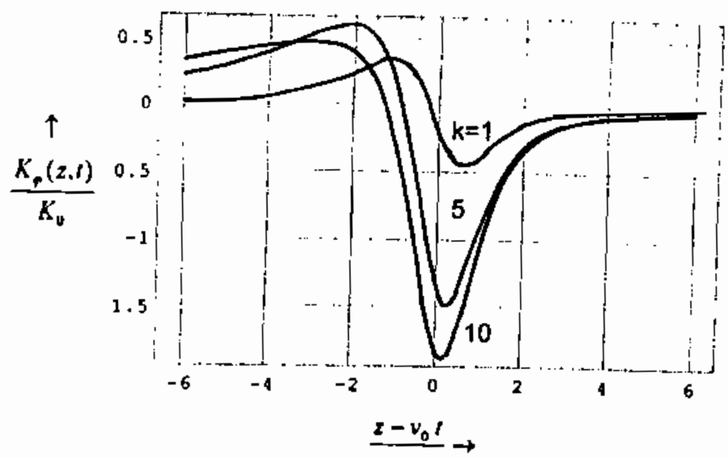

Fig. (2): The surface current density K_φ/K_0 along the cylindrical shell versus the moving coordinate $(z-v_0t)/a$ with $k=\mu_0\sigma h v_0$ as a dimension-less parameter for $b/a=2$

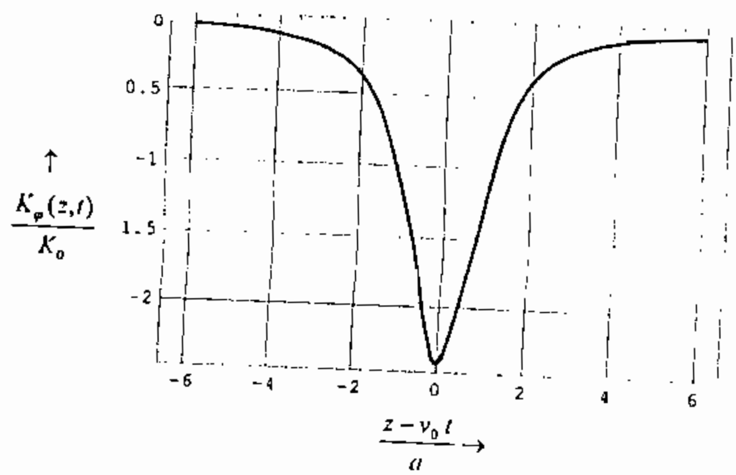

Fig.(3): The surface current density K_φ / K_0 along the cylindrical shell versus the moving point $(z - v_0 t)/a$ at $k = \infty$ for $b/a = 2$

4.2 The accelerated motion

The accelerated motion to be considered is the motion of the current loop when it moves with a time-variable velocity $v(t)$ in the form of a finite ramp function. The current loop moves first with a constant acceleration $a(t) = a_0 = v_0/t_0$ during the time interval $0 \le t \le t_0$, then uniformly to infinity with velocity v_0, figure (4).

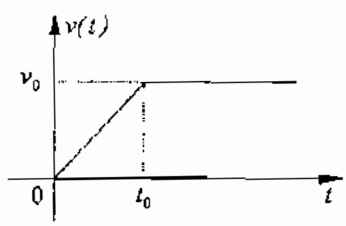

Fig (4). velocity variation as a finite ramp function

Therefore, the surface current density K_φ possesses two components; first K_φ^{I} due to the accelerated motion, and second K_φ^{II} due to the uniform motion. The two components are then calculated separately.

During the accelerated motion in the time interval $0 \le t \le t_0$, the loop moves a distance $z_v = v_0 t^2/2t_0$, so that the coordinate $z' = z - z_v = z - v_0 t^2/2t_0$, and the integration is performed from *zero* to *t*. The first component of the surface current density is then determined by:

$$\frac{K_\varphi^I(z,t)}{K_0} = -2\frac{b}{a}\int_0^\infty \frac{K_1(mb)}{K_1(ma)} f_1(m,z,t)\,d(ma) \qquad \text{for } 0 \le t \le t_0 \qquad (16)$$

with $f_1(m,z,t) = \dfrac{(mv_0)}{t_0}\displaystyle\int_0^t sin\{m[z - \frac{1}{2}\frac{v_0}{t_0}(t-t')^2]\}(t-t')\,exp(-\alpha_m t')\,dt'$

