

**NATURAL CONVECTION HEAT TRANSFER  
FROM A UNIFORM HEAT FLUX PLATE  
WITH ARBITRARY ORIENTATION**

انتقال الحرارة بالحمل من لوح مستو ذو ميل اختياري سخن من كلا جانبيه بقيتي حراري منتظم

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الخلاصة : يتضمن هذا البحث دراسة عملية لانتقال الحرارة بالحمل الحر من لوح من الصلب محدود الابعاد طوله 171مم وعرضه 117مم وسمكه 6 مم سخن من كلا سطحيه بقيتي حراري منتظم. اللوح قابل للحركة حول محور يمر بجانبيه العليا لاطاء زوايا ميل مختلفة من  $90^\circ$  الى  $90^\circ$  مع المحور الرأسي حيث تعنى الاشارة السالبة ميل اللوح في اتجاه صاكس للاتجاه الموجب لبيان انتظام لف تلك التسخين . اوضحت نتائج التجارب أن معامل انتقال الحرارة المتوسط على سطح اللوح لا يعتمد على زاوية ميله على الرأسي أي أن اللوح في الوضع الافقي والمائل يتبع نفس معادلة انتقال الحرارة لنفس اللوح في الوضع الرأسي تحت نفس الظروف .

**ABSTRACT**

This paper represents an experimental investigation on natural convection heat transfer from a finite flat plate. The plate is heated from both sides with a uniform heat flux. The orientation of the plate has different angles of inclination ( $\theta$ ) on the vertical axis. The inclination angle ( $\theta$ ) varies from  $-90^\circ$  to  $90^\circ$ . The plate dimensions are 171 mm height, 117 mm width and 6 mm thickness. The obtained heat transfer results show that there is no a remarkable effect of the angle of inclination on the average Nusselt number values by comparison to those obtained in the case of vertical orientation. During the course of the present experimental work the value of  $Ra^*$  is ranging between  $1.9 \times 10^6$  and  $8 \times 10^7$ . Comparison with the available experimental work is also made. The present results are, in general, higher than the data in comparison by about 8% and this increase may be due to the end effect of the tested plate.

**INTRODUCTION**

In our environment and engineering devices heat transfer by natural convection is frequently encountered. Free convection along vertical, inclined and horizontal surfaces with uniform heat flux and uniform temperature has been extensively studied theoretically and experimentally in laminar, transition and turbulent regions.

Rich [1] measured heat transfer at angles of inclination up to  $40^\circ$  from the vertical axis using a Mach-Zehnder Interferometer. These results indicated that the heat transfer comparatively agrees with same of vertical plate within 10%. In case of the inclined plate Grashof number is multiplied by the term  $\cos(\theta)$ . Subsequent studies of Vliet [2], Hassan and Mohamed [3], Lloyd, Sparrow and Eckert [4], Fujii and Imura [5], Vliet and Ross [6] and Black and Orlic [7] have all supported the suggestions of Rich.

Lee and Lock [8] solved the boundary - layer equation for air, for inclination

angles from  $90^\circ$  to  $-30^\circ$  from the vertical axis. A non-similar transformation method was utilized in [9] to obtain solutions to the flow adjacent to the vertical, inclined or horizontal plates with either prescribed temperature or heat flux thermal boundary conditions.

The influence on heat transfer of induced flow near the leading edge of the plate was numerically studied by [10]. A numerical study [11] shows the effects of trailing edge on the vertical flat plate. These effects deviation of the plate Nusselt number from the classical boundary layer solutions.

A simple correlation for the effect of Prandtl number on Nusselt number is obtained from earlier theoretical analysis [12]. The Nusselt number results have been obtained for natural convection in fluids of different Prandtl numbers along a non-isothermal plate [13].

From the review of literature, one may observe that most studies of flow adjacent to horizontal, inclined and vertical surfaces have been either heat-transfer measurements or visualization for surfaces heated from one side usually the upper one.

