Menofia University
Faculty of Engineering Shebien El-kom
Electrical Engineering Department.
First semester Examination, 2014-2015
Date of Exam : 15 /1/2015

Subject: Eng. Mathematics (4)
Code: BES 311
Year : Third 3rd
Time Allowed : 3 hrs
Total Marks: 70 Marks

Answer all the following questions الامتحان في صفحتين

Question No. 1 (20 marks)

(A) Solve the heat equation:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + x, \qquad 0 < x < 1, \qquad t > 0$$

Where u is the temperature, k is the conductivity of the rod, x is a dimension and t time

Subject to: *B.C.s* $u(0,t) = 1,\ u(1,t) + \frac{\partial u(1,t)}{\partial x} = 2$ *and I.C.* $u(x,0) = x$ (10 Marks)

(B) Consider waves in a resistant medium that satisfy the problem $u_{tt} = c^2 u_{xx} - r u_t + 5xe^{-t}$ Where $0 < x < l,\ t > 0$ With Boundary conditions, $u(0,t) = 1, u(l,t) = 2$ and initial conditions, $u(x,0) = f(x), u_t(x,0) = g(x)$ Where r is a constant, $0 < r < 2\pi c/l$.

Write down the series expansion of the solution (10 Marks)

Question 2 (20 marks)

(A) Given the following system of algebraic equations:

$$x_1 + 3x_3 = 2$$
$$5x_1 + x_2 + 2x_3 = -5$$
$$x_1 + 6x_2 + 2x_3 = -11$$

(i) If you solve this system without ordering the equations. What do you expect? Discuss the convergence of this system through Scarbora criteria.

(ii) Order your equations in an appropriate way. Use Gauss-Siedel iterative method to make two iterations. Use $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$ (5 Marks)

(B) Fit the straight line $y = ax + b$ and also a parabola $y = c + bx + ax^2$ to the following set of observations using least squares method:

x	0	1	2	3	4
y	1	5	10	22	38

Calculate the sum of squares of the residuals in each case and test which curve is more suitable to the data (10 Marks)

(C) Use the method of Secant to find the real root of the equation $x - \cos x = 0$. Take $x^{(0)} = 0,\ x^{(1)} = 1$, make 2 iterations. (5 Marks)

Question 3 (15 marks)

(A) Determine the value of the function $f(x)$ at $x = 1.5$ using the direct method interpolation using first and second order polynomial.

x	0	1	2	3
$f(x)$	1	6	25	55

(5 Marks)

(B) Find the cube root of 12 using Newton-Raphson method, take $x_o = 3$.

(5 Marks)

(C) Prove that the normal equations to the curve $y = ax + b$ using least squares method are $\sum y = a\sum x + nb$ and $\sum xy = a\sum x^2 + b\sum x$. (5 Marks)

Question 4 (15 marks)

(A) If $x = 4 \pm 0.01$, $y = -3 \pm 0.05$, $t = 5 \pm 0.02$ Find the maximum possible error in z where $z = (x^2 + y^2)e^{2t}$ (3 Marks)

(B) Use Euler's and Rung-Kutta 4th order method to solve the differential equation $\frac{dy}{dx} = 2x$ to obtain the value of y at $x = 0.5$; knowing that $y(0) = 1$ (take $h=0.5$ *in both methods*). (7 Marks)

(c) Solve the following equation (Sturm- Louville boundary value problem), $\frac{d^2y}{dx^2} + \lambda y = 0$, where $0 < x < l$, with boundary conditions $y(0) = \dot{y}(l) = 0$ and show that the solution is orthogonal set of function. [note, do it for $\lambda \geq 0$ only) (5 Marks)

This exam measures the following ILOs												
Question Number	Q1-a	Q2-d	Q2-b		Q4-a	Q4-b	Q2-c		Q1-b	Q2-a	Q3-b	
Skills	Q4-b	d2-i	Q3-b		Q3-a	Q3-c			Q3-d	Q4-c		
	Knowledge &understanding skills				Intellectual Skills				Professional Skills			

With my best wishes
Associate Prof. Dr. Islam M. Eldesoky