In the time interval $t \ge t_0$, the second component K_φ^{II} consists of two parts; the first comes from the accelerated motion with integration limits from $(t-t_0)$ to t, the second from the uniform motion with integration limits from *zero* to $(t-t_0)$.

$$\frac{K_\varphi^{II}(z,t)}{K_0} = -2\frac{b}{a}\int_0^\infty \frac{K_1(mb)}{K_1(ma)}[\,f_2(m,z,t) + f_3(m,z,t)\}\,d(ma) \quad \text{for } t \ge t_0 \qquad (17)$$

with $f_2(m,z,t) = \dfrac{(mv_0)}{t_0}\displaystyle\int_{t-t_0}^t sin\{m[z - \frac{1}{2}\frac{v_0}{t_0}(t-t')^2]\}(t-t')\,exp(-\alpha_m t')\,dt'$

and $f_3(m,z,t) = (mv_0)\displaystyle\int_0^{t-t_0} sin\{m[z - v_0(t-t_0-t')]\}\,exp(-\alpha_m t')\,dt'$

The inner integral in the second part of the solution can be evaluated in closed form, and the result is given below:

$$\begin{aligned}(mv_0)\int_0^{t-t_0} & sin\{m[z - v_0(t-t_0-t')]\}\,exp(-\alpha_m t')\,dt' = \\ &= \frac{(mv_0)^2 cos\{m[z - v_0(t-t_0)]\} + (mv_0)\,\alpha_m\, sin\{m[z - v_0(t-t_0)]\}}{(mv_0)^2 + \alpha_m^{\,2}} - \qquad (18)\\ &- \frac{(mv_0)cos(mz) + \alpha_m\, sin(mz)}{(mv_0)^2 + \alpha_m^{\,2}}\,exp[-\alpha_m(t-t_0)]\end{aligned}$$

For $t \gg t_0$ all the terms in (17) that contain the exponential function $exp[-\alpha_m(t-t_0)]$ approach zero and only one term remains, that is the first term in (18), so that expression (17) reduces to (14) for the uniform motion, whereby the correctness of the results obtained is verified.

Figure (5) shows the surface current density versus the coordinate z with time t as a parameter.

4.3 The arbitrary motion of the loop

To determine the induced surface currents on a cylindrical shell due to an arbitrary transient motion, it is convenient to use the superposition integral (DUHAMEL) [10]. This super position integral states that: in a linear system if the characteristic output time response function $y_u(t)$ due to an input unit step function $u(t)$ is already known, then the output response function $y_x(t)$ due to an arbitrary input function $x(t)$ is determined by

$$y_x(t) = x(0)y_u(t) + \int_0^t \frac{dx(\tau)}{d\tau} \cdot y_u(t-\tau)\, d\tau \tag{19}$$

For the present case, the response function due to a step function can easily be obtained from the results of the previous section. By letting the time interval t_0 of the finite ramp function approaches zero, it can be seen easily that the finite ramp function becomes a step function of the form $v(t) = v_0 \cdot u(t)$ and from equation (17) the response function will take the form:

$$\frac{K_\varphi^u(z,t)}{K_0} = -2\frac{b}{a}\int_0^\infty \frac{K_1(mb)}{K_1(ma)} f_3(m,z,t)\,d(ma) \quad ; \quad t \geq 0 \tag{20}$$

with

$$f_3(m,z,t) = \frac{(mv_0)^2 \cos[m(z-v_0t)] + (mv_0)\alpha_m \sin[m(z-v_0t)]}{(mv_0)^2 + \alpha_m^2} - \frac{(mv_0)\cos(mz) + \alpha_m \sin(mz)}{(mv_0)^2 + \alpha_m^2} \exp(-\alpha_m t)$$

For an arbitrary motion of the current loop with the normalized velocity function $v(t)/v_0$, the induced surface current density will be

$$\frac{K_\varphi(z,t)}{K_0} = \frac{v(0)}{v_0}\frac{K_\varphi^u(z,t)}{K_0} + \int_0^t \frac{1}{v_0}\frac{dv(\tau)}{d\tau}\frac{K_\varphi^u(z,t-\tau)}{K_0}\, d\tau \tag{21}$$

Since the formal general solution (21) of the problem does not add much to the cases already treated in the previous sections, further study of the problem will not be continued.