The main objective of the present work is to cover the absence of the experimental investigations in the literature review about the natural convection heat transfer from vertical, inclined and horizontal flat plates heated electrically from both sides with uniform heat flux. This paper represents the change of characteristic of heat transfer coefficient with angle of inclination. The dimensions of the tested plate are 171 mm height, 117 mm width and 6 mm thickness. The plate is able to take arbitrary orientation in which the inclination angle ( $\theta$ ) ranges between  $-90^\circ$  and  $90^\circ$  from the vertical axis. The test plate is designed to give uniform heat flux from its two sides, in which this condition is most practical. The measurements are performed on the foundation of the average heat transfer coefficient and to clarify the effect of the angle of inclination.

## EXPERIMENTAL APPARATUS AND PROCEDURE

The test apparatus used in this study is shown in Fig. (1). It consists of a wooden frame (6)  $1000 \times 1000 \text{ mm}^2$  and 1000 mm height carrying the test plate and other parts. The test plate (11), 171 mm length 117 mm width and 6 mm thickness, is made from stainless steel sheet (12) with 1.5 mm thickness, the back surface area of it is divided to nine imaginary equal rectangular areas, each 57 mm length and 39 mm width. In the center of each area a copper-constantan thermocouple (13), made from 30 gauge wires, is fixed in thin slots (1 mm deep) cut on the under side of the plate. Thus the average of nine local temperatures is obtained. The temperatures are sensed at a depth of about 0.5 mm from the top polished surface. As such these values can be taken to represent the top surface temperatures because of the use of the stainless steel test plate in which the difference between the estimated top surface temperatures and the measured values are found to be, in general, less than  $0.05^\circ\text{C}$ . The plate is heated electrically by means of the heater (14) which consisted of a nichrome heating wire wound around a threaded sheet of mica (15) and sandwiched also between two sheets of mica (16). Each mica sheet has 170 mm length 110 mm width and 0.5 mm thickness. The leading edge of the plate is rounded to avoid disturbance while the trailing edge is packed with plexiglass (8), which is rounded in the direction of flow to avoid disturbance in the trailing edge. The plexiglass is pivoted with two plexiglass supports (7) connected to the main frame to give variable angle

of inclination ( $\theta$ ) between  $-90^\circ$  and  $90^\circ$  using a plexiglass mechanism (1). This mechanism seems to be like the slider crank mechanism. The mechanism has a pointer (10) on the protractor (9), as shown in Fig. (1). The mean bulk air temperatures far from the two sides of the plate are measured using two movable thermocouples facing each side. All the thermocouples are connected to a 6 points temperature recorder (2).

The heat input to the plate is controlled by using an auto-transformer (3) as well as one voltmeter (4) and one ammeter (5). Once the switch of the power source closes, the heating system starts. During the course of this experimental work nearly 90 minutes are needed to reach the steady state condition. This condition is satisfied when the temperature reading does not record any change within a time periods about 15 minutes. Applying the principle of conservation of energy gives:

$$P = Q_{\text{cond.}} + Q_{\text{conv.}} + Q_{\text{rad}}$$

As the power leads, and the thermocouple wires are very small cross section and the various connections in the apparatus are made of insulated materials, then the amount of heat ( $Q_{\text{cond}}$ ) conducted away through these members is very small and, therefore, can be neglected. The heat energy carried away from the plate by radiation ( $Q_{\text{rad}}$ ) is calculated during the course of this work and it has a comparatively small value in which it can be neglected. On the other hand heat energy convected away from the plate by air ( $Q_{\text{conv.}}$ ) can be expressed using Newton's cooling law as follows:

$$q = h (t_w - t_\infty) \tag{2}$$

then the coefficient of natural convection heat transfer can be calculated from the following correlation:

$$h = \frac{P}{A (t_w - t_\infty)} \tag{3}$$

**CONSIDERATIONS, RESULTS AND DISCUSSION**

The experiment was performed in the steady state by ensuring that the data were obtained after the recorded temperature difference between the plate walls and the ambient air was confirmed to be a constant. The ranges of experimental data are shown in table 1, in which the minus sign of  $\theta$  means that the angle of inclination is in the opposite direction to insure the uniformity of heat flux in both the surfaces of the plate.