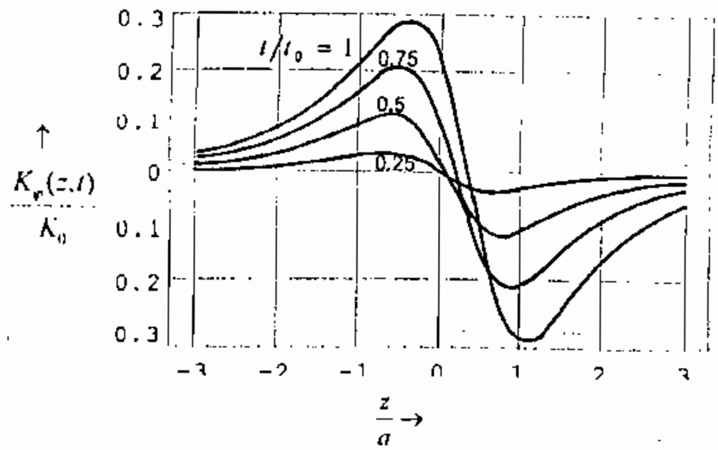

Fig.(5): The surface current density K_φ/K_0 versus the coordinate z/a with time t/t_0 as parameter during the accelerated motion for $(v_0 t_0/a) = 1$, $k = \mu_0 \sigma h v_0 = 1$, and $b/a = 2$

5. The force acting on the moving current loop

In general, the force $\vec{F}$ acting on a current loop of impressed current I_0 and circumference C situated in an external magnetic field of flux density $\vec{B}$ is determined by the following line integral

$$\vec{F} = I_0 \oint_C \vec{d\ell} \times \vec{B} \tag{22}$$

where $\vec{d\ell}$ is a tangent vector on the current loop oriented toward the positive direction of the current. The magnetic flux density $\vec{B}$ is that of the distributed field and is obtained from the expressions (8b) and (11). Since the distributed field is rotationally symmetric and the current loop is placed coaxial with the cylindrical shell, it is obvious that the force acting on the moving current loop is a drag force and has only one effective component in the z-direction. After performing the required contour integration (22), one obtains for the z-component of the force the general expression:

$$F_z = -2\mu_0 I_0^2 b^2 \int_0^\infty \frac{I_1(ma)}{K_1(ma)} K_1^2(mb) \frac{\partial}{\partial z} f(m,z,t)\,|_{z=z_c(t)}\, dm \qquad (23)$$

$$f(m,z,t) = \int_0^t \exp(-\alpha_m t) \frac{d}{dt} \cos\{m[z-(t-t')v(t-t')]\}\, dt'$$

where $z_c(t)$ is position of the moving current loop at the observation time t.

For the special case of a uniform motion with constant velocity v_0 . we obtain with the reference value $F_0 = \mu_0 I_0^2$ the result:

$$\frac{F_z}{F_0} = -2\frac{b^2}{a^2} \int_0^\infty \frac{I_1(ma)}{K_1(ma)} K_1^2(mb)(ma) \frac{(\frac{mv_0}{\alpha_m})}{1+(\frac{mv_0}{\alpha_m})^2}\, d(ma) \qquad (24)$$

A plot of the force function (24) is shown in Fig.(6). It is observed that the force acting on the moving current loop is a drag force which increases linearly for small values of k. The force function reaches an extreme value at $k \approx 3.5$ then it decreases monotonously.

For the case of an accelerated motion, we obtain in the time-interval $0 \le t \le t_0$ the expression:

$$\frac{F_z}{F_0} = -2\frac{b^2}{a^2}\left(\frac{v_0 t_0}{a}\right) \int_0^\infty \frac{I_1(ma)}{K_1(ma)} K_1^2(mb)(ma)^2\, g_1(m,t)\, d(ma) \qquad (25)$$

with $g_1(m,t) = \int_0^{\frac{t}{t_0}} (\frac{t}{t_0} - \frac{t'}{t_0}) \exp[-(\alpha_m t_0)\frac{t'}{t_0}] \cos\{(ma)(\frac{v_0 t_0}{a})[(\frac{t}{t_0} - \frac{1}{2}\frac{t'}{t_0})\frac{t'}{t_0}]\}\, d(\frac{t'}{t_0})$

A plot of this function is shown in Fig.(7). The force function of the transient motion during the time interval $0 \le t \le t_0$ is an increasing function of time, which then remains constant in the time interval $t \ge t_0$ where the acceleration becomes zero.

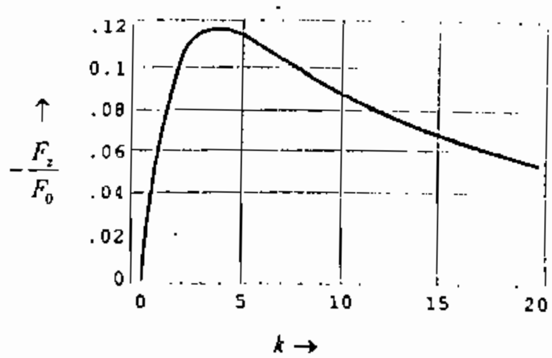

Fig.(6): The force function F_z/F_0 versus the parameter $k = \mu_0 \sigma h v_0$ for the uniform motion with constant velocity for $b/a = 2$

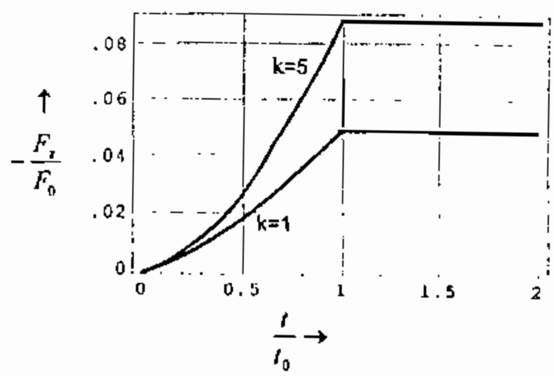

Fig.(7): The force function F_z/F_0 versus the time t/t_0 during the accelerated motion with $k = \mu_0 \sigma h v_0$ as parameter for $(v_0 t_0/a) = 1$, and $b/a = 2$

6. Conclusion

Analysis of the interesting aspects in the study of eddy currents in a cylindrical shell has been developed. The analysis began with a constant velocity motion of a concentric current loop which was then extended to an accelerated motion Eddy currents resulting from the relative motion were expressed as Fourier and convolution integrals which lent themselves readily to computational techniques. Plots of the induced surface current density for both constant and accelerated motion were demonstrated. An extension of the problem to an arbitrary motion of the current loop was also given. The force acting on the moving current loop is also calculated and the results are presented in a graphical form

References

[1] Smythe, W., R.,"Static and Dynamic Electricity",McGraw-Hill 1968

[2] Tegepoulos J, and Kriezis E.,E.,"Eddy currents in linear conducting media", Amesterdam : Elsevier 1985

[3] Stoll,R.L.,"The analysis of eddy currents",Oxford :Clarendon 1974

[4] Lammeraner,J. and Stafl,M.,"Eddy Currents",Lliffe Books Ltd,London,1966

[5] Hannakam ,L ,"Beitrag zur berechnung transienter Wirbelstroeme im massiven Zylinder", Archiv fuer Elektrotechnik Band 52, Heft 5, 1969 (273 –285)

[6] Hannakam, L., "Transienter Skineffekt in zwei duennwandigen Zylindern, hervorgerufen durch eine rotierende Spule",ETZ-A Band 94, 1973 (731 –735)

[7] Bastawros, A.,"Reflected electromagnetic field for a moving permanent magnet", Archiv fuer Elektrotechnik Band 70, 1987 (303 –306)

[8] Jensen,K.,M. and Hannakam, L.,"Current distribution in a rotating cylinder caused by a radially supplied current",Archiv fue Elektrotechnik Band 59, 1977 (253 –259)

[9] Nolle, E. And Hannakam, L.,"Cylindrical ring rotating in the field of a salient poles" Archiv fue Elektrotechnik ,Band 62, 1980 (295 –300)

[10] Stanford Goldman,"Transformation Calculus and Electrical Systems", Prentice-Hall,INC, Englewood Cliffs,N.J.

[11] Chen,D.X., and Mun~oz,J.L., "AC Impedance and Circular permeability of Slab and Cylinder",IEEE Transactions on Magnetics,May 1999,Volume 35,Number 03

[12] Furlani,E.P.,"Analysis of the eddy current field of a rotary bias field device for magneto-optic recording",IEEE Transactions on Magnetics,November 1998, Volume 34, Number 06,[p.

[13] Tozoni,O.V. "New Stable Magnetodynamic Suspension System", IEEE Transactions on Magnetics,March 1999, Volume 35, Number 02 ,[p.1047]