**Table 1. Ranges of experimental data.**

| Parameter                     | Range   | Parameter                | Range                  |
|-------------------------------|---|--------------------------|------------------------|
| $\theta$ (deg.)               | - 90   - 60   - 30   0   15<br>30   45   60   75   90 | $t$ ( $^\circ\text{C}$ ) | 26 : 29                |
| $q$ ( $\text{W}/\text{m}^2$ ) | 11 : 1100   | $Ra^*$                   | $10^6 : 8 \times 10^7$ |
| $t_w$ ( $^\circ\text{C}$ )    | 30 : 144  | $Nu$                     | 11 : 30                |

Average heat transfer coefficient  $h$ , the average Nusselt number  $Nu$  and modified average Grashof number  $Gr^*$  are defined by these correlations:

$$h = \frac{q}{(t_w - t_\infty)} \quad , \quad (4)$$

$$Nu = \frac{h \cdot L_c}{k} \quad , \quad (5)$$

$$Gr^* = \frac{g \beta q L_c^4}{k \nu^2} \quad (6)$$

and

$$L_c = \frac{L_v \cdot L_h}{L_v + L_h} \quad (7)$$

where  $q$  is the average heat flux density on the plate. The wall temperature  $t_w$  is the mean temperature of the plate heated surface. The fluid temperature  $t_\infty$  is the free stream temperature. The calculation of the volumetric thermal expansion coefficients ( $\beta$ ) is based on the average temperature value from  $t_\infty$  to  $(t_\infty + t_w)/2$ . However, the foundation of the thermal conductivity ( $k$ ), kinetic viscosity ( $\nu$ ) and Prandtl number ( $Pr$ ), are based on the temperature value  $t = t_w - 0.25 (t_w - t_\infty)$  by referring to reference [5]. The value of the physical properties are referred to the procedure in Appendix of reference [14].

Fig. (2) indicates the wall temperature for both plate surfaces as a function of angle of inclination ( $\theta$ ). The figure shows that the difference in plate sides temperatures increases with the increase of the inclination angle. It is also seen that the mean temperature of the upper side of the plate is lower than the same of the lower side. One may also observe that the difference in the mean temperatures increases with the heat flux density (i.e. Rayleigh number). In comparison with the results obtained in [5] in which the plate is heated from the lower side or the upper one, it is found that the temperature difference between the present plate side is smaller than the same of [5].

In the present case where the plate is heated from both sides by using single heater the total amount of heat transferred from the plate is divided into two unequal parts, in which the bigger part is transferred through the upper side.

On the light of the above constrain one can conclude that the heat transferred from both plate sides affects each other. This also explains the small difference in the mean temperatures of the heated plate sides as shown in Fig. (2).

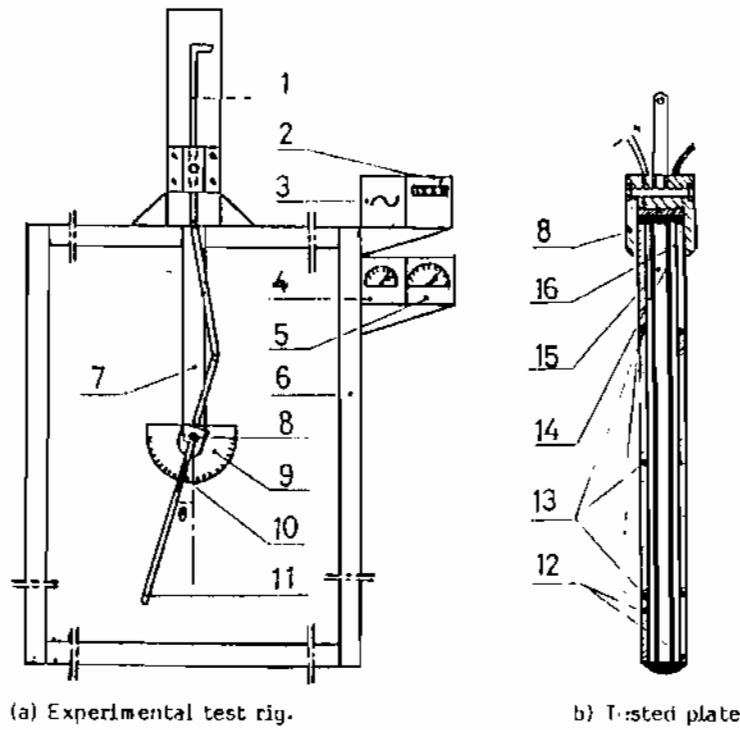


Fig. (1) Experimental test rig.

- |                   |                         |                           |
|-------------------|-------------------------|---------------------------|
| 1- mechanism      | 2- temperature recorder | 3- auto-transformer,      |
| 4- voltmeter      | 5- ammeter              | 6- wooden frame,          |
| 7- support        | 8- packing              | 9- protractor,            |
| 10- pointer       | 11- tested plate        | 12- stainless steel sheet |
| 13- thermocouples | 14- electric heater     | (15 and 16)- mica sheets  |

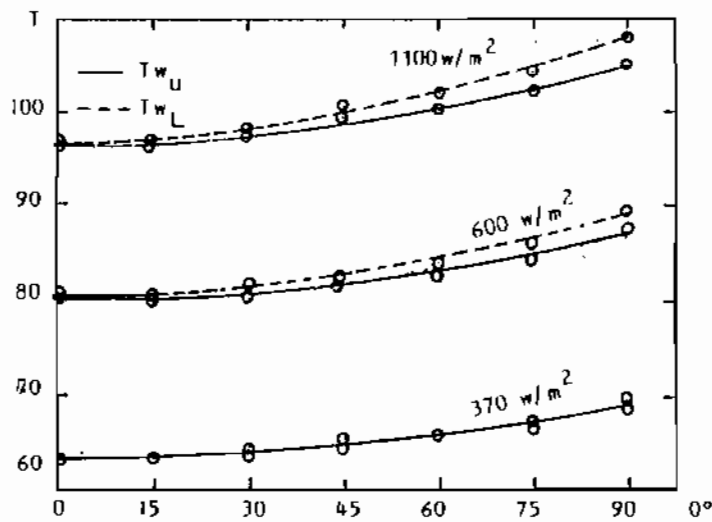


Fig. (2) Examples of the mean wall temperature for different angle of inclination ( $\theta$ ).

The heat transfer data is first obtained with the plate in vertical position (i.e.  $\theta=0^\circ$ ) to check the overall validity of the equipments. The heat transfer results are plotted in terms of the average Nusselt number versus the modified Grashof number multiplied into Prandtl number as shown in Fig. (3) The experimental Nu values is about eight percent higher than the values obtained by Brian [15]. One may conclude that this increase is due to the effect of the leading and trailing edges of the plate.

The average Nusselt number is correlated with the modified Rayleigh number of the heated plate as follows:

$$Nu = 0.76 Ra^{*0.2} \quad (8)$$

where  $1.9 \times 10^6 \leq Ra^* \leq 8 \times 10^7$

The correlation (8) predicts that the values of Nu are in agreement with the obtained results to within  $\pm 9\%$

Secondly the heat transfer data are obtained when the plate is horizontally located. These data are plotted in Fig. (4) as a relation between Nu and  $Ra^*$ . The Figure shows that the correlation obtained for the orientation of the vertical plate still yields for the horizontal orientation.

It is concluded in reference [16] that the heat transfer, by natural convection, from a horizontal plate facing upwards is found to be 30% higher than the same data obtained from the vertical plate of the same size and under the same conditions. It is also concluded that the heat transfer from a horizontal plate facing downwards is found to be 30% lower than the same obtained from the vertical orientation of the same plate. This means that the summation of the heat transfer from the horizontal plate heated from both sides heat transfer from the same plate in vertical orientation at the same conditions.

Finally, the heat transfer data was obtained when the flat plate orientation has different inclination angles ( $-90^\circ \leq \theta \leq 90^\circ$ ). These data are plotted in Fig. (5) as a relation of Nu versus  $Ra^*$  along with the data obtained in case of the vertical plate. One may also observe that the expression for the inclined plate is the same for the vertical one.

## CONCLUSIONS

From the previous discussion one may conclude that:

1. No influence of the flow regime on the heat transfer results for the vertical, horizontal and inclined flat plates heated from both sides in the range  $1.9 \times 10^6 \leq Ra^* \leq 8 \times 10^7$ .
2. The inclination angle ( $\theta$ ) has no effect on the heat transfer results.
3. The following expression yield in vertical, inclined and horizontal orientations:

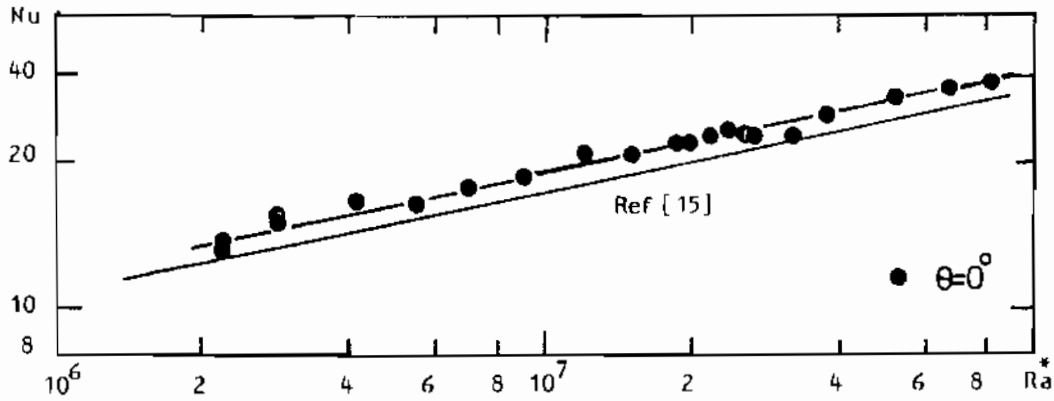


Fig. (3) The variation of Nusselt number versus modified Rayleigh number for the plate in vertical orientation.

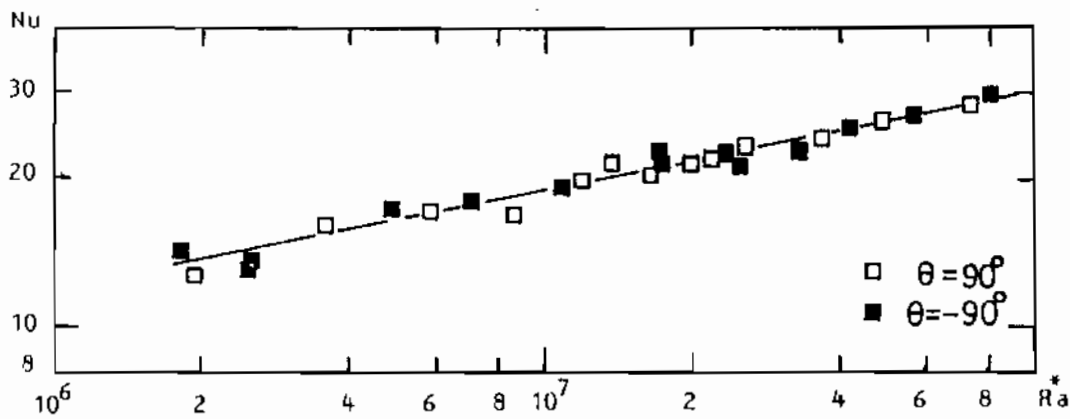


Fig. (4) The variation of Nusselt number versus modified Rayleigh number for the plate in horizontal orientation.

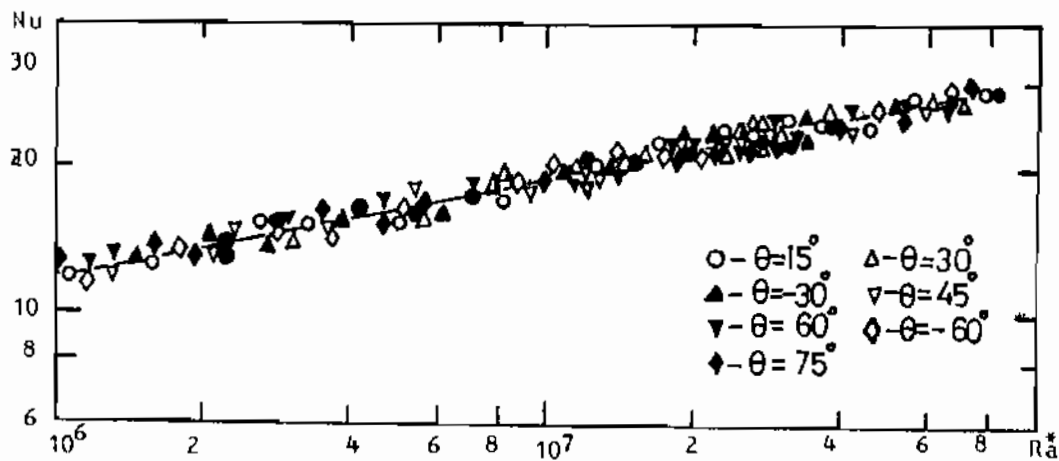


Fig. (5) The variation of Nusselt number versus modified Rayleigh number for the plate in inclined orientation.

$$Nu = 0.76 Ra^{*0.2},$$

$$1.9 \times 10^6 \leq Ra^* \leq 8 \times 10^7 \text{ and } -90 \leq \theta \leq 90$$

with a maximum error of  $\pm 9\%$ .

#### NOHENCCLATURE

- A - plate surface area [ $m^2$ ],  
g - gravitational acceleration [ $m/s^2$ ],  
Gr\*- modified average Grashof number defined by formula (6),  
K - thermal conductivity [ $W/m^{\circ}k$ ],  
 $L_h$  - long horizontal plate dimension [ $m$ ],  
 $L_v$  - vertical plate dimension [ $m$ ];  
 $L_c$  - characteristic length defined by formula (7) [ $m$ ],  
Nu - average Nusselt number defined by formula (5),  
P - Electric power [ $W$ ],  
 $P_r$  - prandtl number,  
q - average wall heat flux [ $W/m^2$ ],  
Ra\*- modified Rayleigh number, ( $Gr^*pr$ );  
 $t_w$  - mean wall temperature [ $^{\circ}C$ ],  
 $t_{wl}$  - lower surface mean temperature [ $^{\circ}C$ ],  
 $t_{wu}$  - upper surface mean temperature [ $^{\circ}C$ ],  
t - mean fluid temperature [ $^{\circ}C$ ].
- Greek Symbols
- $\beta$  - average volumetric thermal expansion coefficient [ $1/^{\circ}k$ ],  
 $\theta$  - angle of inclination of the plate with the vertical, plus and minus signs denote clockwise and anti-clockwise direction respectively [deg.],  
 $\nu$  - kinematic viscosity [ $m^2/s$ ].

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